MULTIDIMENSIONAL COSMOLOGY, CONSTANTS AND TRANSITION TO NEW SI UNITS

V. N. MELNIKOV

*Center for Gravitation and Fundamental Metrology, VNIIMS, Ozernaya 46
Moscow 119361, Russia*

*Institute of Gravitation and Cosmology, Peoples' Friendship University of Russia,
Miklukho-Maklaya 6, Moscow 117198, Russia
melnikov@phys.msu.ru*

Received 19 June 2011

Main current problems of physics, gravitation and cosmology in particular are analyzed. Special attention is paid to results of the theory with extra dimensions and variations of fundamental physical constants. As an example the family of spherically symmetric solutions with horizon with multi-component anisotropic fluid is presented. The metrics of solutions are defined on a manifold that contains a product of $n-1$ Ricci-flat “internal” spaces. A simulation of black brane solutions is considered. For the solution with the fluid matter the post-Newtonian parameters β and γ corresponding to the 4-dimensional section of the metric are found.

Keywords: Multidimensional gravity; dark energy; dark matter; fundamental constants.

PACS numbers: 04.20.Jb; 04.50.+h

1. Introduction

Main problems of modern physics, gravitation and cosmology in particular, are the following:

- unification of known physical interactions and the role of gravitation in it;^{1,2}
- present accelerating expansion of the Universe,^{3,4} dark matter (DM) and dark energy (DE) problems;
- possible variations of fundamental physical constants (FPC)^{2,5} and transition to new definitions of basic units, based on FPC, in the International System (SI).

Unification of all physical interactions is the most promising trend now as all attempts to quantize GR failed and it was proved that it is not renormalizable. This process went through the development of supergravity, strings, superstrings and hopes are for the creation of the so-called M-theory, which does not exist yet. Investigations of gravity in strong fields and construction of gravitational wave detectors speeded up theoretical studies of pulsars, black holes, wormholes, quasars, active galactic nuclei, γ -bursts, early Universe etc., which continue now. We may expect that gravity will be essential not only by itself, but as a missing cardinal link

of theory, unifying all existing physical interactions. In experiments: some crucial next generation gravitational experiments verifying predictions of unified theories (UT) will be important.

We may point out also the outburst of new precise data for cosmological parameters from observational cosmology. It follows that the flat universe is favored, that our Universe is now expanding with acceleration and we know only about 0.04 of the Universe content. This is a real challenge to physics and may lead to a new revolution in our understanding of Nature. Evidence for the dark matter comes from data on gravitational lensing, galactic rotation curves and large-scale structure formation. Evidence for dark energy comes from observations of type Ia supernovae and cosmic microwave background anisotropy.

There are many attempts to explain these data within the standard or new physics. It seems that Λ CDM-model - some cold (dust-like) dark matter plus cosmological constant (universal repulsion field) are in agreement with the present observational data, though their origin is still vague. Among attempts to explain the late-time (after $z = 1000$) accelerating expansion of the Universe are: the cosmological constant, quintessence (some scalar field), k -essence, Chaplygin gas, decay of super-heavy particles - within GR or some modifications of GR, such as modified Newtonian dynamics (MOND), Dirac-Born-Infeld nonlinear theory (DBI), theories with arbitrary functions of the curvature $F(R)$, theories with extra dimensions (multidimensional gravity and cosmology based on Einstein equations in arbitrary dimensions,^{6–8} empirical models like brane worlds (BW), ADD model etc. We may predict that a thorough study of gravity itself and within the unified models will give in the next century and millennium even more applications for our everyday life as electromagnetic theory gave in the 20th century after abstract fundamental investigations of Maxwell, Poincare, Einstein etc., which never dreamed about such applications of their works.

Other important feature – increasing role of fundamental physics studies, gravitation, cosmology and astrophysics in particular, in space experiments. Unique microgravity environments and modern technology outbreak give nearly ideal place for gravitational experiments (no relatively strong gravitational field of the Earth and of gravitational fields of nearby objects due to absence of gravity screening).

In the development of relativistic gravitation and dynamical cosmology after Einstein and Friedmann we may see three stages: 1) investigation of models with matter sources as perfect and viscous fluids; 2) studies of models with sources as different physical fields, starting from electromagnetic and scalar ones, both in classical and quantum cases (our program from 1968, most results are in Ref. 9); 3) Topical now - application of ideas and results of UT for treating fundamental problems of cosmology and BH physics, especially in high energy regimes, using ideas of extra dimensions and p -branes as sources (our program^{6–8} from 1987). Multidimensional gravitational models (MGM) as low energy limits of UT play an essential role in the latter approach. Necessity of studying multidimensional models of gravitation and cosmology is motivated by several reasons.

First, the main trend in modern physics, as we already pointed out, is the unification of 4 known fundamental physical interactions. Different UT models use numbers of extra dimensions: 26, 11, 10, 5-7.

Now, theories with membranes, p -branes and M-theory are being created and studied. As no definite successful UT exists now, it is desirable to study common features of these theories, their applications to solving basic problems of modern gravity and cosmology. Moreover, believing in UT, early stages of the Universe evolution, BH, WH physics, as unique super-high energy regions, are the most proper and natural arena for them.

Second, MGM and scalar-tensor theories (STT)⁹ are theoretical frameworks for describing possible temporal and range variations of FPC (Milne, 1935; Dirac, 1937). Until now they are under study both theoretically and experimentally.

Lastly, applying MGM to basic problems of cosmology and BH physics, we hope to find answers to such long-standing problems as: singular or nonsingular initial states, creation of the Universe, creation of matter and its entropy,¹⁰ acceleration, cosmological constant (or vacuum), origin of inflation and specific scalar fields and their potentials necessary for their realization, isotropization and graceful exit problems, stability and nature of FPC, possible number of extra dimensions, their stable compactification etc.

MGM are certain generalizations of GR (tested reliably for weak fields up to $10^{-4} - 10^{-5}$ and partially in strong fields - binary pulsars). So, it is natural to inquire about their possible observational or experimental windows. These windows are:¹

- possible deviations from the Newton law, or new interactions (ISL),
- possible variations of effective G with a time rate smaller than H ,
- possible existence of scalar (monopole) modes in gravitational waves,
- different behavior of strong field objects (SFO), such as multidimensional BH, WH, AGN etc.,
- standard cosmological tests,
- non-conservation of energy in braneworlds (e.g in the Large Hadron Collider experiments, if found) etc.

Since modern cosmology became unique laboratory for testing unified models of physical interactions at energies far beyond the level of existing and future man-made accelerators and other installations on the Earth, there exists a possibility of using cosmological and astrophysical data for discriminating between future UT.⁵

As no accepted UT exists, we studied simple, but general models based on multidimensional Einstein equations with or without sources as: cosmological constant, perfect and viscous fluids, linear and nonlinear scalar and electromagnetic fields, their possible interactions, dilaton and moduli fields, fields of antisymmetric forms (related to p -branes) etc. We studied also the billiard behavior near the singularity in many dimensions. Our program's main objective was and is to obtain exact self-consistent solutions (integrable models) for these systems in arbitrary D and then to analyze them in cosmological, spherically and axially-symmetric cases. It is a

natural and most reliable way to study highly nonlinear systems. We did it mainly within the Riemannian geometry.

The model we use includes n Einstein spaces of constant curvature with m -component perfect fluid, or fields, form-fields, metric (e.g. cosmological), manifold as a direct product of factor spaces of arbitrary dimensions, energy-momentum tensor of fluids or fields (scalar, electromagnetic, forms). In a harmonic time gauge Einstein multidimensional equations are equivalent to Lagrange equations with some Lagrangian in a mini-superspace metric, depending on dimensions of factor spaces with some potential. Diagonalization of the metric and reduction to σ -model and Toda-like systems lead further to Liouville, Abel, (Generalized) Emden-Fowler equations¹⁰ etc.

Behavior of extra spaces in our solutions may be: constant, dynamically compactified, torus, large, but with barriers, walls etc.

We realized the program with extra arbitrary dimensions (from 1988) in cosmology and obtained exact general solutions in DD⁶⁻⁸ for sources:

- Λ , (Λ plus φ), singled out nonsingular, dynamically compactified, inflationary ones;¹¹
- perfect fluid (PF plus φ) with nonsingular, inflationary solutions;
- viscous fluid with nonsingular solutions, generation of mass, entropy, quintessence and coincidence in 2-component model;
- with stochastic behavior near the singularity, billiards in Lobachevsky space, $D=11$ critical, φ destroys billiards (1994);
- Ricci-flat solutions above for any n , also with curvature in one factor-space and with curvatures in 2 factor-spaces only for $N = 10, 11$;¹²
- scalar and dilaton fields, forms of arbitrary rank, inflationary, bouncing, Λ generation; first billiards for p -branes;
- quantum variants (WDW-equation¹³) for all above cases;
- dilatonic fields with potentials,¹⁴ studied also billiards for this case;
- present acceleration and $G\dot{}$ in many above mentioned cases, their combination (as a new test of viable cosmological models⁵).

We obtained also general solutions depending on r in DD:

- generalized Schwarzschild, Tangerlini (singled out BH's), also with φ (no BH's in this case);
- generalized Reissner-Nordstrom (singled out BH's), plus φ (no BH's);
- multi-temporal;
- dilaton-like interaction of φ and electromagnetic fields (BH's exist only for special case); stability studies were done (stable only in BH case above);
- same with dilaton-forms interaction, stability was obtained only in some cases, e.g. for one form in particular;
- PPN parameters were calculated for the above solutions, effective $G(r)$ and non-Newtonian corrections as well.

We worked out also the theoretical part of the new space project SEE to measure G , $G\dot{}$ and test ISL at lab and Earth ranges and new lab experiments to measure G and test ISL with higher precision than present.

2. Fundamental Physical Constants

One of possible windows of extra dimensions is the problem of possible variations of FPC. In any physical theory we meet with constants which characterize the stability properties of different types of matter: of objects, processes, classes of processes and so on. These constants are important because they arise independently in different situations, cannot be calculated from other constants and have the same value, at any rate within accuracies we have gained nowadays. That is why they are called FPC. It is impossible to define strictly this notion because the constants, mainly dimensional, are present in definite physical theories. So, we may talk not about an absolute choice of FPC, but only about a choice corresponding to the present state of physical sciences.

FPC are known with different accuracies. The most precisely determined constant was and remains the speed of light c : its accuracy was 10^{-10} and now it is defined with null accuracy. Atomic constants, $e, \hbar, m$ and others are determined with errors 10^{-8} , G - up to 10^{-4} or even worse, Weinberg angle θ_w - up to 10^{-3} ; the accuracy of H is about 0.02. Other cosmological parameters (also could be considered as FPC): Ω varies within several percent; for Λ - its corresponding energy density exceeds the matter density (0.7 of the total mass).

As to the nature of FPC, there are several approaches. One of them - speed of light c , velocities of usual bodies are smaller than c , so it plays the role of asymptotic limit. The same sense have $\hbar$ - as a minimal quantum of action and e - as a minimal observable charge.

FPC or their combinations may be considered also as natural scales determining the basic units. Earlier they were chosen more or less arbitrarily, i.e., the second, meter and kilogram. Now the first two are based on stable (quantum) phenomena. Their stability is ensured by physical laws which include FPC. In 2005 there appeared suggestions¹⁵ for a new reproducible realization of a kg , fixing values of $\hbar$, N_A , e and k and to redefine other basic SI units. But, up till now it was not possible to solve all problems of basic units realization and to reach necessary uncertainties in measurements of above mentioned FPC.¹⁶

An exact knowledge of FPC and precision measurements are necessary for testing main physical theories, extension of our knowledge of nature and, in the long run, for practical applications of fundamental theories.

As to classification of FPC, we may set them now into 4 groups according to their generality: 1) universal constants, such as $\hbar$, which divides all phenomena into quantum and non-quantum ones (micro- and macro-worlds) and to a certain extent c , which divides all motions into relativistic and non-relativistic ones; 2) constants of interactions like α (or e), θ_w , Λ_{QCD} and G ; 3) constants of elementary constituents of matter like m_e, m_W, m_χ , etc., and 4) transformation multipliers such as k, I and partly c . Of course, this division into classes is not absolute.

As to the number of FPC, there are two opposite tendencies: the number of "old" FPC is usually diminishing when a new, more general theory is created,

but at the same time new fields of science arise, new processes are discovered in which new constants appear. So, we may come to some minimal choice which is characterized by one or several FPC, maybe to Planck parameters - combinations of c , $\hbar$ and G : The role of these parameters: - m_L defines the energy of unification of four known fundamental interactions and - L is the scale where classical notions of space-time lose their meaning. Other ideas on minimal number of FPC exist: 2 or none.

The problem of FPC variations is a part of the gravitational-relativistic metrology (GRM), first formulated in Ref. 17. Main subjects of GRM are: a) general relativistic models for different astronomical scales: Earth, Solar System, galaxies, clusters of galaxies, cosmology - for time transfer, VLBI, space dynamics, relativistic astrometry etc.; b) development of generalized gravitational theories and UT for testing their effects in experiments; c) measurement of FPC, G in particular, and testing their stability in space and time, testing EP, ISL etc.; d) measurement of fundamental cosmological parameters as FPC: cosmological models studies, measurements and observations; dark matter, dark energy, H , Ω , Λ , q and n ; e) study of gravitational waves (creation third generation GW-detectors, their networks; study of GW sources); f) basic standards (clocks), atomic, molecular, and neutron interferometry, atomic force spectroscopy etc. in fundamental gravitational experiments (especially in space) for testing GR, STT and other theories up to 10^{-6} (rotational, torsional and second order effects).

As to PPN parameters β and γ , we are now at the level of $10^{-4} - 10^{-5}$ (Cassini data); limit on the Brans-Dicke parameter: $\omega_{BD} > 40000$.

3. Variations of FPC

There appeared some evidence for a varying fine structure constant on a cosmological time scale:¹⁸ $d(\ln\alpha)/dt = (6.4 \pm 1.4) \times 10^{-16} \text{y}^{-1}$ (in linear parametrization), but data from some other groups do not confirm this variations! The same situation is for m_e/m_p possible variation. Moreover, there appeared data on anisotropy of variations of α : different sign of variations in the Northern and Southern Hemispheres.¹⁶

Great progress is observed recently in A-clocks. Measurement of optical clocks frequency ratios with single Al^+ and Hg^+ ions, 2008, gave relative variation of α at the level $d(\ln\alpha)/dt = (-1.6 \pm 2.3) \times 10^{-17} \text{y}^{-1}$ (the same order as from Oklo data (10^9 y ago)). All these data at different z cannot be compared directly. For that one needs the full universe evolution.

As to G , there are three problems, which origin lies mainly in UT predictions: 1) absolute G measurements, 2) possible time variations of G , 3) possible range variations of G - non-Newtonian, or new interactions. The most recent and precise (less than 50 ppm) G measurements do not agree and some differ from the CODATA value (2006): $G = 6.67428(67) \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$. This situation means that either the limit of terrestrial accuracies has been reached or we have some new physics entering the measurement procedure. The first means that maybe we should turn

to space experiments to measure G , and the second means that a more thorough study of theories generalizing Einstein's GR or UT is necessary.

As to possible time variations of G , the most reliable data are based on 1) lunar laser ranging (LLR) and 2) spacecraft and planets ranging. They are at the level of 5×10^{-13} . Here, once more we see that there is a need for corresponding theoretical and experimental studies. Probably, future space missions like Earth SEE satellite, STEP or PHOBOS-GRUNT-2011 (Russia-China, Russian Zenit rocket, two satellites: Russian lander and Chinese orbiter, 800-80000 km) and LLR will be decisive steps in solving the problem of temporal variations of G and determining the fate of different theories.

Different theoretical schemes lead to variations of the effective gravitational constant: 1. Empirical models and theories of Dirac type, where G is replaced by $G(t)$. 2. Numerous scalar-tensor theories of Jordan-Brans-Dicke type where G depending on a scalar field appears. 3. Gravitational theories with the conformal scalar field.^{9,19} 4. General STT²⁰ (particular models of Miyazaki and Fujii). 5. Multidimensional UT with dilaton fields and effective scalar fields appearing in our 4-dimensional space-time from additional dimensions.⁶⁻⁸

Temporal variations of FPC are connected with each other in multidimensional models of unification of interactions. So, experimental tests on $\dot{\alpha}/\alpha$ may at the same time be used for estimation of $\dot{G}/G$ and vice versa. Moreover, variations of G are related also to the cosmological parameters ρ (or Ω) and q , which gives in principle opportunities of raising the precision of their determination. So, $\dot{G}$ is now an additional test of cosmological models!⁵ As variations of FPC are closely connected with the behavior of internal scale factors (volume), it is also a direct probe of properties of extra dimensions and the corresponding theories.

Nearly all modified theories of gravity and UT predict deviations from the Newton law (ISL) or composition-dependent violations of the Equivalence Principle (EP) due to appearance of new possible massless or massive particles (partners). Experimental data exclude violations of ISL or existence of these particles at nearly all ranges except less than 100 nm and at meter and hundreds of meters ranges. In the GR G is a true constant, massless gravitons are mediators of the gravitational interaction, they obey second-order differential equations and interact with matter with a constant strength G . If any of these requirements is violated, we come in general to deviations from the Newton law with a range (generalization of GR). We analyzed several classes of such theories.²¹ In all these theories some effective or real masses appear leading to Yukawa-type or power-law deviations from the Newton law, characterized by strength and range. p -brane models also predict non-Newtonian additional interactions both of Yukawa or power-law types, which is intensively discussed nowadays.

We obtained new experimental limits on possible violation of ISL based on precession of orbits method due to non-Newtonian interaction of two celestial bodies, using modern tracking data of satellites, planets and pulsar data (Pitjeva) in planets

(satellites) radii ranges (1-4 order better at different ranges).²² The forbidden area in parameters of Yukawa additional interaction is extended at ranges $10^5 - 10^{15}$ m.

Here, as an example of multidimensional solution we give spherically-symmetric solutions with a horizon (e.g., black brane ones) defined on product manifolds containing several Ricci-flat factor-spaces (with diverse signatures and dimensions).

4. Black Brane Solutions

These solutions appear either in models with antisymmetric forms and scalar fields^{23–29} or in models with multi-component anisotropic fluid (MCAF).^{30–32} For black brane solutions with 1-dimensional factor-spaces (of Euclidean signatures), see Ref. 33 and references therein.

These and more general brane cosmological and spherically symmetric solutions were obtained by reduction of the field equations to the Lagrange equations corresponding to Toda-like systems.^{24,34} An analogous reduction for models with multicomponent anisotropic fluids was performed earlier in Ref. 35, 36.

Here we consider spherically-symmetric solutions with horizon in two models: i) with scalar field and fields of forms, ii) with multi-component anisotropic fluid (MCAF), when certain relations on fluid parameters are imposed.

We start with a model governed by the action

$$S = \int d^D x \sqrt{|g|} \{ R[g] - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \sum_{a \in \Delta} \frac{\theta_a}{n_a!} \exp[2\lambda_a(\varphi)] (F^a)^2 \}, \quad (1)$$

where $g = g_{MN}(x) dx^M \otimes dx^N$ is a metric, $\varphi = (\varphi^\alpha) \in \mathbb{R}^l$ is a vector of scalar fields, $(h_{\alpha\beta})$ is a constant symmetric non-degenerate $l \times l$ matrix ($l \in \mathbb{N}$), $\theta_a = \pm 1$, $F^a = dA^a$ is a n_a -form ($n_a \geq 1$), λ_a is a 1-form on $\mathbb{R}^l$: $\lambda_a(\varphi) = \lambda_{\alpha a} \varphi^\alpha$, $a \in \Delta$, $\alpha = 1, \dots, l$. Here Δ is some finite set. In the models with one time all $\theta_a = 1$ when the signature of the metric is $(-1, +1, \dots, +1)$.

In Refs. 26–28 we have obtained a family of black brane solutions to field equations corresponding to the action (1). These solutions are defined on the manifold

$$M = (R_0, +\infty) \times (M_0 = S^{d_0}) \times (M_1 = \mathbb{R}) \times \dots \times M_n, \quad (2)$$

and have the following form

$$g = \left(\prod_{s \in S} H_s^{2h_s d(I_s)/(D-2)} \right) \left\{ f^{-1} dR \otimes dR + R^2 g^0 - \left(\prod_{s \in S} H_s^{-2h_s} \right) f dt \otimes dt + \sum_{i=2}^n \left(\prod_{s \in S} H_s^{-2h_s \delta_{i I_s}} \right) g^i \right\}, \quad (3)$$

$$\exp(\varphi^\alpha) = \prod_{s \in S} H_s^{h_s \chi_s \lambda_{\alpha s}^\alpha}, \quad (4)$$

$$F^a = \sum_{s \in S} \delta_{a_s}^a \mathcal{F}^s, \quad (5)$$

where $f = 1 - 2\mu/R^d$,

$$\mathcal{F}^s = -\frac{Q_s}{R^{d_1}} \left(\prod_{s' \in S} H_{s'}^{-A_{ss'}} \right) dR \wedge \tau(I_s), \quad s \in S_e, \quad (6)$$

$$\mathcal{F}^s = Q_s \tau(\mathbf{I}_s), \quad s \in S_m. \quad (7)$$

Here $Q_s \neq 0$ ($s \in S$) are charges, $R_0 > 0$, $R_0^d = 2\mu > 0$, $d = d_0 - 1$. In (3) g^0 is a canonical metric on unit sphere $M_0 = S^{d_0}$ and g^i is a Ricci-flat metric on M_i , $i = 2, \dots, n$ and $\delta_{iI} = \sum_{j \in I} \delta_{ij}$ is the indicator of i belonging to I : $\delta_{iI} = 1$ for $i \in I$ and $\delta_{iI} = 0$ otherwise. We also denote $g^1 = -dt \otimes dt$.

The brane set S is by definition $S = S_e \cup S_m$, $S_v = \cup_{a \in \Delta} \{a\} \times \{v\} \times \Omega_{a,v}$, $v = e, m$ and $\Omega_{a,e}, \Omega_{a,m} \subset \Omega$, where $\Omega = \Omega(n)$ is the set of all non-empty subsets of $\{1, \dots, n\}$, i.e. all branes do not “live” in M_1 .

Any brane index $s \in S$ has the form $s = (a_s, v_s, I_s)$, where $a_s \in \Delta$, $v_s = e, m$ and $I_s \in \Omega_{a_s, v_s}$. The sets S_e and S_m define electric and magnetic branes correspondingly. In (4) $\chi_s = +1, -1$ for $s \in S_e, S_m$ respectively.

All branes contain the time manifold $M_1 = \mathbb{R}$, i.e. $1 \in I_s$, $\forall s \in S$.

All manifolds M_i , $i > 1$, are assumed to be oriented and connected and the volume d_i -forms $\tau_i \equiv \sqrt{|g^i(y_i)|} dy_i^1 \wedge \dots \wedge dy_i^{d_i}$, are well-defined for all $i = 1, \dots, n$. Here $d_i = \dim M_i$, $i = 0, \dots, n$, $d_0 > 1$, $d_1 = 1$, and for any $I = \{i_1, \dots, i_k\} \in \Omega$, $i_1 < \dots < i_k$, we denote $\tau(I) \equiv \tau_{i_1} \wedge \dots \wedge \tau_{i_k}$, $d(I) = \sum_{i \in I} d_i$.

Forms $\mathcal{F}^s$ correspond to electric and magnetic branes for $s \in S_e, S_m$ respectively. In (7) $\mathbf{I} = \{0, \dots, n\} \setminus I$.

The parameters h_s appearing in the solution satisfy the relations: $h_s = (B_{ss})^{-1}$, where $B_{ss'} = d(I_s \cap I_{s'}) + \frac{d(I_s)d(I_{s'})}{2-D} + \chi_s \chi_{s'} \lambda_{\alpha a_s} \lambda_{\beta a_{s'}} h^{\alpha\beta}$, $s, s' \in S$, with $(h^{\alpha\beta}) = (h_{\alpha\beta})^{-1}$ and $D = 1 + \sum_{i=1}^n d_i$. Here we assume that (i) $B_{ss} \neq 0$, $s \in S$, and (ii) $\det(B_{ss'}) \neq 0$, i.e. the matrix $(B_{ss'})$ is a non-degenerate one. In the solution $(A_{ss'}) = (2B_{ss'}/B_{s's'})$. Here some ordering in S is assumed.

Functions $H_s = H_s(z) > 0$, $z = 2\mu/R^d \in (0, 1)$ obey the equations

$$\frac{d}{dz} \left(\frac{(1-z)}{H_s} \frac{dH_s}{dz} \right) = B_s \prod_{s' \in S} H_{s'}^{-A_{ss'}}, \quad (8)$$

equipped with the boundary conditions: 1. $H_s(1-0) = H_{s0} \in (0, +\infty)$, 2. $H_s(+0) = 1$, $s \in S$. Here $B_s = K_s \varepsilon_s Q_s^2 / (2d\mu)^2$ and $\varepsilon_s = (-\varepsilon[g])^{(1-\chi_s)/2} \varepsilon(I_s) \theta_{a_s}$, $s \in S$, $\varepsilon[g] \equiv \text{sign det}(g_{MN})$.

Equations (8) are equivalent to Toda-type equations. First boundary condition guarantees the existence of a regular horizon at $R = R_0$ ($R_0^d = 2\mu$) if $H_s(R)$ is

smooth in the vicinity of $R = R_0$. Second condition ensures an asymptotical (for $R \rightarrow +\infty$) flatness of the $(2 + d_0)$ -dimensional section of the metric.

The solutions under consideration take place if certain restrictions on the sets of branes are obeyed.²⁶ These restrictions guarantee the block-diagonal form of the energy-momentum tensor.

5. Solutions With a Horizon in the Model With MCAF

5.1. The model

Now we consider a family of spherically symmetric solutions to Einstein equations with an anisotropic matter source $R_N^M - \frac{1}{2}\delta_N^M R = k^2 T_N^M$, defined on the manifold $M = \mathbb{R}_* \times (M_0 = S^{d_0}) \times (M_1 = \mathbb{R}) \times \dots \times M_n$, with the block-diagonal metrics

$$g = e^{2\gamma(u)} du \otimes du + \sum_{i=0}^n e^{2\beta^i(u)} g^i. \quad (9)$$

Here $\mathbb{R}_* \subseteq \mathbb{R}$ is an open interval. The manifold M_i with the metric g^i , $i = 1, 2, \dots, n$, is a Ricci-flat space of dimension d_i and g^0 is the standard metric on the unit sphere S^{d_0} , so that $\text{Ric}[g^0] = (d_0 - 1)g^0$; u is a radial variable, κ^2 is the gravitational constant, $d_1 = 1$ and $g^1 = -dt \otimes dt$.

The energy-momentum tensor is adopted in the following form for each component of the fluid:

$$(T_N^M) = \text{diag}(-\rho^s, \mathbf{p}_0^s \delta_{k_0}^{m_0}, \mathbf{p}_1^s \delta_{k_1}^{m_1}, \dots, \mathbf{p}_n^s \delta_{k_n}^{m_n}), \quad (10)$$

where ρ_0^s and $\mathbf{p}_i^s$ are the effective density and pressures respectively, depending on the radial variable u .

We assume that the following “conservation laws” $\nabla_M T_N^{(s)M} = 0$ are valid for all components. We also impose the following equations of state

$$\mathbf{p}_i^s = \left(1 - \frac{2U_i^s}{d_i}\right) \rho_0^s, \quad (11)$$

where U_i^s are constants, $i = 0, 1, \dots, n$.

The physical density and pressures are related to the effective ones (ρ_0^s , $\mathbf{p}_i$) by the formulae $\rho^s = -\mathbf{p}_1^s$, $p_u^s = -\rho_0^s$, $p_i^s = \mathbf{p}_i^s$ ($i \neq 1$). In what follows we put $\kappa = 1$ for simplicity.

5.2. The solutions

We will make the following assumptions: 1°. $U_0^s = 0$, 2°. $U_1^s = 1$, 3°. $(U^s, U^s) = U_i^s G^{ij} U_j^s > 0$, 4°. $2(U^s, U^l)/(U^l, U^l) = A_{sl}$, where $A = (A_{sl})$ is non-degenerate matrix, $G^{ij} = \frac{\delta^{ij}}{d_i} + \frac{1}{2-D}$, are components of the matrix inverse to the matrix of the minisuperspace metric¹³ $(G_{ij}) = (d_i \delta_{ij} - d_i d_j)$, $i, j = 0, 1, \dots, n$ and $D = 1 + \sum_{i=0}^n d_i$ is the total dimension.

The conditions 1^o and 2^o in brane terms mean that brane “lives” in the time manifold M_1 and does not “live” in M_0 . Due to assumptions 1^o and 2^o and the equations of state (11), the energy-momentum tensor reads as follows:

$$(T^{(s)}_N)^M = \text{diag}(-\rho^s, \rho^s \delta_{k_0}^{m_0}, -\rho^s, p_2^s \delta_{k_2}^{m_2}, \dots, p_n^s \delta_{k_n}^{m_n}). \quad (12)$$

Under the conditions mentioned above the following black-brane-like solutions to the Hilbert-Einstein equations were obtained³⁷

$$g = J_0 \left(\frac{dR^2}{1 - 2\mu/R^d} + R^2 g^0 \right) - J_1 \left(1 - \frac{2\mu}{R^d} \right) dt^2 + \sum_{i=2}^n J_i g^i, \quad (13)$$

$$\rho^s = -\frac{A_s}{J_0 R^{2d_0}} \prod_{l=1}^m H_l^{-A_{sl}}, \quad (14)$$

which may be derived by analogy with the black brane solutions.^{27,28}

$$J_i = \prod_{s=1}^m H_s^{-2h_s U^{si}}, \quad (15)$$

$i = 0, 1, \dots, n$, $\mu > 0$ is integration constant and $U^{si} = G^{ij} U_j^s$, $h_s = K_s^{-1}$, $K_s = (U^s, U^s)$.

Functions $H_s = H_s(z) > 0$, $z = 2\mu/R^d \in (0, 1)$ obey the equations (8) equipped with the boundary conditions: i) $H_s(1 - 0) = H_{s0} \in (0, +\infty)$, ii) $H_s(+0) = 1$, $s = 1, \dots, m$. A special orthogonal case when $(U^s, U^l) = 0$, for $s \neq l$, was considered earlier in Ref. 31 (for $m = 1$ see Ref. 30) More general solutions in orthogonal case (with more general condition instead of 2^o) were obtained in Ref. 32.

For the Hawking temperature corresponding to the solution (13) we get:

$$T_H = \frac{d}{4\pi(2\mu)^{1/d}} \prod_{s=1}^m H_{s0}^{-h_s}. \quad (16)$$

5.3. Post-Newtonian approximation

Let $d_0 = 2$. Here we consider the 4-dimensional section of the metric (13), in the range $R > 2\mu$. One could imagine that some astrophysical objects may be described by the 4-dimensional physical metric. For post-Newtonian approximation we restrict ourselves by the first two powers of $1/R$, i. e. $H_s = 1 + \frac{P_s}{R} + \frac{P_s^{(2)}}{R^2} + o(1/R^2)$, for $R \rightarrow +\infty$, $s \in S$. Introducing a new radial variable ρ by the relation $R = \rho(1 + (\mu/2\rho))^2$, ($\rho > \mu/2$), the 4-dimensional section of the metric (13) in the 3-dimensional conformally-flat form

$$g^{(4)} = J_0 \left\{ -Y_1 \frac{(1 - (\mu/2\rho))^2}{(1 + (\mu/2\rho))^2} dt \otimes dt + \left(1 + \frac{\mu}{2\rho} \right)^4 \delta_{ij} dx^i \otimes dx^j \right\}, \quad (17)$$

where

$$J_0 = \prod_{s \in S} H_s^{-2h_s U^{s0}}, \quad Y_1 = \prod_{s \in S} H_s^{-2h_s} \quad (18)$$

and $\rho^2 = |x|^2 = \delta_{ij} x^i x^j$ ($i, j = 1, 2, 3$).

For possible physical applications, one should calculate the post-Newtonian parameters β and γ (Eddington parameters) using the following standard relations $g_{00}^{(4)} = -(1 - 2V + 2\beta V^2) + O(V^3)$, $g_{ij}^{(4)} = \delta_{ij}(1 + 2\gamma V) + O(V^2)$, $i, j = 1, 2, 3$, where $V = GM/\rho$ is Newton's potential and M is the gravitational mass. Then, we deduce the formulas $GM = \mu + \sum_{s \in S} h_s P_s (1 + U^{s0})$ and

$$\beta - 1 = \frac{1}{2(GM)^2} \sum_{s \in S} h_s (P_s^2 + 2\mu P_s - 2P_s^{(2)})(1 + U^{s0}), \quad (19)$$

$$\gamma - 1 = -\frac{1}{GM} \sum_{s \in S} h_s P_s (1 + 2U^{s0}). \quad (20)$$

6. Conclusions

Main current problems of physics, gravitation and cosmology in particular were analyzed. Special attention was paid to results of the theory with extra dimensions and variations of fundamental physical constants. Here, we have presented a family of spherically symmetric solutions with horizon in the models: i) with scalar field and fields of forms, ii) with multi-component anisotropic fluid. The metric of any solution contains $(n - 1)$ Ricci-flat "internal" space metrics. As in Ref. 26-28 the solutions are defined up to solutions of non-linear differential equations (equivalent to Toda-like ones) with certain boundary conditions imposed. For certain equations of state (with $p_i = \pm \rho$) the metric of the solution coincides with the metric of intersecting black branes (in a model with antisymmetric forms without dilatons). In fluid-matter case we have calculated the post-Newtonian parameters β and γ corresponding to the 4-dimensional section of the metric (for $d_0 = 2$).

Acknowledgments

This work was supported in part by Russian Foundation for Basic Research (Grant Nr. 09-02-00677-a) and by the FTsP "Nauchnie i nauchno-pedagogicheskie kadry innovatsionnoy Rossii" for 2009-2013.

References

1. V. N. Melnikov, in *2000 NASA/JPL Conference on Fundamental Physics in Micro-gravity*, NASA Document D-21522, 2001, p. 4.1.
2. V. N. Melnikov, in *Albert Einstein Century Int. Conf.*, eds. J.-M. Alimi and A. Fuzfa, (AIP Conf. Proc. 861, AIP, NY, 2006), p. 109.
3. A. G. Riess et al., *Astron. J.* **116**, 1009 (1998).

4. S. Perlmutter et al., *Astroph. J.* **517**, 565 (1999).
5. V. N. Melnikov, *Grav. Cosmol.* **13**, 81 (2007).
6. V. N. Melnikov, *Multidimensional Classical and Quantum Cosmology and Gravitation. Exact Solutions and Variations of Constants* (Rio de Janeiro, CBPF-NF-051/93, 1993); in *Cosmology and Gravitation*, ed. M. Novello (Editions Frontieres, Singapore, 1994), p. 147.
7. V. N. Melnikov, *Multidimensional Cosmology and Gravitation*, Rio de Janeiro, CBPF-MO-002/95, 1995; in *Cosmology and Gravitation. II*, ed. M. Novello (Editions Frontieres, Singapore, 1996), p. 465.
8. V. N. Melnikov, *Exact Solutions in Multidimensional Gravity and Cosmology III* (Rio de Janeiro, CBPF-MO-03/02, 2002).
9. K. P. Staniukovich and V. N. Melnikov, *Hydrodynamics, Fields and Constants in the Theory of Gravitation* (Moscow, Energoatomizdat, 1983) (in Russian); V. N. Melnikov, *Fields and Constants in the Theory of Gravitation* (Rio de Janeiro, CBPF MO-02/02, 2002).
10. V. R. Gavrilov and V. N. Melnikov, *Teor. Mat. Fiz.* **114**, 454 (1998).
11. U. Bleyer, V. D. Ivashchuk, V. N. Melnikov and A. I. Zhuk, *Nucl. Phys. B* **429**, 77 (1994).
12. V. R. Gavrilov, V. D. Ivashchuk and V. N. Melnikov, *Class. Quant. Grav.* **13**, 3039 (1996).
13. V. D. Ivashchuk, V. N. Melnikov and A. I. Zhuk, *Nuovo Cimento B* **104**, 575 (1989).
14. H. Dehnen, V. R. Gavrilov and V. N. Melnikov, *Gravit. Cosmol.* **9**, 189 (2003).
15. I. M. Mills et al., *Metrologia* **43**, 227 (2006).
16. K. A. Bronnikov and V. N. Melnikov, *Izmerit. Technika* **11**, 3 (2010).
17. V. N. Melnikov, in: *Gravitational Measurements, Fundamental Metrology and Constants*, eds. V. de Sabbata and V.N. Melnikov (Kluwer Acad. Publ., Dordrecht, 1988), p. 283.
18. J. Webb et al., *Phys. Rev. Lett.* **87**, 091301 (2001).
19. K. A. Bronnikov, V. N. Melnikov and K. P. Staniukovich. ITP-68-69 (Kiev, 1968).
20. K. A. Bronnikov, V. N. Melnikov and M. Novello. *Gravit. Cosmol., Suppl. II* **8**, 18 (2002).
21. V. de Sabbata, V. N. Melnikov and P. I. Pronin, *Prog. Theor. Phys.* **88**, 623 (1992).
22. N. I. Kolosnitsyn and V. N. Melnikov. *GRG* **36**, 1619 (2004).
23. K. A. Bronnikov, V. D. Ivashchuk and V. N. Melnikov *Grav. Cosmol.* **3**, 203 (1997).
24. V. D. Ivashchuk, V. N. Melnikov, *J. Math. Phys.* **39**, 2866 (1998).
25. V. D. Ivashchuk and V. N. Melnikov, in *Lecture Notes in Physics*, Vol. 537, eds. S. Cotsakis and G.W. Gibbons (Berlin, Springer, 2000), p. 214.
26. V. D. Ivashchuk and V. N. Melnikov, *Grav. Cosmol.* **5**, 313 (1999).
27. V. D. Ivashchuk and V. N. Melnikov, *Grav. Cosmol.* **6**, 27 (2000).
28. V. D. Ivashchuk and V. N. Melnikov, *Class. Quantum Grav.* **17**, 2073 (2000).
29. V. D. Ivashchuk and V. N. Melnikov, *Class. Quantum Grav.* **18**, R87 (2001).
30. V. D. Ivashchuk, V. N. Melnikov and A. B. Selivanov, *Grav. Cosmol.* **7**, 308 (2001).
31. V. D. Ivashchuk, V. N. Melnikov and A. B. Selivanov, *Grav. Cosmol.* **9**, 50 (2003).
32. H. Dehnen and V. D. Ivashchuk, *J. Math. Phys.* **45**, 4726 (2004).
33. N. Ohta, *Phys. Lett. B* **403**, 218 (1997).
34. V. D. Ivashchuk and S.-W. Kim, *J. Math. Phys.* **41**, 444 (2000).
35. V. D. Ivashchuk and V. N. Melnikov, *Int. J. Mod. Phys. D* **3**, 795 (1994).
36. V. R. Gavrilov, V. D. Ivashchuk and V. N. Melnikov, *J. Math. Phys.* **36**, 5829 (1995).
37. V. D. Ivashchuk, *Phys. Lett. B* **693**, 399 (2010).