ON DEGENERATE COMPRESSED ATOMS AND COMPRESSED NUCLEAR MATTER CORES OF STELLAR DIMENSIONS

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The relativistic generalization of the Feynman, Metropolis and Teller treatment of compressed atoms is obtained. Each atomic configuration is confined by a Wigner-Seitz cell and is characterized by a positive electron Fermi energy. There exists a limiting configuration, reached when the Wigner-Seitz cell radius equals the radius of the nucleus, with a maximum value of the electron Fermi energy $(E_e^F)_{max}$, here expressed analytically in the ultra-relativistic approximation. The treatment is then extrapolated to compressed nuclear matter cores of stellar dimensions with $A \simeq (m_{\text{Planck}}/m_n)^3 \sim 10^{57}$ or $M_{core} \sim M_\odot$. A new family of equilibrium configurations exists for selected values of the electron Fermi energy varying in the range $0 < E_e^F \leq (E_e^F)_{max}$. Such configurations fulfill global but not local charge neutrality. They have electric fields on the core surface, increasing for decreasing values of the electron Fermi energy reaching values much larger than the critical value $E_c = m_e^2 c^3 / (e\hbar)$, for $E_e^F = 0$.

Keywords: Thomas-Fermi model; degenerate Fermi gas; neutron star core.

1. Introduction

In Refs 1, 2 we describe a degenerate system of N_n neutrons, N_p protons and N_e electrons constrained to a constant density distribution for the protons and we solve the corresponding relativistic Thomas-Fermi equation and derive for the neutrons the distribution following the implementation of the beta equilibrium condition. Using then the existence of scaling laws we have extended the results from heavy nuclei to the case of nuclear matter cores of stellar dimensions. In both these treatments we had assumed the Fermi energy of the electrons $E_e^F = 0$. The aim of this article is to proceed with this dual approach and to consider first the case of compressed atoms and then, using the existence of scaling laws, the compressed nuclear matter cores of stellar dimensions with a positive value of their electron Fermi energies.

2. The relativistic generalization of the Feynman-Metropolis-Teller treatment for compressed atoms

The Thomas-Fermi model assumes that the electrons of an atom constitute a fully degenerate gas of fermions confined in a spherical region by the Coulomb potential of a point-like nucleus of charge $+eN_p$.³⁴ Feynman, Metropolis and Teller have shown that this model can be used to derive the equation of state of matter at high pressures by considering a Thomas-Fermi model confined in a Wigner-Seitz cell of radius R_{WS} .⁵ In the relativistic generalization of the Thomas-Fermi equation the point-like approximation of the nucleus must be abandoned since the relativistic equilibrium condition

$$E_e^F = \sqrt{(P_e^F c)^2 + m_e^2 c^4} - m_e c^2 - eV(r) = \text{constant} \geq 0, \quad (1)$$

which defines the electron Fermi energy E_e^F , would lead to a non-integrable expression for the electron density near the origin.⁶⁷ In Eq. (1) P_e^F is the electron Fermi momentum related to the electron number density n_e by $P_e^F = (3\pi^2 \hbar^3 n_e)^{1/3}$, V is the Coulomb potential and m_e is the electron mass. Consequently we adopt an extended nucleus.

In particular, following the previous treatments (see e.g. Ref. 2), we have assumed a constant distribution of protons confined in a radius R_c defined by

$$R_c = \Delta \frac{\hbar}{m_\pi c} N_p^{1/3}, \quad (2)$$

where m_π is the pion mass and Δ is a parameter such that $\Delta \approx 1$ ($\Delta < 1$) corresponds to nuclear (supranuclear) densities when applied to ordinary nuclei. The proton density can be written as

$$n_p(r) = \frac{N_p}{\frac{4}{3}\pi R_c^3} \theta(R_c - r) = \frac{3}{4\pi} \frac{m_\pi^3 c^3}{\hbar^3} \frac{1}{\Delta^3} \theta(R_c - r), \quad (3)$$

where $\theta(x)$ is the Heaviside function.

The overall Coulomb potential satisfies the Poisson equation

$$\nabla^2 V(r) = -4\pi e [n_p(r) - n_e(r)], \quad (4)$$

where the electron number density n_e is given by

$$n_e(r) = \frac{(P_e^F)^3}{3\pi^2 \hbar^3} = \frac{1}{3\pi^2 \hbar^3 c^3} \left[e^2 \hat{V}^2(r) + 2m_e c^2 e \hat{V}(r) \right]^{3/2}, \quad (5)$$

and $e\hat{V} = eV + E_e^F$.

Defining $x = r/\lambda_\pi$, $\chi/r = e\hat{V}(r)/(c\hbar)$, Eq. (4) can be written as (see Ref. 8)

$$\frac{1}{3x} \frac{d^2 \chi(x)}{dx^2} = -\frac{\alpha}{\Delta^3} \theta(x_c - x) + \frac{4\alpha}{9\pi} \left[\frac{\chi^2(x)}{x^2} + 2 \frac{m_e}{m_\pi} \frac{\chi}{x} \right]^{3/2}, \quad (6)$$

where $x_c = R_c/\lambda_\pi$, $\lambda_\pi = \hbar/(m_\pi c)$.

The neutron density $n_n(r)$, related to the neutron Fermi momentum $P_n^F = (3\pi^2\hbar^3n_n)^{1/3}$, is determined by imposing the condition of beta equilibrium

$$\begin{aligned} E_n^F &= \sqrt{(P_n^F c)^2 + m_n^2 c^4} - m_n c^2 \\ &= \sqrt{(P_p^F c)^2 + m_p^2 c^4} - m_p c^2 + eV(r), \end{aligned} \quad (7)$$

which in turn is related to the proton density n_p and the electron density by Eqs. (4), (5). In Eq. (7) m_n is the neutron mass, m_p is the proton mass and P_p^F is the proton Fermi momentum related to the proton density n_p by $P_p^F = (3\pi^2\hbar^3n_p)^{1/3}$. Integrating numerically these equations we have obtained a new generalized relation between A and N_p for any value of A .²

In this case Eq. (6) has to be integrated with the boundary conditions $\chi(0) = 0$, $\chi(x_{WS}) = x_{WS}\chi'(x_{WS})$, $x_{WS} = R_{WS}/\lambda_\pi$. The electron Fermi energy can be written as

$$E_e^F = m_\pi c^2 \frac{\chi(x_{WS})}{x_{WS}}. \quad (8)$$

3. Comparison and contrast with approximate treatments

There exists in the literature a variety of semi-qualitative approximations adopted in order to describe the electron component of a compressed atom (see e.g. Ref. 9 for applications of the uniform approximation and e.g. Refs. 10–15 for applications of the Salpeter¹⁶ approximate treatment).

We shall see how the relativistic treatment of the Thomas-Fermi equation affects the current analysis of compressed atoms in the literature by introducing qualitative and quantitative differences which deserve attention.

3.1. Relativistic FMT treatment vs. relativistic uniform approximation

One of the most used approximations in the treatment of the electron distribution in compressed atoms is the one in which, for a given nuclear electric charge $+eN_p$, the Wigner-Seitz cell radius R_{WS} is defined by

$$N_p = \frac{4\pi}{3} R_{WS}^3 n_e, \quad (9)$$

where $n_e = (P_e^F)^3/(3\pi^2\hbar^3)$.

In Fig. 1, (Left panel), we have plotted the electron number density obtained from Eq. (5) where the Coulomb potential is related to the function χ , which is obtained from numerical integration of the relativistic Thomas-Fermi equation (6) for helium at a given compression. We have normalized the electron density to the average electron number density $n_0 = 3N_e/(4\pi R_{WS}^3) = 3N_p/(4\pi R_{WS}^3)$ as given by Eq. (9).

We can see in Fig. 1 how our treatment, based on the numerical integration of the relativistic Thomas-Fermi equation (6) and imposing the condition of beta

Fig. 1. (Left panel) The electron number density n_e in units of the average electron number density $n_0 = 3N_e/(4\pi R_{WS}^3)$ is plotted as a function of the dimensionless radial coordinate $x = r/\lambda_\pi$ for $x_{WS} = 9.7$ in both the relativistic Feynman, Metropolis, Teller approach and the uniform approximation for helium. (Right panel) The electron Fermi energies in units of $m_\pi c^2 N_p^{4/3}$ for helium, carbon and iron are plotted as a function of the ratio $R_{WS}/(\lambda_\pi N_p^{-1/3})$ respectively in the non-relativistic and in the relativistic Feynman-Metropolis-Teller (FMT) treatment. The dimensionless quantities have been chosen in order to obtain an universal curve in the non relativistic treatment. The relativistic treatment leads to results of the electron Fermi energy dependent on the nuclear composition and systematically smaller than the non-relativistic ones. The electron Fermi energy can attain arbitrary large values, in the non relativistic treatment, as the point-like nucleus is approached.

equilibrium (7), leads to electron density distributions markedly different from the constant electron density approximation.

Taking into account the global neutrality condition of the Wigner-Seitz cell $eV(R_{WS}) = 0$, the electron Fermi energy in the uniform approximation can be written as

$$E_e^F \simeq \left[-\frac{m_e}{m_\pi} + \sqrt{\left(\frac{m_e}{m_\pi}\right)^2 + \left(\frac{9\pi}{4}\right)^{2/3} \frac{N_p^{2/3}}{x_{WS}^2}} \right] m_\pi c^2. \quad (10)$$

The uniform approximation becomes exact in the limit when the electron Fermi energy acquires its maximum value as given by

$$(E_e^F)_{max} \simeq \left[-\frac{m_e}{m_\pi} + \sqrt{\left(\frac{m_e}{m_\pi}\right)^2 + \left(\frac{3\pi^2}{2}\right)^{2/3} \left(\frac{N_p}{A}\right)^{2/3}} \right] m_\pi c^2, \quad (11)$$

which is attained when R_{WS} coincides with the nuclear radius R_c . Here, the maximum electron Fermi energy (11) is obtained by replacing in Eq. (10) the value of the normalized Wigner-Seitz cell radius $x_{WS} = x_c = R_c/\lambda_\pi \approx [(3/2)\pi]^{1/3} A^{1/3}$.

3.2. Relativistic FMT treatment vs. Salpeter approximate treatment

Corrections to the uniform distribution were also studied by Salpeter¹⁶ and his approximations are largely applied in physics (see e.g. Refs. 10, 11) and astrophysics (see e.g. Refs. 12–15).

Keeping the point-like nucleus assumption, Salpeter¹⁶ studied the corrections to the above models due to the inhomogeneity of the electron distribution inside the Wigner-Seitz cell. He expressed an analytic formula for the total energy of a Wigner-Seitz cell based on Coulomb corrections to the uniform distribution of electrons. However, despite to be very interesting in identifying piecewise contributions to the total pressure, the validity of the Salpeter approach needs a verification by a more general treatment. For instance, the failure of the Salpeter formulas can be seen at densities of the order of $\sim 10^2 - 10^3 \text{ g cm}^{-3}$ for nuclei with large N_p , as in the case of iron, where the pressure becomes negative (see Ref. 17). Therefore, the problem of solving the relativistic Thomas-Fermi equation within the Feynman, Metropolis, Teller approach becomes a necessity, since this approach gives all the possible Coulomb and relativistic contributions automatically and correctly.

3.3. Relativistic FMT treatment vs. non-relativistic FMT treatment

We now compare and contrast the Fermi energy, given by Eq. (8), of a compressed atom in the non-relativistic and the relativistic limit (see Fig. 1, (Right panel)).

There are major differences:

1) The electron Fermi energy in the relativistic treatment is strongly dependent on the nuclear composition, while the non-relativistic treatment presents a universal behavior. In the limit of low densities the relativistic curves approach the universal non-relativistic curve. In the non relativistic treatment the ratio $E_e^F / (m_\pi c^2 N_p^{4/3})$ does not depend on the number of protons N_p if the Wigner-Seitz cell radius R_{WS} is multiplied by $N_p^{1/3}$. This universality is lost in the relativistic treatment since there is no way to eliminate the dependence of the electron Fermi energy on the nuclear composition (see Eq. (6) and Ref. 6).

2) The relativistic treatment leads to values of the electron Fermi energy consistently smaller than the ones of the non-relativistic treatment.

3) While in the non-relativistic treatment the electron Fermi energy can reach, by compression, infinite values as $R_{WS} \rightarrow 0$, in the relativistic treatment it reaches a perfectly finite value given by Eq. (11) attained when R_{WS} coincides with the nuclear radius R_c .

4. Application to nuclear matter cores of stellar dimensions

We turn now to nuclear matter cores of stellar dimensions of $A \simeq (m_{\text{Planck}}/m_n)^3 \sim 10^{57}$ or $M_{\text{core}} \sim M_\odot$.

Following the treatment presented in Ref. 2, we use the existence of scaling laws and proceed to the ultra-relativistic limit of Eq. (6). For positive values of the electron Fermi energy E_e^F , we introduce the new function $\phi = 4^{1/3} (9\pi)^{-1/3} \chi \Delta / x$ and the new variable $\hat{x} = kx$ where $k = (12/\pi)^{1/6} \sqrt{\alpha} \Delta^{-1}$, as well as the variable $\xi = \hat{x} - \hat{x}_c$ in order to describe better the region around the core radius.

Eq. (6) becomes

$$\frac{d^2\hat{\phi}(\xi)}{d\xi^2} = -\theta(-\xi) + \hat{\phi}(\xi)^3, \quad (12)$$

where $\hat{\phi}(\xi) = \phi(\xi + \hat{x}_c)$ and the curvature term $2\hat{\phi}'(\xi)/(\xi + \hat{x}_c)$ has been neglected.

The Coulomb potential energy is given by

$$eV(\xi) = \left(\frac{9\pi}{4}\right)^{1/3} \frac{1}{\Delta} m_\pi c^2 \hat{\phi}(\xi) - E_e^F, \quad (13)$$

corresponding to the electric field

$$E(\xi) = -\left(\frac{3^5\pi}{4}\right)^{1/6} \frac{\sqrt{\alpha}}{\Delta^2} \frac{m_\pi^2 c^3}{e\hbar} \hat{\phi}'(\xi), \quad (14)$$

and the electron number-density

$$n_e(\xi) = \frac{1}{3\pi^2 \hbar^3 c^3} \left(\frac{9\pi}{4}\right) \frac{1}{\Delta^3} (m_\pi c^2)^3 \hat{\phi}^3(\xi). \quad (15)$$

In the core center we must have $n_e = n_p$. From Eqs. (3) and (15) we then have that, for $\xi = -\hat{x}_c$, $\hat{\phi}(-\hat{x}_c) = 1$.

In order to consider a compressed nuclear density core of stellar dimensions, we then introduce a Wigner-Seitz cell determining the outer boundary of the electron distribution which, in the new radial coordinate ξ is characterized by ξ^{WS} . In view of the global charge neutrality of the system the electric field goes to zero at $\xi = \xi^{WS}$. This implies, from Eq. (14), $\hat{\phi}'(\xi^{WS}) = 0$.

We now turn to the determination of the Fermi energy of the electrons in this compressed core. The function $\hat{\phi}$ and its first derivative $\hat{\phi}'$ must be continuous at the surface $\xi = 0$ of the nuclear density core.

This boundary-value problem can be solved analytically and indeed Eq. (12) has the first integral,

$$2[\hat{\phi}'(\xi)]^2 = \begin{cases} \hat{\phi}^4(\xi) - 4\hat{\phi}(\xi) + 3, & \xi < 0, \\ \hat{\phi}^4(\xi) - \phi^4(\xi^{WS}), & \xi > 0, \end{cases} \quad (16)$$

with boundary conditions at $\xi = 0$:

$$\begin{aligned} \hat{\phi}(0) &= \frac{\hat{\phi}^4(\xi^{WS}) + 3}{4}, \\ \hat{\phi}'(0) &= -\sqrt{\frac{\hat{\phi}^4(0) - \hat{\phi}^4(\xi^{WS})}{2}}. \end{aligned} \quad (17)$$

Having fulfilled the continuity condition we integrate Eq. (16) obtaining for $\xi \leq 0$

$$\hat{\phi}(\xi) = 1 - 3 \left[1 + 2^{-1/2} \sinh(a - \sqrt{3}\xi) \right]^{-1}, \quad (18)$$

Fig. 2. The electric field in units of the critical field for vacuum polarization $E_c = m_e^2 c^3 / (e \hbar)$ is plotted as a function of the coordinate ξ , for different values of the electron Fermi energy in units of the pion rest energy. The solid line corresponds to the case of null electron Fermi energy. To an increase of the value of the electron Fermi energy it is found a reduction of the peak of the electric field.

where the integration constant a has the value

$$\sinh(a) = \sqrt{2} \left(\frac{11 + \hat{\phi}^4(\xi^{WS})}{1 - \hat{\phi}^4(\xi^{WS})} \right). \quad (19)$$

In the interval $0 \leq \xi \leq \xi^{WS}$, the field $\hat{\phi}(\xi)$ is implicitly given by

$$F \left(\arccos \frac{\hat{\phi}(\xi^{WS})}{\hat{\phi}(\xi)}, \frac{1}{\sqrt{2}} \right) = \hat{\phi}(\xi^{WS})(\xi - \xi^{WS}), \quad (20)$$

where $F(\varphi, k)$ is the elliptic function of the first kind, and $F(0, k) \equiv 0$. For $F(\varphi, k) = u$, the inverse function $\varphi = F^{-1}(u, k) = \text{am}(u, k)$ is the well known Jacobi amplitude. In terms of it, we can express the solution (20) for $\xi > 0$ as,

$$\hat{\phi}(\xi) = \hat{\phi}(\xi^{WS}) \left\{ \cos \left[\text{am} \left(\hat{\phi}(\xi^{WS})(\xi - \xi^{WS}), \frac{1}{\sqrt{2}} \right) \right] \right\}^{-1}. \quad (21)$$

In the present case of $E_e^F > 0$ the ultra-relativistic approximation is indeed always valid up to $\xi = \xi^{WS}$ for high compression factors, i.e. for $R_{WS} \simeq R_c$. In the case $E_e^F = 0$, $\xi^{WS} \rightarrow \infty$, there is a breakdown of the ultra-relativistic approximation when $\xi \rightarrow \xi^{WS}$.

We can now estimate two crucial quantities of the solutions: the Coulomb potential at the center of the configuration and the electric field at the surface of the core

$$eV(0) \simeq \left(\frac{9\pi}{4}\right)^{1/3} \frac{1}{\Delta} m_\pi c^2 - E_e^F, \quad (22)$$

$$E_{\max} \simeq 2.4 \frac{\sqrt{\alpha}}{\Delta^2} \left(\frac{m_\pi}{m_e}\right)^2 E_c |\hat{\phi}'(0)|, \quad (23)$$

where $E_c = m_e^2 c^3 / (e\hbar)$ is the critical electric field for vacuum polarization. These functions depend on the value $\hat{\phi}(\xi^{WS})$ via Eqs. (16)–(20). At the boundary $\xi = \xi^{WS}$, due to the global charge neutrality, both the electric field $E(\xi^{WS})$ and the Coulomb potential $eV(\xi^{WS})$ vanish. From Eq. (13), we determine the value of $\hat{\phi}(\xi)$ at $\xi = \xi^{WS}$

$$\hat{\phi}(\xi^{WS}) = \Delta \left(\frac{4}{9\pi}\right)^{1/3} \frac{E_e^F}{m_\pi c^2}, \quad (24)$$

as a function of the electron Fermi energies E_e^F . From the above Eq. (24), one can see that there exists a solution, characterized by the value of electron Fermi energy

$$\frac{(E_e^F)_{\max}}{m_\pi c^2} = \frac{1}{\Delta} \left(\frac{9\pi}{4}\right)^{1/3}, \quad (25)$$

such that $\hat{\phi}(\xi^{WS}) = 1$. From Eq. (20) and $\xi = 0$, we also have

$$\xi^{WS}(\hat{\phi}(\xi^{WS})) = \left\{ \frac{1}{\hat{\phi}(0)} F \left[\arccos \left(4 - \frac{3}{\hat{\phi}(0)} \right), \frac{1}{\sqrt{2}} \right] \right\}. \quad (26)$$

For $\hat{\phi}(\xi^{WS}) = 1$, from Eq. (17) follows $\hat{\phi}(0) = 1$ hence Eq. (26) becomes

$$\xi^{WS}(\hat{\phi}(0)) = F \left[0, \frac{1}{\sqrt{2}} \right]. \quad (27)$$

It is well known that if the inverse Jacobi amplitude $F[0, 1/\sqrt{2}]$ is zero, then

$$\xi^{WS}(\hat{\phi}(\xi^{WS}) = \hat{\phi}(0) = 1) = 0. \quad (28)$$

Indeed from $\hat{\phi}(\xi^{WS}) = 1$ follows $\hat{\phi}(0) = 1$ and $\xi^{WS} = 0$. When $\xi^{WS} = 0$ from Eq. (17) follows $\hat{\phi}'(0) = 0$ and, using Eq. (23), $E_{\max} = 0$. In other words for the value of E_e^F fulfilling Eq. (24) no electric field exists on the boundary of the core and from Eq. (15) and Eqs. (2), (3) it follows that indeed this is the solution fulfilling both global $N_e = N_p$ and local $n_e = n_p$ charge neutrality.

5. Conclusions

We have generalized to the relativistic regime the classic work of Feynman, Metropolis and Teller, solving a compressed atom by the Thomas-Fermi equation in a Wigner-Seitz cell.

There are marked differences between the relativistic and the non-relativistic treatments.

The first and the most general one is that the existence of a finite size nucleus introduces necessarily a limit to the compressibility: the dimension of the Wigner-Seitz cell can never be smaller than the nuclear size.

The second is the clear difference of the electron distribution as a function of the radius as contrasted to the uniform approximation often adopted in the literature (see e.g. Ref. 9) which we have explicitly shown in the Fig. 1 (Left panel).

The third, one of the most relevant, is that the relativistic Feynman-Metropolis-Teller treatment allows to treat globally and in generality the electrodynamical interaction within the atom and the relativistic corrections leading to a softening of the dependence of the electron Fermi energy on the compression factor. It is then possible to derive a consistent equation of state for compressed matter which overcomes some of the difficulties of existing treatments describing the electrodynamical effects by a sequence of approximations which have lead to the occurrence of unphysical regimes e.g. the existence of negative pressure as in the Salpeter approach.¹⁷ As a direct application of this treatment we have reconsidered the study of white dwarfs within the relativistic Feynman, Metropolis, Teller approach and evaluate their effects on the value of the radii, of the masses of the equilibrium configurations as well as on the numerical value of the critical mass.¹⁸

We have then extrapolated these results to the case of nuclear matter cores of stellar dimensions for $A \approx (m_{\text{Planck}}/m_n)^3 \sim 10^{57}$ or $M_{\text{core}} \sim M_{\odot}$. The aim here is to explore the possibility of obtaining for these systems a self-consistent solution presenting global and not local charge neutrality. The results generalize the considerations presented in the previous article corresponding to a nuclear matter core of stellar dimensions with null Fermi energy of the electrons.² The ultra-relativistic approximation allows to obtain analytic expressions for the fields. An entire family of configurations exist with values of the Fermi energy of the electrons ranging from zero to a maximum value $(E_e^F)_{\text{max}}$ which is reached when the Wigner-Seitz cell coincides with the core radius. The configuration with $E_e^F = (E_e^F)_{\text{max}}$ corresponds to the configuration with $N_p = N_e$ and $n_p = n_e$. For this limiting value of the Fermi energy the system fulfills both the global and the local charge neutrality and correspondingly no electrodynamical structure is present in the core. All the other configurations presents overcritical electric fields close to their surface. The configuration with $E_e^F = 0$ has the maximum value of the electric field at the core surface, well above the critical value E_c . All these cores with overcritical electric fields are stable against the vacuum polarization process due to the Pauli blocking by the degenerate electrons.¹⁹

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