MEASURING PARTICLE MASSES WITH m_{T2} AT THE LHC

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We review some recent works on m_{T2} collider variable, which is applied for pair-produced mother particles decaying to visible particles and a pair of invisible particles at the LHC.

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1. Introduction

The Large Hadron Collider (LHC) is about to explore TeV energy scale, where new physics beyond the Standard Model (SM) likely reveals itself.^{1,2} Among various new physics proposals, weak scale supersymmetry (SUSY)³ is perhaps the most promising candidate, which provides a solution for gauge hierarchy problem and complies with gauge coupling unification. Furthermore, with R-parity conservation, the lightest supersymmetric particle (LSP) becomes a natural candidate for the non-baryonic dark matter in the Universe.

Once SUSY signals are discovered through event excess beyond the SM backgrounds in inclusive search channels, the next step will be the measurements of SUSY particle masses and their physical properties in various exclusive decay chains. It might be then possible to reconstruct SUSY theory, in particular SUSY breaking mechanism from the experimental information on SUSY particle properties.

In this paper, we review some recent works on mass determination with an event variable m_{T2} ⁴ at the LHC. The event variable m_{T2} is applied to the process, where massive particles are pair produced in hadron collider and each massive particle subsequently decays to some visible particles and one LSP.

In Sec. 2, we show that the gluino m_{T2} can be utilized to measure the gluino mass and the lightest neutralino mass separately, thereby providing a good first look at the pattern of sparticle masses experimentally.^{5,6}

We investigate the possibility to measure the top quark mass using m_{T2} at the LHC experiment, in Sec. 3. Monte Carlo studies of m_{T2} are performed with the events corresponding to the dilepton decays of $t\bar{t}$ produced at the LHC with 10 fb^{-1} integrated luminosity. Our analysis suggests that the top quark mass can be determined by the m_{T2} variable with a reasonable accuracy.⁷

2. Gluino m_{T2}

If light enough, gluinos would be pair produced copiously in proton-proton collision ($pp \rightarrow \tilde{g}\tilde{g}$), and each gluino decays into two quarks and one LSP ($\tilde{g} \rightarrow qq\tilde{\chi}_1^0$) through three-body decay induced by an exchange of off-shell squark or two body cascade decay with intermediate on-shell squark. For each gluino decay $\tilde{g} \rightarrow qq\tilde{\chi}_1^0$, the following transverse mass can be constructed

$$m_T^2(m_{qqT}, m_\chi, \mathbf{p}_T^{qq}, \mathbf{p}_T^\chi) = m_{qqT}^2 + m_\chi^2 + 2(E_T^{qq} E_T^\chi - \mathbf{p}_T^{qq} \cdot \mathbf{p}_T^\chi), \quad (1)$$

where m_{qqT} and $\mathbf{p}_T^{qq}$ are the transverse invariant mass and transverse momentum of the qq system, respectively, while m_χ and $\mathbf{p}_T^\chi$ are the *trial* mass and transverse momentum of the LSP, respectively. Here transverse momenta are measured in laboratory frame with respect to the proton beam direction.

The transverse energies of the qq system and LSP are defined as $E_T^{qq} \equiv \sqrt{|\mathbf{p}_T^{qq}|^2 + m_{qqT}^2}$ and $E_T^\chi \equiv \sqrt{|\mathbf{p}_T^\chi|^2 + m_\chi^2}$. With two such gluino decays in each event, the gluino stransverse mass m_{T2} is defined as

$$m_{T2} \equiv \min_{\mathbf{p}_T^{\chi(1)} + \mathbf{p}_T^{\chi(2)} = \mathbf{p}_T^{miss}} \left[\max\{m_T^{(1)}, m_T^{(2)}\} \right], \quad (2)$$

with the minimization over all possible splitting of the observed missing transverse energy $\mathbf{p}_T^{miss}$ into two assumed transverse momenta, $\mathbf{p}_T^{\chi(1)}$ and $\mathbf{p}_T^{\chi(2)}$.

From Eq. (2), one finds

$$m_{T2} \leq m_{\tilde{g}} \quad \text{for} \quad m_\chi = m_{\tilde{\chi}_1^0}, \quad (3)$$

where m_χ and $m_{\tilde{\chi}_1^0}$ denote the *trial* LSP mass and the *true* LSP mass, respectively. Therefore, if $m_{\tilde{\chi}_1^0}$ is known, one can determine the gluino mass $m_{\tilde{g}}$ from the endpoint measurement of m_{T2} distribution:

$$m_{T2}^{\max} \equiv \max_{\text{all events}} [m_{T2}]. \quad (4)$$

However $m_{\tilde{\chi}_1^0}$ might not be known in advance, and then $m_{T2}^{\max}(m_\chi)$ can be considered as a function of the trial LSP mass m_χ , satisfying $m_{T2}^{\max}(m_\chi = m_{\tilde{\chi}_1^0}) = m_{\tilde{g}}$. One can show that the $m_{T2}^{\max}$ for $m_\chi \leq m_{\tilde{\chi}_1^0}$ is given by

$$m_{T2}^{\max}(m_\chi) = \frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}} + \sqrt{\left(\frac{m_{\tilde{g}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{g}}}\right)^2 + m_\chi^2} \quad \text{for} \quad m_\chi \leq m_{\tilde{\chi}_1^0}. \quad (5)$$

Note that $m_{T2}^{\max}(m_\chi = m_{\tilde{\chi}_1^0}) = m_{\tilde{g}}$ as required. On the other hand, $m_{T2}^{\max}$ for $m_\chi \geq m_{\tilde{\chi}_1^0}$ is obtained as, for gluino three-body decay,

$$m_{T2}^{\max}(m_\chi) = (m_{\tilde{g}} - m_{\tilde{\chi}_1^0}) + m_\chi \quad \text{for} \quad m_\chi \geq m_{\tilde{\chi}_1^0}, \quad (6)$$

which again gives $m_{T2}^{\max}(m_\chi = m_{\tilde{\chi}_1^0}) = m_{\tilde{g}}$.

The experimental feasibility of measuring $m_{\tilde{g}}$ and $m_{\tilde{\chi}_1^0}$ through $m_{T2}^{\max}$ has investigated by a Monte Carlo study, which indicates that $m_{\tilde{g}}$ and $m_{\tilde{\chi}_1^0}$ can be determined

rather accurately by the crossing behavior of $m_{T2}^{\max}$. As a specific example, we have examined a parameter point in the minimal anomaly mediated SUSY-breaking⁸ (mAMSB) scenario with heavy squarks, which gives

$$m_{\tilde{g}} = 780.3 \text{ GeV}, \quad m_{\tilde{\chi}_1^0} = 97.9 \text{ GeV},$$

and a few TeV masses for sfermions. We have generated a Monte Carlo sample of SUSY events for proton-proton collision at 14 TeV by PYTHIA.⁹ The event sample corresponds to 300 fb^{-1} integrated luminosity. We have also generated SM backgrounds such as $t\bar{t}$, $W/Z + \text{jet}$, $WW/WZ/ZZ$ and QCD events, with less equivalent luminosity. The generated events have been further processed with a modified version of fast detector simulation program PGS,¹⁰ which approximates an ATLAS or CMS-like detector with reasonable efficiencies and fake rates.

The following event selection cuts are applied to have a clean signal sample for gluino m_{T2} : at least 4 jets with $P_{T1,2,3,4} > 200, 150, 100, 50 \text{ GeV}$; missing transverse energy $E_T^{\text{miss}} > 250 \text{ GeV}$; transverse sphericity $S_T > 0.25$; no b-jets and no leptons. The four hardest jets are divided into two groups of dijets as follows. The highest momentum jet and the other jet which has the largest $|p_{\text{jet}}|\Delta R$ with respect to the leading jet are chosen as the two ‘seed’ jets for division. Here p_{jet} is the jet momentum and $\Delta R \equiv \sqrt{\Delta\phi^2 + \Delta\eta^2}$, i.e. a separation in azimuthal angle and pseudorapidity plane. Each of the remaining two jets is associated to a seed jet which makes the smallest opening angle. Then, each group of the dijets is considered to originate from the same gluino.

Fig. 1. The gluino m_{T2} distribution with $m_{\chi} = 90 \text{ GeV}$ for the benchmark point of mAMSB with heavy squarks.

Figure 1 shows the resulting distribution of m_{T2} for the trial LSP mass $m_{\chi} = 90 \text{ GeV}$. The blue histogram corresponds to the SM background. Fitting with a linear function with a linear background, we get the endpoint $778.0 \pm 2.3 \text{ GeV}$, which appears as the crossing point of two linear functions. The measured $m_{T2}^{\max}$ as a

function of m_χ is shown in Fig. 2. Fitting the data points with the curves (5) and (6), we obtain $m_{\tilde{g}} = 776.3 \pm 1.3$ GeV and $m_{\tilde{\chi}_1^0} = 97.3 \pm 1.7$ GeV, which are quite close to the true values, $m_{\tilde{g}} = 780.3$ GeV and $m_{\tilde{\chi}_1^0} = 97.9$ GeV. This demonstrates that the gluino m_{T2} can be very useful for measuring the gluino and LSP masses experimentally.

Fig. 2. $m_{T2}^{\max}$ as a function of the trial LSP mass m_χ for the benchmark point of mAMSB with heavy squarks.

3. Top Quark Mass with m_{T2}

When the Large Hadron Collider (LHC) is turned on, it will serve as a ‘top quark factory’.^{1,2} The cross section for $t\bar{t}$ pair production at the LHC is estimated to be 833 pb at the NLO calculation,¹¹ implying roughly 8 million $t\bar{t}$ pairs per year at low luminosity run ($10 \text{ fb}^{-1}/\text{year}$). Such a large number of $t\bar{t}$ events will enable us to measure the top quark mass with high precision.

Precision measurement of the top quark mass m_t is desirable in many respects. For example, it would help to constrain the allowed Higgs boson mass in the Standard Model (SM). In general, it would affect the constraints on the allowed parameter space of various models of new physics at the TeV scale, including the Minimal Supersymmetric Standard Model and technicolor-like models.

The top quark mass measurement can be performed through various methods in different channels, which have their own advantage/disadvantage with different systematic uncertainties. In the overall, the accuracy of m_t measured at the LHC is expected to be around 1 GeV.¹²

In the SM, top quark decays mostly into a b-quark and a W boson. The W boson then decays hadronically ($W \rightarrow qq'$) or leptonically ($W \rightarrow l\nu$). Depending on the W boson decay mode, the $t\bar{t}$ events are divided into three channels, *i.e.*, the dilepton channel (both W bosons decay leptonically), the lepton plus jets channel

(one W boson decays leptonically and the other hadronically) and the pure hadronic channel (both W bosons decay hadronically).

The dilepton channel has a small branching fraction compared to the lepton plus jets channel and the pure hadronic channel. It also involves two missing neutrinos, which makes a direct event-by-event measurement of m_t not possible. However, it has a cleaner environment, e.g. less combinatorial background and less jet energy scale dependence, compared to other channels, therefore various approaches for an indirect measurement of m_t with dilepton channel have been investigated.¹²

Let us consider a $t\bar{t}$ pair production and its subsequent decay at the LHC:

$$pp \rightarrow t\bar{t} \rightarrow bW^+ \bar{b}W^-. \quad (7)$$

In case that one of the W bosons decays into leptons, one can consider the associated transverse mass of $t \rightarrow bl\nu$, which is defined as

$$m_T^2 = m_{bl}^2 + m_\nu^2 + 2(E_T^{bl} E_T^\nu - \mathbf{p}_T^{bl} \cdot \mathbf{p}_T^\nu), \quad (8)$$

where m_{bl} and $\mathbf{p}_T^{bl}$ denote the invariant mass and transverse momentum of the bl system, respectively, while m_ν and $\mathbf{p}_T^\nu$ are the mass and transverse momentum of the missing neutrino, respectively. The transverse energies of the bl system and neutrino are defined as

$$E_T^{bl} \equiv \sqrt{|\mathbf{p}_T^{bl}|^2 + m_{bl}^2} \quad \text{and} \quad E_T^\nu \equiv \sqrt{|\mathbf{p}_T^\nu|^2 + m_\nu^2}. \quad (9)$$

A method to determine m_t without using the hadronic decay of W is to construct m_{T2} for the dilepton decay

$$t\bar{t} \equiv t^{(1)} t^{(2)} \rightarrow b^{(1)} l^{(1)} \nu^{(1)} b^{(2)} l^{(2)} \nu^{(2)}. \quad (10)$$

Although each neutrino momentum cannot be measured in this case, still the total missing transverse momentum $\mathbf{p}_T^{\text{miss}} = \mathbf{p}_T^{\nu(1)} + \mathbf{p}_T^{\nu(2)}$ can be determined experimentally. The m_{T2} variable of each event is defined as

$$m_{T2} \equiv \min_{\mathbf{p}_T^{\nu(1)} + \mathbf{p}_T^{\nu(2)} = \mathbf{p}_T^{\text{miss}}} \left[\max\{m_T^{(1)}, m_T^{(2)}\} \right], \quad (11)$$

where $m_T^{(i)}$ ($i = 1, 2$) is the transverse mass of $t^{(i)} \rightarrow b^{(i)} l^{(i)} \nu^{(i)}$, and the minimization is performed over the *trial* neutrino momenta $\mathbf{p}_T^{\nu(i)}$ constrained as

$$\mathbf{p}_T^{\nu(1)} + \mathbf{p}_T^{\nu(2)} = \mathbf{p}_T^{\text{miss}}. \quad (12)$$

The above definition of m_{T2} indicates that m_{T2} for $m_\nu = 0$ is bounded above by m_t in the approximation ignoring the decay width of top quark. One might then determine m_t as

$$\begin{aligned} m_t &= m_{T2}^{\text{max}}(m_\nu = 0) \\ &\equiv \max \left[m_{T2}(m_{bl}^{(1)}, \mathbf{p}_T^{bl(1)}, m_{bl}^{(2)}, \mathbf{p}_T^{bl(2)}, m_\nu = 0) \right]. \end{aligned} \quad (13)$$

In fact, because of nonzero decay width, there can be certain amount of events which give m_{T2} exceeding the physical top quark mass m_t . Our Monte Carlo study suggests

that such events do not spoil the sharp edge structure of the m_{T2} distribution with which one can determine m_t rather precisely.

Measuring the top mass using m_{T2} , when applied to real data, will suffer from a variety of uncertainty factors such as backgrounds, event selection cuts, finite jet energy resolution and combinatorial background. In order to check the feasibility of the m_{T2} method at the LHC, we have generated Monte Carlo samples of $t\bar{t}$ events by PYTHIA with the CTEQ5L parton distribution function (PDF). The event sample corresponds to 10 fb^{-1} integrated luminosity.

The generated events have been further processed with a modified version of fast detector simulation program PGS.

The dilepton events are selected by requiring (A) only two isolated leptons of opposite charge with $p_T > 25 \text{ GeV}$ and $|\eta| < 2.5$, (B) dilepton invariant mass with $|m_{ll} - m_Z| > 5 \text{ GeV}$, (C) large missing transverse energy $E_T^{\text{miss}} > 40 \text{ GeV}$, and (D) at least two b-tagged jets with $p_T > 30 \text{ GeV}$ and $|\eta| < 3.0$. After this selection, 5133 events are survived among the 5.5×10^6 generated $t\bar{t}$ events (in which 1.8×10^5 are the dilepton events, considering only electrons and muons), leading to a selection efficiency of about 2.8% for the dilepton channel signal events.

With two b-jets and two leptons in each selected event, there are two possible combinations for bl pairing. We calculated m_{T2} variable for each of the two possible bl combinations, and chose the smaller one as the final m_{T2} . This procedure closely follows the idea proposed in Ref. 13.

Perhaps the most reliable way to determine m_t using m_{T2} is to employ the template binned likelihood fit. For this, we attempted to fit the m_{T2} distribution of the ‘nominal data’ (which was generated with $m_t = 170.9 \text{ GeV}$) to ‘templates’. Here, a template means a simulated m_{T2} distribution with an input top quark mass different from 170.9 GeV. The templates were generated with input top quark mass between 166 GeV and 176 GeV, in steps of 1 or 0.5 GeV, using the same PYTHIA+PGS Monte Carlo programs as the case of nominal data sample.

Fig. 3. (Color online) Three representative m_{T2} distributions for the nominal data (points) and two templates with $m_t = 166 \text{ GeV}$ (blue solid) and $m_t = 176 \text{ GeV}$ (red solid), respectively.

Figure 3 shows three representative m_{T2} distributions for the nominal data (points) and two templates with $m_t = 166 \text{ GeV}$ (blue solid) and $m_t = 176 \text{ GeV}$

(red solid), respectively. Each template distribution is normalized to make the total number of events is same as that of the nominal data. One can notice that those three m_{T2} distributions are well separated from each other, showing the sensitivity of the m_{T2} distribution to the input top quark mass.

We fit the m_{T2} distribution of nominal data to templates in the range $100 \text{ GeV} < m_{T2} < 180 \text{ GeV}$. The result of the likelihood fit for m_{T2} distributions is shown in Fig. 4, where the negative logarithm of the likelihood ratio $\mathcal{L}/\mathcal{L}_{\max}$ as a function of m_t is depicted. The $\mathcal{L}_{\max}$ is the maximum likelihood which was determined as the minimum of a parabola fit to the $-\ln\mathcal{L}$ distribution. The top quark mass resulting from our template likelihood fit is given by

$$m_t = 170.3 \pm 0.3 \text{ GeV}, \quad (14)$$

which reproduces well the input top quark mass with a small statistical error. The above result does not include systematic error.

Fig. 4. The negative logarithm of the likelihood ratio $\mathcal{L}/\mathcal{L}_{\max}$ as a function of m_t for the m_{T2} fit.

4. Conclusion

We have examined the collider observable gluino m_{T2} applied to pair produced gluinos each of which is decaying to two quarks and one invisible neutralino LSP. Analytic expression of $m_{T2}^{\max}$ (= maximum of the gluino m_{T2} over all events) as a function of the *trial* LSP mass m_χ has been discussed with an observation that $m_{T2}^{\max}$ has a *kink structure*, i.e. a continuous but not differentiable cusp, at $m_\chi =$ true LSP mass, from which the gluino mass and the LSP mass can be determined simultaneously with good accuracy.

We also examined the possibility to determine the top quark mass using the m_{T2} distribution of the dileptonic decay channel of $t\bar{t}$ events at the LHC. Our analysis suggests that the top quark mass can be extracted by the m_{T2} variable

with a reasonable accuracy, although more complete analysis of systematic errors is required to estimate the the precision more accurately.

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