Acknowledgment of priority

Frobenius numbers of Pythagorean triples

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After we have published the formula

$$(m-1)(m^2-n^2) + (m-1)(2mn) - (m^2+n^2)$$

for Frobenius numbers of Pythagorean triples $(m^2-n^2, 2mn, m^2+n^2)$ in the *International Journal of Number Theory*, Professor Leonid G. Fel emailed us that he had also found the same formula in his article entitled “Frobenius problem for semi-groups $S(d_1, d_2, d_3)$ ” which was published in *Funct. Anal. Other Math.* **1** (2006) 119–157.

In his article, he gave a general algorithm to find the Frobenius numbers for three numbers. His method starts from finding $A = (a_{ij}) \in \text{Mat}_{3 \times 3}(\mathbb{Z})$ with $a_{ii} \geq 2$ such that $A(d_1 \ d_2 \ d_3)^T = \mathbf{0}$. For the Pythagorean triple, the matrix A can be constructed explicitly (Example 3 at pp. 149–150). That is,

$$\begin{pmatrix} k_1 + k_2 & -k_1 + k_2 & -k_1 + k_2 \\ -k_2 & k_1 & -k_2 \\ -k_1 & -k_2 & k_1 \end{pmatrix} \begin{pmatrix} k_1^2 - k_2^2 \\ 2k_1 k_2 \\ k_1^2 + k_2^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

Using this matrix, he obtained the same formula

$$F^{\text{Pth}}(\mathbf{d}^3) = k_1[k_1^2 - k_2^2 + 2(k_1 k_2 - k_1 - k_2)]$$

as ours.

Our method could not yield a general solution for arbitrary three numbers. It proves the formula for Pythagorean triples directly.

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