Addendum

On automorphism-invariant modules

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Let R be an associative ring with 1. A submodule N of M over the ring R is called an *automorphism-invariant submodule* of M , if $\sigma(N) \subseteq M$, for every automorphism σ of M . A module is called an *automorphism-invariant module* if it is an automorphism-invariant submodule of its injective hull [4]. M is called *N -pseudo-injective* if, for any submodule A of N , every monomorphism in $\text{Hom}_R(A, M)$ can be extended to an element of $\text{Hom}_R(N, M)$. M is called *pseudo-injective* if it is pseudo- M -injective (see [3] and [1]).

Remark. Assume that a ring R contains an element x which is not a left zero-divisor in R and x is not left invertible in R . Then, clearly $f : xR \rightarrow R$ given by $f(xa) = a$ for all $a \in R$ is an R -isomorphism of right R -modules. This mapping cannot be extended to an endomorphism g of R_R . Otherwise, there exists an element $b = g(1)$ such that $bx = g(1)x = g(x) = f(x) = 1$, a contradiction. Therefore, R is not right pseudo-injective. This implies R is not right automorphism-invariant, since automorphism-invariant modules coincide with pseudo-injective modules by [2, Theorem 16] (We thank Professor Askar Tuganbaev for pointing out this remark to us).

It follows from the above remark that any polynomial ring (or power series ring) with identity is not right automorphism-invariant. Thus Theorem 2.18 in [5] should be ignored.

References

- [1] H. Q. Dinh, A note on pseudo-injective modules, *Commun. Algebra* **33** (2005) 361–369.
- [2] N. Er, S. Singh and A. K. Srivastava, Rings and modules which are stable under automorphisms of their injective hulls, *J. Algebra* **379** (2013) 223–229.
- [3] S. K. Jain and S. Singh, Quasi-injective and pseudo-injective modules, *Canad. Math. Bull.* **18**(3) (1975) 359–365.
- [4] T. K. Lee and Y. Zhou, Modules which are invariant under automorphisms of their injective hulls, *J. Algebra Appl.* **12**(2) (2013) 1250159 (9 pages).
- [5] T. C. Quynh and M. T. Koşan, On automorphism-invariant modules, *J. Algebra Appl.* **14**(5) (2015) 1550074 (11 pages).