CORRECTIONS TO THE FINE STRUCTURE CONSTANT IN (D+1)-DIMENSIONAL GLOBAL MONOPOLE SPACETIME

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In this paper we use the Generalized Uncertainty Principle in order to obtain the corrections to the fine structure constant in $(D + 1)$ -dimensional global monopole spacetime. We also discuss the case $D = 3$ corresponding to the $(3+1)$ -dimensional global monopole spacetime.

Keywords: Global monopole; Fine structure constant; Higher dimensional spacetime.

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1. Introduction

The Standard Cosmological Model predicts that the Universe has experienced a chain of phase transitions with spontaneous symmetry breaking. During this process the so called topological defects arised, as for example, monopoles, strings and domain walls¹. In particular, global monopoles appear in models with broken global $SO(3)$ symmetry². The model consists of the Higgs triplet of scalar fields with Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{1}{4}\lambda(\phi^a\phi^a - \eta^2)^2, \quad (1)$$

where $a = 1, 2, 3$.

The gravitational field corresponding to a global monopole is represented by the following spherically symmetric line element²

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (2)$$

with the functions A and B given by

$$B = A^{-1} = 1 - 8\pi \frac{G\eta^2}{c^4} - \frac{2GM(r)}{rc^2}, \quad (3)$$

where the parameter η^2 is related with the linear energy density.

The scale of the monopole's core is defined by the Compton wavelength of Higgs bosons, $\varrho \sim \lambda^{-1/2}\eta^{-1}$. The parameter $M = \lim_{r \rightarrow \infty} M(r)$ which characterizes the monopole's mass, assumes values in the interval $0 \leq 8\pi\eta^2 < 1$ and is given, approximately, by³

$$M \approx -\frac{6\pi\eta}{\sqrt{\lambda}}. \quad (4)$$

Therefore asymptotically the spacetime of a global monopole corresponds to the Schwarzschild spacetime with negative mass and additional solid angle deficit $32\pi^2\eta^2$. The spacetime of point-like monopole is obtained from the above metric by disregarding internal structure of the global monopole but preserving the solid angle deficit. It is described by the following line element

$$ds^2 = -c^2 b_3^2 dt^2 + \frac{dr^2}{b_3^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (5)$$

where the parameter $b_3^2 = 1 - \frac{8\pi G\eta^2}{c^4}$, and the subindex in b indicates the dimension of the space. The scalar curvature is given by $R = \frac{1-b_3^2}{r^2 b_3^2}$. Therefore, this spacetime is not flat, but surprisingly there is no Newtonian potential associated to it.

This metric can be written in a different form if we make the following change in time coordinates $t \rightarrow \tau = t/b_3$. As a result, it turns into

$$ds^2 = -c^2 d\tau^2 + \frac{dr^2}{b_3^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (6)$$

The main characteristic of this spacetime is the existence of a solid angle deficit $4\pi(1-b_3^2)$. The typical value of the solid angle deficit in the framework of the GUT ($\eta^2 c^2 L_{pl} \sim 10^{16} GeV$) is proportional to $1 - b_3^2 = \frac{8\pi G\eta^2}{c^4} \sim 10^{-5}$.

Taking into account that our proposal is to consider the higher dimensional global monopole spacetime, let us write the generalization of the metric given by eq. (6), for $(D+1)$ -dimensional spacetime. In this case, the Euclidean version of the global monopole line element is given by

$$ds^2 = d\tau^2 + \frac{dr^2}{b_D^2} + r^2 d\Omega_{D-1}^2, \quad (7)$$

where $D \geq 3$ and b_D is a parameter which codifies the presence of a $(D+1)$ -dimensional global monopole. The coordinates are $(\tau, r, \theta_1, \theta_2, \dots, \theta_{D-2}, \phi)$. In this case, the solid angle associated to a hypersphere with unity radius is $\Omega_{b_D} = b_D^2 \frac{2\pi^{D/2}}{\Gamma(\frac{D}{2})}$ which is smaller than the solid angle associated to a hypersphere embedded in flat spacetime.

2. Corrections to the Fine Structure Constant in Higher Dimensions

In order to obtain the fine structure constant in the $(D + 1)$ -dimensional global monopole spacetime, let us consider a Hydrogen atom in this background. Thus, the electrostatic force on the electron is given by

$$\vec{F}_{totD} = \left(-\frac{1}{\Omega_{D-1}\epsilon_{D,0}} \frac{e^2}{r^{D-1}} + \frac{S(b_D)}{\Omega_{D-1}\epsilon_{D,0}} \frac{e^2}{r^{D-1}} \right) \hat{r}_D, \quad (8)$$

where the first term corresponds to the generalization of the usual Coulomb force to D -dimensional space. The second term arises from the geometrical and topological features of the spacetime and corresponds to a generalization of a result obtained in the framework of a $(3 + 1)$ -dimensional global monopole⁴. The factor $S(b_D)$ is also a generalization of previous result⁴ and is given by

$$S(b_D) = \frac{1}{2} \sum_{n=1}^{\infty} \frac{(\pi^2 \Delta_D)^n}{(n!)^2} |B_{2n}| (1 - 2^{-2n}) \quad (9)$$

with $\Delta_D = 1 - b_D^2$ and B_n being the Bernoulli numbers. Thus, we can obtain the Bohr radius in this background, which can be written as

$$r_{B,D} = \left(\frac{b_D^2 \Omega_{D-1} \epsilon_{D,0} \hbar^2}{m e^2} \right)^{\frac{1}{4-D}} \times (1 - S(b_D))^{4-D}, \quad (10)$$

where m and e are the mass and charge of the electron, respectively. Note that when $D = 3$ and $b_3 = 1$, which means absence of global monopole, we obtain the so-called Bohr radius $r_{B,3} = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 0.0529nm$ in three dimensional space. This expression for the Bohr radius takes into account that the usual quantization of the angular momentum is now given by $L_{n,D} = \frac{n\hbar}{b_D}$.

The fine structure constant in D -dimensional space was already obtained⁷. It is given by the following expression

$$\alpha_D = \hbar^{2-D} e^{D-1} [\Omega_{D-1} \epsilon_{D,0}]^{(1-D)/2} c^{D-4} G_D^{(3-D)/2} \quad (11)$$

where $\epsilon_{D,0}$ is the permittivity constant and $\Omega_{D-1} = \frac{2\pi^{D/2}}{\Gamma(\frac{D}{2})}$ is the solid angle associated with a hypersphere in D dimensions. From this result, we can obtain a corresponding generalized one in the sense that the global monopole is present. This can be written as

$$\hat{\alpha}_D = \hbar^{2-D} e^{D-1} [b_D^2 \Omega_{D-1} \epsilon_{D,0}]^{(1-D)/2} c^{D-4} G_D^{(3-D)/2}, \quad (12)$$

where the parameter b_D^2 was introduced to take into account the presence of the $(D + 1)$ -dimensional global monopole. For $D = 3$ and $b_3 = 1$, eq. (12) turns into $\alpha = e^2/4\pi\epsilon\hbar c \approx 1/137$.

The so-called Generalized Uncertainty Principle⁵ is expected to be satisfied when quantum gravitational effects are important. The interest in this Principle has been motivated by studies in string theory⁶ and gravity⁵. In this context, a minimal length scale is expected to be of the order of Planck length L_p . This fact leads to corrections to the usual Heisenberg Uncertainty Principle, resulting in the Generalized Uncertainty Principle, which can be expressed as

$$\Delta x_i \Delta p_i \geq \hbar \left[1 + \beta^2 L_{p,D}^2 \left(\frac{\Delta p_i}{\hbar} \right)^2 \right]. \quad (13)$$

where β is a numerical factor of order unity that depends on the specific model and $L_{p,D}$ is the Planck length in D dimensions, which is given by $L_{p,D} = (\hbar G_D / c^3)^{1/D-1}$. Thus, in a three dimensional space eq. (13) implies that the lower bound on the length scale is of the order of Planck's length, $L_p = \sqrt{(G\hbar/c^3)} \approx 10^{-33} \text{cm}$, and this plays an important role as a fundamental scale.

Thus, from eq. (13) we can obtain the uncertainty for the momentum which is given by the following relation

$$\frac{\Delta p_i}{\hbar} = \frac{\Delta x_i}{2\beta L_{p,D}^2} \left(1 - \sqrt{1 - \frac{4\beta^2 L_{p,D}^2}{\Delta x_i^2}} \right). \quad (14)$$

Now, let us consider that the uncertainty in position is equal to the Bohr radius given by eq. (10). Thus, we can write the following relation

$$\begin{aligned} \frac{\Delta x_i}{L_{p,D}} &= \left(\frac{b_D^2 \Omega_{D-1} \epsilon_{D,0} M_{p,D}^3 G_D}{m e^2} \right)^{\frac{1}{4-D}} \\ &\times (1 - S(b_D))^{4-D}, \end{aligned}$$

where $M_{p,D}$ is the Planck mass in D dimensional space, which is given by $M_{p,D} = (\hbar^{D-2} / c^{D-4} G_D)^{1/D-1}$.

Comparing the Generalized Uncertainty Principle and the Heisenberg Uncertainty Principle, we conclude that the first one can be written formally as

$$\Delta x_i \Delta p_i \geq \hbar_{eff}, \quad (15)$$

where

$$\hbar_{eff} = \hbar \left[1 + \beta^2 L_{p,D}^2 \left(\frac{\Delta p_i}{\hbar} \right)^2 \right]. \quad (16)$$

Substituting Eqs. (15) into (14) and the result into (16), we get

$$\begin{aligned} \hbar_{eff} &= \hbar \left[1 + \frac{1}{4\beta^2} \left(\frac{b_D^2 \Omega_{D-1} \epsilon_{D,0} M_{p,D}^3 G_D}{m e^2} \right)^{\frac{2}{4-D}} (1 - S(b_D))^{2(4-D)} \right. \\ &\quad \times \left. \left(1 - \sqrt{1 - 4\beta^2 \left(\frac{m e^2}{b_D^2 \Omega_{D-1} (1 - S(b_D))^{2(4-D)} \epsilon_{D,0} M_{p,D}^3 G_D} \right)^{\frac{2}{4-D}}} \right)^2 \right]. \quad (17) \end{aligned}$$

If we consider a global monopole in the Grand Unified Theory(GUT) framework, the factor $\frac{b_D}{(1-S_D)}$ should be approximately equal to unity.

Therefore, the term $\frac{me^2}{\Omega_{D-1}\epsilon_{D,0}M_{p,D}^3G_D}$ is much less than one, and therefore can be used as the expansion parameter. Expanding eq. (18) in terms of this quantity, we get

$$\hbar_{eff} \simeq \hbar \left[1 + \beta^2 \left(\frac{me^2}{b_D^2 \Omega_{D-1} \epsilon_{D,0} M_{p,D}^3 G_D} \right)^{\frac{2}{4-D}} \times (1 - S(b_D))^{-2(4-D)} \right]. \quad (18)$$

Note that in the absence of a global monopole, the result given by eq.(18) corresponds to the result obtained by Nasser⁷ in D -dimensional space. Applying this result given by eq. (18) to $3 + 1$ dimensional global monopole spacetime, we get

$$\hbar_{eff} \simeq \hbar(1 + 9.53 \times 10^{-50} \beta^2). \quad (19)$$

Combining eqs. (12) and (18), we find that the structure constant in a $(D + 1)$ -dimensional global monopole spacetime is given by the expression

$$\hat{\alpha}_{D,eff} = \hbar_{eff}^{2-D} e^{D-1} [b_D^2 \Omega_{D-1} \epsilon_{D,0}]^{(1-D)/2} c^{D-4} G_D^{(3-D)/2}, \quad (20)$$

which in $3 + 1$ -dimensional global monopole spacetime, turns to

$$\alpha_{3,eff} = \frac{e^2}{4\pi\epsilon_0\hbar c} (1 - 9.53 \times 10^{-50} \beta^2). \quad (21)$$

3. Concluding Remarks

The corrections to the Planck constant and to the fine structure constant are very close to the ones obtained in D -dimensional flat spacetime. The difference between $\hbar_{eff} - \hbar$ in relation to $\hbar$ is of the order of 10^{-49} . Therefore, the values of the fine structure constant evaluated in the framework of the Generalized Uncertainty Principle and in the presence of a $(D + 1)$ -dimensional global monopole are very close to the value of the fine structure constant obtained in the framework of the Heisenberg Uncertainty Principle in flat spacetime.

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