Addendum

Chain equivalences for symplectic bases, quadratic forms and tensor products of quaternion algebras

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We show how the chain lemma for quaternion algebras over fields of char $F \neq 2$ and $I_q^3 F = 0$ can be strengthened so that up to two slots are changed at a time.

Theorem 0.1 [1, Theorem 5.2]. *If char $F \neq 2$ and $I_q^3 F = 0$ then every two isomorphic tensor products of n quaternion algebras are connected by a finite number of steps, where each step is either*

$$(1) \quad (c_k, d_k) \otimes (c_{k+1}, d_{k+1}) \rightarrow (c_k, (\beta^2 - c_k c_{k+1})d_k) \otimes (c_{k+1}, (\beta^2 - c_k c_{k+1})d_{k+1})$$

for some $k \in \{1, \dots, n-1\}$ and $\beta \in F$, or one of the following types:

$$(2) \quad (c_k, d_k) \rightarrow ((\alpha^2 - \beta^2 d_k)c_k, d_k),$$

$$(3) \quad (c_k, d_k) \rightarrow (c_k, (\alpha^2 - \beta^2 c_k)d_k),$$

for some $k \in \{1, \dots, n\}$ and $\alpha, \beta \in F$.

Proof. The proof is essentially the proof of [1, Theorem 5.2], with the following observation: There is one additional type of step in the original theorem

$$(c_1, d_1) \otimes \dots \otimes (c_n, d_n) \rightarrow (c_1, (\beta^2 - \prod_{r=1}^n c_r)d_1) \otimes \dots \otimes (c_n, (\beta^2 - \prod_{r=1}^n c_r)d_n).$$

This step corresponds to a change of the last two entries $\langle a_{n+1}, b_{n+1} \rangle$ in the corresponding quadratic form. Now, we can avoid such changes, because both the initial

and final quadratic forms can be chosen so that they have -1 in the last entry, and therefore according to the Witt cancellation theorem they are connected by a chain of steps that leave the last entry unchanged. $\square$

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Reference

- [1] A. Chapman, hain equivalences for symplectic bases, quadratic forms and tensor products of quaternion algebras, *J. Algebra Appl.* **14**(3) (2015) 1550030 (9 pages).