Journal of Nonlinear Mathematical Physics, Vol. 19, No. 4 (2012) 1292001 (3 pages)

© R. Boll

DOI: 10.1142/S1402925112920015

CORRIGENDUM

CLASSIFICATION OF 3D CONSISTENT QUAD-EQUATIONS

[J. Nonlinear Math. Phys., Vol. 18, No. 3 (2011) 337–365]

RAPHAEL BOLL

*Institut für Mathematik, MA 7-2, Technische Universität Berlin
Str. des 17. Juni 136 10623 Berlin, Germany
boll@math.tu-berlin.de*

Received 21 August 2012

Accepted 21 August 2012

Published 31 December 2012

Unfortunately, the list of type H^6 equations given in Sec. 2.2 on page 340 of the paper is incomplete: one quad-equation is missing and in another equation, there is one summand missing. The correct list is as follows:

Every quad-equation of type H^6 is equivalent modulo $(\text{Möb})^4$ to one of the following quad-equations characterized by the quadruples of discriminants:

- $(0, 0, 0, 0)$:

$$Q = x_1 + x_2 + x_3 + x_4$$

- $(1, 0, 1, \delta^2)$:

$$Q = x_1 + x_3 + x_2(x_4 + \delta x_1)$$

- $(1, 4x_2, 1, 1)$:

$$Q = x_2 + x_1x_3 + x_1x_4 + x_3x_4$$

- $(x_1^2, x_2^2 + 4\delta_1\delta_2\delta_3, x_3^2, x_4^2)$:

$$Q = x_1x_3 + x_2x_4 + \delta_1x_2x_3 + \delta_2x_3x_4 + \delta_3$$

As a consequence, in Theorems 3.9–3.11 several 3D consistent systems have to be replaced by new ones. In Theorem 3.9 on page 357 the systems (3.19)–(3.20) and

(3.21)–(3.22) can be unified into the following system:

- $(x^2 + 4\delta_1\delta_2\delta_3, x_1^2, x_{12}^2, x_2^2)$:

$$\begin{aligned} A(x, x_1, x_2, x_{12}; \delta_1, \delta_2) &= xx_{12} + x_1x_2 + \delta_1x_1x_{12} + \delta_2x_2x_{12} + \delta_3, \\ B(x, x_2, x_3, x_{23}; \delta_1, \delta_2, \alpha) &= \alpha(xx_2 + x_3x_{23}) - (xx_{23} + x_2x_3) \\ &\quad + (\alpha^2 - 1) \left(\delta_2x_2x_{23} - \frac{\delta_1\delta_3}{\alpha} \right), \end{aligned} \quad (3.19)$$

$$\begin{aligned} \bar{A} &= A(x_3, x_{13}, x_{23}, x_{123}; \delta_1, \delta_2), & \bar{B} &= B(x_1, x_{12}, x_{13}, x_{123}; 0, \delta_2, \alpha), \\ C &= B(x, x_1, x_3, x_{13}; \delta_2, \delta_1, \alpha^{-1}), & \bar{C} &= B(x_2, x_{12}, x_{23}, x_{123}; 0, \delta_1, \alpha^{-1}), \\ K &= A(x, x_{13}, x_{23}, x_{12}; \delta_1\alpha^{-1}, \delta_2\alpha), & \bar{K} &= A(x_3, x_1, x_2, x_{123}; \delta_1\alpha^{-1}, \delta_2\alpha), \end{aligned} \quad (3.20)$$

and there is one new system to be listed in Theorem 3.9:

- $(4x, 1, 1, 1)$:

$$\begin{aligned} B(x, x_2, x_3, x_{23}; \alpha) &= (x - x_3)(x_2 - x_{23}) + \alpha(x + x_3 - 2x_2x_{23}) \\ &\quad - \alpha^2(x_2 + x_{23}) - \alpha^3, \\ \bar{B}(x_1, x_{12}, x_{13}, x_{123}; \alpha) &= (x_1 - x_{13})(x_{12} - x_{123}) - \alpha(x_1 + x_{12} + x_{13} + x_{123}) + \alpha^2, \\ K(x, x_{13}, x_{23}, x_{12}; \alpha) &= x + x_{13}x_{23} + x_{13}x_{12} + x_{23}x_{12} + \alpha(x_{13} - x_{23}) - \alpha^2, \end{aligned} \quad (3.21)$$

$$\begin{aligned} A &= K(x, x_1, x_2, x_{12}; 0), & \bar{A} &= K(x_3, x_{13}, x_{23}, x_{123}; 0), \\ C &= B(x, x_1, x_3, x_{13}; -\alpha), & \bar{C} &= \bar{B}(x_2, x_{12}, x_{23}, x_{123}; -\alpha), \\ \bar{K} &= K(x_3, x_1, x_2, x_{123}; \alpha). \end{aligned} \quad (3.22)$$

Similarly, in Theorem 3.10 on page 359 the systems (3.29)–(3.30), (3.31)–(3.32) and (3.33)–(3.34) can be unified into the following system:

- $(x^2 + 4\delta_1\delta_2\delta_3, x_1^2, x_2^2, x_{12}^2)$:

$$\begin{aligned} A(x, x_1, x_2, x_{12}; \delta_1, \delta_2) &= xx_1 + x_2x_{12} + \delta_1x_1x_{12} + \delta_2x_1x_2 + \delta_3, \\ B(x, x_2, x_3, x_{23}; \delta_1, \delta_2, \alpha) &= \alpha(xx_2 + x_3x_{23}) - (xx_3 + x_2x_{23}) \\ &\quad + (\alpha^2 - 1) \left(\delta_2x_2x_3 - \frac{\delta_1\delta_3}{\alpha} \right), \end{aligned} \quad (3.29)$$

$$\begin{aligned} \bar{A} &= A(x_{23}, x_{123}, x_3, x_{13}; \delta_1, \delta_2), & \bar{B} &= B(x_{12}, x_1, x_{123}, x_{13}; 0, \delta_2, \alpha), \\ C &= A(x, x_1, x_3, x_{13}; \delta_1\alpha^{-1}, \delta_2\alpha), & \bar{C} &= A(x_{23}, x_{123}, x_2, x_{12}; \delta_1\alpha^{-1}, \delta_2\alpha), \\ K &= B(x, x_{12}, x_{13}, x_{23}; \delta_2, \delta_1, \alpha^{-1}), & \bar{K} &= B(x_2, x_1, x_{123}, x_3; 0, \delta_1, \alpha^{-1}), \end{aligned} \quad (3.30)$$

and there is one new system to be listed here:

- $(4x, 1, 1, 1)$:

$$\begin{aligned} B(x, x_2, x_3, x_{23}; \alpha) &= (x - x_{23})(x_2 - x_3) + \alpha(x + x_{23} - 2x_2x_3) \\ &\quad - \alpha^2(x_2 + x_3) - \alpha^3, \\ \bar{B}(x_1, x_{12}, x_{13}, x_{123}; \alpha) &= (x_1 - x_{123})(x_{12} - x_{13}) \\ &\quad - \alpha(x_1 + x_{12} + x_{13} + x_{123}) + \alpha^2, \end{aligned} \quad (3.31)$$

$$\begin{aligned} C(x, x_1, x_3, x_{13}; \alpha) &= x + x_1x_3 + x_1x_{13} + x_3x_{13} - \alpha(x_3 - x_{13}) - \alpha^2 \\ A = C(x, x_1, x_2, x_{12}; 0), \quad \bar{A} &= C(x_{23}, x_{123}, x_3, x_{13}; 0), \\ \bar{C} = C(x_{23}, x_{123}, x_2, x_{12}; \alpha), \quad K &= B(x, x_{12}, x_{13}, x_{23}; -\alpha), \\ \bar{K} = \bar{B}(x_1, x_2, x_3, x_{123}; -\alpha). \end{aligned} \quad (3.32)$$

Finally, in Theorem 3.11 on page 360 the systems (3.41)–(3.42) and (3.43)–(3.44) can be unified into the following system:

- $(x^2 + 4\delta_1\delta_2\delta_3\delta_4, x_1^2, x_2^2, x_{12}^2)$:

$$\begin{aligned} A &= xx_{12} + x_1x_2 + \delta_3\delta_4x_1x_{12} + \delta_2x_2x_{12} + \delta_1, \\ \bar{A} &= x_3x_{123} - x_{13}x_{23} + \delta_2x_{23}x_{123} + \delta_1, \\ B &= xx_{23} + x_2x_3 + \delta_2x_2x_{23} + \delta_4x_3x_{23} + \delta_1\delta_3, \\ \bar{B} &= x_1x_{123} - x_{12}x_{13} + \delta_4x_1x_{12} + \delta_2x_{12}x_{123}, \\ C &= xx_1 - x_3x_{13} + \delta_4x_1x_3 + \delta_3x_1x_{13} - \delta_1\delta_2, \\ \bar{C} &= x_2x_{12} + x_{23}x_{123} + \delta_4x_{12}x_{23} - \delta_3x_{12}x_{123}, \\ K &= xx_{12} - x_{13}x_{23} + \delta_3x_{12}x_{13} - \delta_2\delta_4x_{12}x_{23} + \delta_1, \\ \bar{K} &= x_1x_2 + x_3x_{123} - \delta_3x_1x_{123} + \delta_1. \end{aligned} \quad (3.41)$$

The complete proofs of these theorems are now published in my PhD thesis “Classification and Lagrangian structure of 3D consistent quad-equations” (see <http://opus.kobv.de/tuberlin/volltexte/2012/3628/>).