THE DEGENERACY PROBLEM IN LAMBDA-CDM COSMOLOGICAL MODELS

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Recent measurements of the cosmic microwave background radiation (CMB) from the WMAP satellite led to formulate a successful concordance cosmological model, termed Λ CDM. This model satisfactorily explains the origin and structure of the CMB temperature fluctuations, from small to large angular scales, and moreover it accurately fits –with only six parameters– the CMB angular power spectrum. Despite of their triumphs in describing the observed WMAP data, we notice that some Λ CDM cosmological parameters can attain, due to their error bars, slightly different values and this degree of freedom could produce a significant impact in our understanding of the primordial universe. We are talking about the degeneracy problem, that is cosmological models with parameters that are a little bit different from those given by the Λ CDM model but fits equally well the angular power spectrum of the CMB data. Our interest here is to investigate the Gaussian statistical property, at large angular scales, in two sets of Monte Carlo CMB maps produced by seeding them with slightly different Λ CDM angular power spectra.

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1. Introduction

Successive data releases from the Wilkinson Microwave Anisotropy Probe (WMAP)¹ have confirmed the validity of the concordance cosmological model, best known as Λ CDM. The model is obtained through the best-fitting of the cosmic microwave background radiation (CMB) angular power spectrum, as a result it provides the best values for the cosmological parameters, which of course are not sharp values but values within small intervals due to the error and uncertainty limits. For this, it is interesting to investigate synthetic CMB maps obtained from angular power spectra generated by very similar (but not equal) cosmological parameters that fit the WMAP data as well as the Λ CDM model. One such parameter is the spectral index of primordial fluctuations n_s . The main difficulty to establish the correct spectral index value is that it determines the form of the angular power spectra

at the largest scales (i.e. low multipoles), precisely the region where the cosmic variance uncertainty is large and dominates other effects.

The importance to determine, with the highest degree of accuracy, the value of the spectral index n_s is because it is related to physical processes of the primordial universe (remember that the CMB radiation is -with present techniques- the oldest cosmological observable). And because such processes probably left imprints in the statistical attributes of the observed CMB, it deserves a meticulous analysis the relationship between the spectral index and the Gaussian properties of the CMB temperature field. For instance, the value $n_s = 1$ means that the CMB temperature fluctuations correspond to a statistically Gaussian random field, therefore small variations from this value should be correlated with small deviations of Gaussianity in the data.

For these reasons, besides the determination of the spectral index n_s by fitting the CMB angular power spectrum, the released data from the WMAP¹ is nowadays under rigorous scrutiny for the possible deviations from Gaussianity²⁻⁷ and from statistical isotropy⁸⁻¹⁸ in the CMB data, properties that could be, in some sense, related. Moreover, a detection or non-detection of primordial non-Gaussianity in the CMB data is crucial to discriminate among inflationary models, and to test alternative scenarios of the early universe.¹⁹

We shall investigate the effects caused on the statistical properties of two sets of Monte Carlo CMB maps produced by seeding them with slightly different angular power spectra, each one obtained using the same cosmological parameters as the Λ CDM model except that we consider different spectral indexes values: $n_s = 0.96$ and 1.00 , respectively. To detect the presence of Gaussian deviations in these CMB maps we use two recently published non-Gaussian indicators,²⁰ based on the application of skewness and kurtosis in large-angle patches of CMB temperature maps, which allow us to construct the **S**- and **K**-maps (for details see Sec. 3) from which we calculate their angular power spectra (APS). After that, we compare the APS of the two sets of **S**-maps obtained by applying our skewness tool to both sets of Monte Carlo CMB maps. Similar procedures are done with the two sets of **K**-maps. Our results show that, in the mean, the Gaussianity property of these sets of Monte Carlo maps is different, and this fact seems to be crucial when one has to quantify the confidence level of some data analysis result. That is to say, the statistical-significance evaluation of a result concerning a WMAP CMB map is model dependent.

This work is organized as follows. In Sec. 2 we explain how to produce the sets of simulated Monte Carlo CMB maps based on the Λ CDM angular power spectrum, and also the ILC WMAP map, used in the analyses. In Sec. 3 we describe our method to analyze the statistical properties of the CMB maps. Finally, in Sec. 4 we summarize our results and formulate our conclusions and perspectives for future works.

2. Λ CDM Angular Power Spectra and Monte Carlo CMB data

The degeneracy problem in the concordance cosmological model consists on having two or more APS similar to that one of the Λ CDM model obtained with slightly different cosmological parameter value(s), but all of them consistent with current data because of the error bars.

Here we shall study two Λ CDM APS with the spectral indexes attaining slightly different values: $n_s = 0.96, 1.00$. For this, we use the CMBFAST tool²² to generate the corresponding APS seeds that are then used in the SYNFAST algorithm²³ to produce two sets of 1 000 random realizations each. As we show in Fig. 1, left panel, the Λ CDM APS corresponding to $n_s = 0.96$ is very similar to the APS obtained with the same cosmological parameters as the concordance model, except that $n_s = 1.00$. Moreover, we also observe in the right panel that both APS fits suitably the APS from WMAP data. The comparison of the statistical properties between these two sets of Monte Carlo CMB maps constitute the target of our analysis.

It is well known that foregrounds introduce non-Gaussianities in the CMB data. For these the WMAP science team have performed substantial efforts to minimize the presence of foregrounds in the measured CMB data. The recently released seven-year foreground-reduced ILC map represents well a CMB map that is consistent with Gaussianity, after the application of the KQ75-7yr mask.¹ We find interesting to use this WMAP map to test the mean statistical properties of these two sets of Monte Carlo CMB maps.

In the next section we shall describe our method to analyze the statistical features of the CMB maps, both Monte Carlo and WMAP ones. The comparison of the statistical properties between these two sets of Monte Carlo CMB maps, together with the analysis of the ILC map, will be performed in the last section.

3. The Method for Statistical Analysis

In this section we briefly present how to construct two non-Gaussianity indicators. For a detailed discussion of these indicators we refer the readers to Ref. 20.

Consider a discrete set of points $\{j = 1, \dots, N_c\}$ uniformly distributed on the sphere S^2 as the center of spherical caps of a given aperture γ , and calculate for each cap j the skewness and kurtosis values

$$S_j \equiv \frac{1}{N_p \sigma_j^3} \sum_{i=1}^{N_p} (T_i - \bar{T}_j)^3, \quad K_j \equiv \frac{1}{N_p \sigma_j^4} \sum_{i=1}^{N_p} (T_i - \bar{T}_j)^4 - 3,$$

where N_p is the number of pixels in the j^{th} cap, T_i is the temperature at the i^{th} pixel, $\bar{T}_j$ is the CMB mean temperature on the j^{th} cap, and σ_j is the standard deviation for each j . The sets of values $\{S_j, j = 1, \dots, N_c\}$ and $\{K_j, j = 1, \dots, N_c\}$ can then be viewed as a measure of the non-Gaussianity in the direction (θ_j, ϕ_j) (which corresponds to the center of the j^{th} cap). Patching together the $\{S_j\}$ and $\{K_j\}$ values, we obtain the functions $S = S(\theta, \phi)$ and $K = K(\theta, \phi)$ defined on

Fig. 1. (Left panel) Angular power spectra for Λ CDM models with slightly different values for the spectral index: $n_s = 0.96$ (dotted curve) and $n_s = 1.00$ (dashed curve). (Right panel) The same angular power spectra appearing in the left panel plus the angular power spectrum from WMAP-7yr data (continuous noisy curve).

S^2 . Throughout this work the sky map representation of the functions $S = S(\theta, \phi)$ and $K = K(\theta, \phi)$ are named, respectively, **S**–map and **K**–map. Clearly, one can expand each of these functions in their spherical harmonics and calculate their corresponding angular power spectrum. For the skewness function $S = S(\theta, \phi) = \{S_j, j = 1, \dots, N_c\}$, for example, one has

$$S(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} b_{\ell m} Y_{\ell m}(\theta, \phi) \implies S_{\ell} = \frac{1}{2\ell+1} \sum_m |b_{\ell m}|^2,$$

where S_{ℓ} is the corresponding angular power spectrum. Similar expressions hold for the $K = K(\theta, \phi)$.

Thus, to obtain quantitative information for each multipole component ℓ about the non-Gaussianity at large-angles (low ℓ) from the **S**– and **K**–maps, which were obtained from the CMB temperature maps, we calculate their power spectra S_{ℓ} and K_{ℓ} . Finally, to compare the statistical properties of the spectra $\{S_{\ell}\}$ and $\{K_{\ell}\}$, obtained from the analyses of the different Monte Carlo CMB sets, we compare their corresponding mean $\overline{S}_{\ell}$ and $\overline{K}_{\ell}$ and 2-sigma values (i.e., 95% CL). Here we concentrate on the large angular scales, that is, for $\ell = 1 - 10$.

4. Results and Conclusions

In this section we report the results of Gaussianity analyses performed by applying our indicators, discussed in the previous section, to the Monte Carlo and WMAP data described in Sec. 2.

In Fig. 2 we show the APS of the **S**–maps and **K**–maps calculated from the Monte Carlo CMB maps, which were produced by the APS with slightly different spectral indexes, namely $n_s = 0.96$ and 1.00 . As expected, the similarity in the CMB APS implies in the similarity of the angular power spectra obtained from the **S**– and **K**–maps. We also exhibit in these plots the APS from the **S**– and **K**–maps obtained from the ILC-7yr WMAP map (with the corresponding mask to ensure Gaussianity properties, see Sec. 2).

Our main conclusion is that, as observed in Fig. 2, Monte Carlos generated through APS with a different spectral index n_s have different statistical properties and therefore they show different confidence level when used to evaluate the statistical significance of some result obtained from WMAP data. For instance, the confidence level of the APS $\{S_\ell\}$ and $\{K_\ell\}$ for the WMAP ILC-7yr foreground-reduced map reveal that it is more likely non-Gaussian ($n_s = 0.96$) than Gaussian ($n_s = 1.00$). In order to quantify the possible correlation between data from WMAP and data from the sets of Monte Carlo maps we use the reduced χ^2 -goodness-of-fit test which gives an overall measure between two data sets by quantifying how related they are. For this we shall calculate the value $\chi^2/dof \equiv \chi^2_{red}$, where dof stands for degree of freedom, between pairs of data sets regarding the APS values S_ℓ and K_ℓ , for $\ell = 1 - 10$, calculated from the ILC map versus the mean-APS $\bar{S}_\ell$ and $\bar{K}_\ell$ obtained from the two sets of Monte Carlo CMB maps, those produced with $n_s = 0.96$ and $n_s = 1.00$ (see Fig. 2).

Our results are: $\chi^2_{red} = 1.8(1.3)$ between the APS of the **S**–map (**K**–map) from the ILC versus the mean APS of the **S**–maps (**K**–maps) obtained from Monte Carlo CMB maps with $n_s = 0.96$. Similarly, we obtain $\chi^2_{red} = 4.7(4.3)$ between the APS of the **S**–map (**K**–map) from the ILC versus the mean APS of the **S**–maps (**K**–maps) obtained from Monte Carlo CMB maps with $n_s = 1.00$. Remember that a χ^2 value near 1 means a high correlation between analyzed data, while larger values means poorer correlations. Thus, our results indicate that is more probable that (the $\{S_\ell\}$ and $\{K_\ell\}$ APS from) the WMAP ILC map plus the KQ75-7yr mask corresponds to the case $n_s = 0.96$, with a p -value of $\sim 15\%$, than to the Gaussian case $n_s = 1.00$, with a p -value of less than 0.1% .

Summarizing, our APS analyses of **S**– and **K**–maps produced from the two sets of Monte Carlo CMB maps (seeded by Λ CDM spectra with $n_s = 0.96, 1.00$, respectively) show that, in the average, these data contain different amounts of non-Gaussianity. Furthermore, Monte Carlos seeded on the Λ CDM concordance model spectrum ($n_s = 0.96$) can not be considered as the absolute reference for *Gaussian* properties in CMB maps. Additionally, our results show that, in the average, the Gaussianity property of these sets of Monte Carlo maps is different, and this fact seems to be crucial when one has to quantify the confidence level of some result. That is to say, *the statistical-significance evaluation of any result concerning WMAP data is model dependent*.

It would be interesting to test the Gaussian properties of these two sets of Monte Carlo CMB maps using some statistical tools available in the literature.

Fig. 2. Angular Power Spectra for the **S**– and **K**–maps, left and right panel, respectively. The dot-dashed (dashed) line corresponds to the 2σ level (95% confidence level) of the **S**– and **K**–map’s power spectra, maps obtained by applying our statistical tools to 1000 Monte Carlo CMB maps produced from a Λ CDM model with spectral index $n_s = 0.96$ ($n_s = 1.00$).

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