DARK ENERGY AND EMERGENT SPACETIME

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A natural geometric framework of noncommutative spacetime is symplectic geometry rather than Riemannian geometry. The Darboux theorem in symplectic geometry then admits a novel form of the equivalence principle such that the electromagnetism in noncommutative spacetime can be regarded as a theory of gravity. Remarkably the emergent gravity reveals a noble picture about the origin of spacetime, dubbed as emergent spacetime, which is radically different from any previous physical theory all of which describe what happens in a given spacetime. In particular, the emergent gravity naturally explains the dynamical origin of flat spacetime, which is absent in Einstein gravity: A flat spacetime is not free gratis but a result of Planck energy condensation in a vacuum. This emergent spacetime picture, if it is correct anyway, turns out to be essential to resolve the cosmological constant problem, to understand the nature of dark energy and to explain why gravity is so weak compared to other forces.

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1. Einstein Gravity and The Cosmological Constant Problem

In general relativity, gravitation arises out of the dynamics of spacetime being curved by the presence of stress-energy and the equations of motion for the metric fields of spacetime are determined by the distribution of matter and energy:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu}. \quad (1)$$

The Einstein equations (1) describe how the geometry of spacetime on the left-hand side (LHS) is determined dynamically, at first sight, in harmony with matter fields on the right-hand side (RHS).

However there is a deep conflict between the spacetime geometry described by general relativity and matter fields described by quantum field theory (QFT). If spacetime is flat, i.e., $g_{\mu\nu} = \eta_{\mu\nu}$, the LHS of Eq. (1) identically vanishes and so the energy-momentum tensor of matter fields should vanish, i.e., $T_{\mu\nu} = 0$. In other words, a flat spacetime is free gratis, i.e., costs no energy. But the concept of empty space in Einstein gravity is in an acute contrast to the concept of vacuum in QFT where the vacuum is not empty but full of quantum fluctuations. As a result, the vacuum is extremely heavy whose weight is roughly of Planck mass, i.e., $\rho_{\text{vac}} \sim M_P^4$.

The conflict rises to the surface that gravity and matters respond differently to the vacuum energy and perplexingly brings about the notorious cosmological constant (CC) problem. Indeed the clash manifests itself as a mismatch of symmetry between gravity and matters.¹ To be precise, if we shift a matter Lagrangian $\mathcal{L}_M$ by a constant Λ , that is,

$$\mathcal{L}_M \rightarrow \mathcal{L}_M - 2\Lambda, \quad (2)$$

it results in the shift of the matter energy-momentum tensor by $T_{\mu\nu} \rightarrow T_{\mu\nu} - \Lambda g_{\mu\nu}$ in the Einstein equation (1) although the equations of motion for matters are invariant under the shift (2). Definitely the Λ -term in Eq. (2) will appear as the CC in Einstein gravity and it affects the spacetime structure. For example, a flat spacetime is no longer a solution of Eq. (1).

Let us sharpen the problem arising from the conflict between the geometry and matters. In QFT there is no way to suppress quantum fluctuations in a vacuum. Fortunately the vacuum energy due to the quantum fluctuations, regardless of how large it is, does not make any trouble to QFT thanks to the symmetry (2). However the general covariance requires that gravity couples universally to all kinds of energy. Therefore the vacuum energy $\rho_{\text{vac}} \sim M_P^4$ will induce a highly curved spacetime whose curvature scale R would be $\sim M_P^2$ according to Eq. (1). If so, the QFT framework in the background of quantum fluctuations must be broken down due to a large back-reaction of background spacetime. But we know that it is not the case. The QFT is well-defined in the presence of the vacuum energy $\rho_{\text{vac}} \sim M_P^4$ and the background spacetime still remains flat. So far there is no experimental evidence for the vacuum energy to really couple to gravity^a while it is believed that the vacuum energy is real as experimentally verified by the Casimir effect.

Which side of Eq. (1) is the culprit giving rise to the incompatibility? After consolidating all the suspicions inferred above, we throw a doubt on the LHS of Eq. (1), especially, on the result that a flat spacetime is free gratis, i.e., costs no energy. It would be remarked that such a result is not compatible with the inflation scenario either which shows that a huge vacuum energy in a highly nonequilibrium state is required to generate an extremely large spacetime. Note that Einstein gravity is not completely background independent since it assumes the prior existence of a spacetime manifold.^b In particular, the flat spacetime is a geometry of special relativity rather than general relativity and so it is assumed to be *a priori* given without reference to its dynamical origin. This reasoning implies that the negligence about the dynamical origin of flat spacetime defining a local inertial frame in general relativity might be a core root of the incompatibility inherent in Eq. (1).

^aContrary to the Einstein gravity, the emergent gravity reveals a completely different picture:² A vacuum itself does not gravitate and only fluctuations around the vacuum generate gravitational fields. After some thought, one may then realize that the emergent gravity will tame the previous conflict between the geometry and matters. See Sec. 3.

^bHere we refer to a background independent theory where any spacetime structure is not *a priori* assumed but defined from the theory.

All in all, it is tempted to infer that a flat spacetime may be not free gratis but a result of Planck energy condensation in a vacuum. Surprisingly, if that inference is true, it appears as the Hóly Gráil to cure several notorious problems in theoretical physics; for example, to resolve the CC problem, to understand the nature of dark energy and to explain why gravity is so weak compared to other forces. After all, the problem is to formulate a physically viable theory (i.e., a background independent theory) to correctly explain the dynamical origin of flat spacetime. It turns out^{2,3,4} that the emergent gravity from noncommutative (NC) geometry precisely realizes the desired property.

2. Noncommutative Spacetime and Emergent Gravity

Consider the electromagnetism on a D-brane whose data are given by (M, g, B) where M is a smooth manifold equipped with a metric g and a symplectic structure B . The dynamics of $U(1)$ gauge fields on the D-brane is described by open string field theory whose low energy effective action is given by DBI action. The DBI action predicts two important results: (I) The triple (M, g, B) comes only into the combination $(M, g + \kappa B)$, which embodies a generalized geometry continuously interpolating between symplectic geometry ($|\kappa B g^{-1}| \gg 1$) and Riemannian geometry ($|\kappa B g^{-1}| \ll 1$). (II) The electromagnetic force $F = dA$ appears only as the deformation of symplectic structure $\Omega(x) = (B + F)(x)$. Including the $U(1)$ gauge fields, the data of ‘D-manifold’ are given by $(M, g, \Omega) = (M, g + \kappa \Omega)$.

Consider an another D-brane whose ‘D-manifold’ is described by different data $(N, G, B) = (N, G + \kappa B)$. A question is whether there exist a diffeomorphism $\phi : N \rightarrow M$ such that $\phi^*(g + \kappa \Omega) = G + \kappa B$ on N . In order to answer the question, note that a D-brane whose worldvolume M supports a symplectic structure B respects the so-called Λ -symmetry in addition to $\text{Diff}(M)$, defined by

$$(B, A) \rightarrow (B - d\Lambda, A + \Lambda), \quad (3)$$

where the gauge parameter Λ is a one-form on M . Consider the symmetry transformation which is a combination of the Λ -transformation (3) followed by a diffeomorphism $\phi : N \rightarrow M$. The action transforms the “DBI metric” $g + \kappa B$ on M according to $g + \kappa B \rightarrow \phi^*(g + \kappa \Omega)$. The crux is that there “always” exists a diffeomorphism $\phi : N \rightarrow M$ such that $\phi^*(\Omega) = B$, which is precisely the Darboux theorem in symplectic geometry. Then we arrive at a remarkable fact that two different DBI metrics $g + \kappa \Omega$ and $G + \kappa B$ are diffeomorphic each other, i.e., $\phi^*(g + \kappa \Omega) = G + \kappa B$, where $G = \phi^*(g)$. Note that this property holds for any pair (g, B) of Riemannian metric g and symplectic structure B .

Since the symplectic structure B is nondegenerate at any point $y \in M$, we can invert this map to obtain the map $\theta \equiv B^{-1} : T^*M \rightarrow TM$. This cosymplectic structure $\theta \in \wedge^2 TM$ is called the Poisson structure of M which defines a Poisson bracket $\{\cdot, \cdot\}_\theta$. A NC spacetime then arises from quantizing the symplectic manifold

(M, B) with its Poisson structure, treating it as a quantum phase space, i.e.,

$$\langle B_{ab} \rangle_{\text{vac}} = (\theta^{-1})_{ab} \Leftrightarrow [y^a, y^b]_{\star} = i\theta^{ab}. \quad (4)$$

The above argument reveals a superb physics in NC spacetime. There “always” exists a coordinate transformation to locally eliminate the electromagnetic force $F = dA$ as long as a manifold M supports a symplectic structure B , i.e., M becomes a NC space. That is, the NC spacetime admits a novel form of the equivalence principle, known as the Darboux theorem, for the geometrization of the electromagnetism. Since it is always possible to find a coordinate transformation $\phi \in \text{Diff}(M)$ such that $\phi^*(B+F) = B$, the relationship $\phi^*(g+\kappa(B+F)) = G+\kappa B$ clearly shows that the electromagnetic fields in the DBI metric $g+\kappa(B+F)$ now appear as a new metric $G = \phi^*(g)$. One may also consider the inverse relationship $\phi_*(G+\kappa B) = g+\kappa(B+F)$ which implies that a nontrivial metric G in a vacuum (4) can be interpreted as an inhomogeneous condensation of gauge fields on a ‘D-manifold’ with metric g . It might be pointed out that the relationship in the case of $\kappa = 2\pi\alpha' = 0$ is the familiar diffeomorphism in Riemannian geometry and so it says nothing marvelous.

We observed that the Darboux theorem for symplectic structures immediately leads to the diffeomorphism between two different DBI metrics. In terms of local coordinates $\phi : y \mapsto x = x(y)$, the diffeomorphism then reads as²

$$(g + \kappa\Omega)_{\alpha\beta}(x) = \frac{\partial y^a}{\partial x^\alpha} \left(G_{ab}(y) + \kappa B_{ab}(y) \right) \frac{\partial y^b}{\partial x^\beta} \quad (5)$$

where $\Omega = B + F$ and

$$G_{ab}(y) = \frac{\partial x^\alpha}{\partial y^a} \frac{\partial x^\beta}{\partial y^b} g_{\alpha\beta}(x). \quad (6)$$

Eq. (5) conclusively shows how NC gauge fields manifest themselves as a spacetime geometry. To expose the intrinsic connection between NC gauge fields and spacetime geometry, let us represent the coordinate transformation in Eq. (5) as

$$x^a(y) = y^a + \theta^{ab} \hat{A}_b(y). \quad (7)$$

Note that $F(x) = 0$ or equivalently $\hat{A}_a(y) = 0$ corresponds to $G_{ab} = g_{ab}$ as it should be. Clearly Eq. (6) embodies how the metric on M is deformed by the presence of NC gauge fields.⁴

Now we will elucidate how the Darboux theorem in symplectic geometry materializes as a novel form of the equivalence principle such that the electromagnetism in NC spacetime can be regarded as a theory of gravity. An important observation is that, for a given Poisson algebra $(C^\infty(M), \{\cdot, \cdot\}_\theta)$, there exists a natural map $C^\infty(M) \rightarrow TM : f \mapsto X_f$ between smooth functions in $C^\infty(M)$ and vector fields in TM such that

$$X_f(g) = \{g, f\}_\theta \quad (8)$$

for any $g \in C^\infty(M)$. Actually the 1–1 correspondence between $C^\infty(M)$ and $\Gamma(TM)$, i.e., vector fields in TM , is the Lie algebra homomorphism in the sense that

$$X_{\{f,g\}_\theta} = -[X_f, X_g]. \quad (9)$$

The covariant and background independent coordinates $x^a(y) \in C^\infty(M)$ in Eq. (7) are smooth functions in M and so they can be mapped to vector fields in TM according to Eq. (8). Let us denote the vector fields as $V_a \in \Gamma(TM)$ and their dual vector space as $D^a \in \Gamma(T^*M)$. It can be shown² that the vector fields V_a and the one-forms D^a are related to the orthonormal frames (vielbeins) $E_a \in \Gamma(TM)$ and $E^a \in \Gamma(T^*M)$ by $V_a = \lambda E_a$ and $E^a = \lambda D^a$, respectively, where $\lambda^2 = \det V_a^b$. Therefore we see that the Darboux theorem in symplectic geometry implements a deep principle to realize a Riemannian manifold as an emergent geometry from NC gauge fields (7) through the correspondence (8) whose metric is given by⁵

$$ds^2 = g_{ab} E^a \otimes E^b = \lambda^2 g_{ab} D_\mu^a D_\nu^b dy^\mu \otimes dy^\nu. \quad (10)$$

Let us point out that the emergent gravity can be generalized to full NC gauge fields in two different ways which are dual to each other. The first notable point is that the correspondence (8) between a Poisson algebra $(C^\infty(M), \{\cdot, \cdot\}_\theta)$ and vector fields in $\Gamma(TM)$ can be generalized to a NC C^* -algebra $(\mathcal{A}_\theta, [\cdot, \cdot]_\star)$ by considering an adjoint action of NC gauge fields $\hat{D}_a(y) \in \mathcal{A}_\theta$ as follows^{2,3,4}

$$ad_{\hat{D}_a}[\hat{f}](y) \equiv -i[\hat{D}_a(y), \hat{f}(y)]_\star = V_a^\mu(y) \frac{\partial f(y)}{\partial y^\mu} + \mathcal{O}(\theta^3) \quad (11)$$

where the leading term exactly recovers the vector fields in Eq. (8). The second point is that every NC space can be represented as a theory of operators in a Hilbert space $\mathcal{H}$, which consists of NC C^* -algebra $\mathcal{A}_\theta$, and any operator in $\mathcal{A}_\theta$ or any NC field can be represented as a matrix whose size is determined by the dimension of $\mathcal{H}$. For the Moyal NC space as an example of Eq. (4), one gets $N \times N$ matrices in the $N \rightarrow \infty$ limit. In this sense, the emergent geometry arising from the vector fields in the NC space (4) can be understood as a dual geometry of large N matrices in $\mathcal{H}$ according to the large N duality or AdS/CFT correspondence.^{2,5}

3. Emergent Spacetime and Dark Energy

We are ready to clarify how the emergent gravity outlined in the previous section reveals a noble picture about the origin of spacetime, dubbed as emergent spacetime, which is radically different from any previous physical theory all of which describe what happens in a given spacetime. We will consider the Moyal NC space in Eq. (4), i.e., $M = \mathbb{R}^{2n}$ with $B = \text{constant}$, for simplicity.^c In this case, the background NC space (4), where every physical processes take place, is described by $x^a(y) = y^a$ in Eq. (7). The corresponding vector fields are given by $V_a = \frac{\partial}{\partial y^a}$ according to Eq. (8) or Eq. (11) and so the metric for the background (4) is flat, i.e., $g_{ab} = \delta_{ab}$.

^cSee Ref. 2 for a general NC spacetime. Interestingly the emergent gravity based on NC geometry endows a natural concept of “emergent time” since a symplectic manifold (M, B) always admits a Hamiltonian dynamical system on M defined by a Hamiltonian vector field X_H , i.e., $\iota_{X_H} B = dH$, described by $\frac{df}{dt} = X_H(f) = \{f, H\}_\theta$ for any $f \in C^\infty(M)$. In a general case where the symplectic structure is changing along the dynamical flow as in Eq. (5), the dynamical evolution must be locally defined on every local Darboux chart as expected on a general ground.

We will take the picture prescribed in the footnote c for emergent time. Then one can see that the background (4) gives rise to a flat spacetime, i.e., $g_{\mu\nu} = \eta_{\mu\nu}$.² A tangible difference from Einstein gravity arises at this point. The flat spacetime is not coming from an empty space but emerges from a uniform condensation of gauge fields in a vacuum (4). Note that the uniform condensation (4) should appear as the CC in Einstein gravity and so a flat spacetime is not allowed.

Since gravity emerges from NC gauge fields, the parameters, g_{YM}^2 and $|\theta|$, defining, e.g., 4-dimensional NC gauge theory should be related to the Newton constant G in emergent gravity. A simple dimensional analysis shows that $\frac{G\hbar^2}{c^2} \sim g_{YM}^2|\theta|$. This relation immediately leads to the fact that the energy density of the vacuum (4) is $\rho_{\text{vac}} \sim |B_{ab}|^2 \sim M_P^4$ where $M_P = (8\pi G)^{-1/2} \sim 10^{18} \text{ GeV}$ is the Planck mass. Therefore the emergent gravity finally reveals a remarkable picture that the huge Planck energy M_P is actually the origin of a flat spacetime. In other words, a flat spacetime is not free gratis but a result of Planck energy condensation in a vacuum. Hence a vacuum energy does not gravitate unlike as Einstein gravity.⁶

If gauge field fluctuations are turned on, i.e., $F \neq 0$, the spacetime metric will no longer be flat as can be seen from Eq. (10). However, the flat spacetime should be very robust against any perturbations since the vacuum (4) was triggered by the Planck energy condensation, the maximum energy in Nature. Then the gravity generated by the deformations of the background (4) will be very weak since the spacetime vacuum is very solid with a stiffness of the Planck scale. So the dynamical origin of flat spacetime is intimately related to the weakness of gravitational force.

Recall that the Planck energy condensation in the vacuum (4) causes the spacetime to be NC and the NC spacetime is the essence of emergent gravity. The UV/IR mixing in the NC spacetime then implies that any UV fluctuations of the Planck scale L_P will be necessarily paired with IR fluctuations of a typical scale L_H . These vacuum fluctuations around the flat spacetime will add a tiny energy $\delta\rho$ to the vacuum so that the total energy density is equal to $\rho \sim M_P^4 + \delta\rho$. A simple dimensional analysis and a symmetry consideration, e.g., the cosmological principle, lead to the estimation of the vacuum fluctuation as $\rho \sim M_P^4(1 + \frac{L_P^2}{L_H^2})$.¹ Since the first term in ρ does not gravitate, the second term $\delta\rho \sim \frac{1}{L_P^2 L_H^2}$ will be a leading contribution to the deformation of spacetime curvature, leading to possibly a de Sitter phase. Interestingly this energy of vacuum fluctuations, $\delta\rho \sim \frac{1}{L_P^2 L_H^2}$, is in good agreement with the observed value of current dark energy.^{1,6}

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