Enhancing the Stability of the Switched Systems Using the Saltation Matrix

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Received 9 March 2018

Accepted 17 November 2018

Published 4 January 2019

Controlling switched systems is a difficult task, even when dealing with piecewise linear systems (CPWLs), which consist of a set of linear differential equations and a set of switching conditions. This difficulty is largely due to the loss of linearity in the entire system, and it is necessary to solve differential and algebraic equations to determine the solution. In this paper, a new method to tune the parameters of the controllers applied to switched systems is derived using information from the saltation matrix, particularly its induced norm. First, the parameters are tuned using classical methods, and then, after analyzing the norm of the saltation matrix, a new set of parameters that guarantees the stability of the period-1 orbit is obtained. The method is validated using analytical solutions for two different systems (boost and boost-flyback power converters) and is also experimentally validated for the boost-flyback power converter.

Keywords: Monodromy matrix; saltation matrix; matrix norm; current-mode-control; power converters.

1. Introduction

A switched system is a system that changes its topology according to an event. For this reason, switched systems are defined by a set of ordinary differential equations and a set of switching conditions. Switched systems are present in chemical

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processes,¹ aeronautics,² physics,³ communications,⁴ automotive industry,⁵ renewable energies^{6,7} and power electronics,^{8,9} among many others. As electronic devices have gained an important place in modern life, the design of new and robust controllers plays a critical role in the development of these devices. Although the relation between stability and other mathematical concepts, such as the measure matrix, matrix norm, and so on, has been known for many years,^{10–12} its use has been minimal in the design and computation of controllers compared with other tools, such as PID, robust and adaptive controllers.

One of the most popular methods to analyze switched systems is to describe them as Filippov systems.¹³ If the system switches among linear topologies, it is called a piecewise linear system (PWL). If the system has a period- T orbit, it is possible to determine its stability through the Floquet exponents by using the Filippov theory, the information of the periodic orbit and the saltation matrix.¹⁴

Power converters are a type of switched system that can be used to analyze, compute and validate new controllers since their experimental set-up allows for the testing of analyses and results. Boost and boost-flyback converters allow for output voltages that are more than twice the input voltage.¹⁵ These converters can be used in different applications, such as fuel cells¹⁶ and photovoltaic sources.¹⁷ Boost converters have a simple topology and have been studied with different controllers. One of the most frequently used controller is the peak current mode control, which uses an outer voltage loop and an inner current.¹⁸ When the switch is on for a period of time greater than 50% of the period T , the converters tend to subharmonic or chaotic solutions. Therefore, it is necessary to attach a compensation ramp to avoid these types of solutions.^{19,20} Although this compensation ramp adds stability to the system, as the slope increases, the response tends to be slower.

A simple control strategy using the saltation matrix applied to a buck power converter is described in Ref. 21; in this study, a high frequency and low amplitude signal is added to the compensation ramp to enhance the stability range of the period-1 orbit. In the same paper, a boost power converter is analyzed and controlled, thereby increasing the value of the slope of the compensation ramp. However, as this value increases, the response of the system deteriorates, mainly when light loads are present.²² In Ref. 23, a controller is proposed that forces the system to have more than two switchings during a period in an attempt to avoid subharmonic behaviors in complex converters; however, as the switching time increases, faster devices are needed and the durability of the elements decreases. Other studies that include both fast scale and slow scale instabilities are reported in Refs. 24 and 25; these instabilities appear due to the use of large parameters or high frequency phenomena.

Taking into account previous works and the possibilities provided by the matrix norm concept, it proposes modifying the saltation matrix modifying the switching function by changing the controllers. This change is achieved using the matrix norm and computing the new values of the controllers so that a change in the norm of the

saltation matrix is obtained and the stability of the period-1 orbit is guaranteed over a wider range. The main contribution of this paper is to extend the stability range of the period-1 orbit in switched systems by re-tuning the initial parameters of the controller and employing the matrix norm of the saltation matrix as the objective of control as opposed to using a new controller. Tuning is carried out offline.

The paper is organized as follows: in Sec. 2, mathematical concepts, such as the monodromy matrix, saltation matrix and matrix norm are introduced. In Sec. 3, the stability of the period-1 orbit in a boost and boost-flyback converter is analyzed using the monodromy matrix. In Sec. 4, the method to re-tune the control parameters and the analytical results are presented. In Sec. 5, an experimental set-up is used to validate the proposed approach. Finally, in Sec. 6, the main conclusions are shown.

2. Mathematical Preliminaries

This section introduces two important concepts of this paper: the saltation matrix and the matrix norm induced by the vector norm.

2.1. Monodromy and saltation matrices

The monodromy matrix is a powerful tool that is used to determine the stability of a periodic orbit in discontinuous systems.^{14,26} The monodromy matrix is the fundamental matrix of a system of ODEs evaluated at the period T of the coefficients of the system. The evolution of a period-1 orbit during a period T of a system with two topologies is schematically depicted in Fig. 1(a). The system starts at $t = 0$ and evolves according to the flux ϕ_1 in the subspace V_- . When an event occurs at $t = t_p$, the system changes its topology and evolves with the flux ϕ_2 until $t = T$, according to the dynamics defined in the subspace V_+ . At $t = T$, the topology changes again

Fig. 1. Typical periodic solution of a switched system.

and the cycle is repeated. The condition to change topologies is given in a general form as $h(x(t)) = 0$ and is represented as the hypersurface Σ . A system with two topologies can be described using the Filippov theory in the following way:

$$\dot{X}(t) \in F(X(t)) = \begin{cases} f_-(t, X(t)) & X \in V_- \\ \overline{\text{co}}\{f_-(t, X(t)), f_+(t, X(t))\} & X \in \Sigma \\ f_+(t, X(t)) & X \in V_+ \end{cases}, \quad (2.1)$$

where the vector fields before and after the computation are f_- and f_+ , respectively, and $\overline{\text{co}}\{f_-, f_+\}$ is the convex set between the vector fields, defined as

$$\overline{\text{co}}\{f_-, f_+\} = \{(1 - q)f_- + qf_+, \forall q \in [0, 1]\}. \quad (2.2)$$

The period-1 solution of a piece-wise linear system with two topologies is

$$\begin{aligned} x(t_\Sigma) &= e^{A_1(t_\Sigma)}x(0) + \int_0^{t_\Sigma} e^{A_1(t_\Sigma-\tau)}B_1 \, d\tau \\ x(T) &= e^{A_2(T-t_\Sigma)}x(t_\Sigma) + \int_{t_\Sigma}^T e^{A_2(T-\tau)}B_2 \, d\tau, \end{aligned} \quad (2.3)$$

where t_Σ is the switching instant. To analyze the stability of the period-1 orbit, it is necessary to know the value of the periodic orbit, the switching instants (t_Σ) and the values of the corresponding state at these times ($x(t_\Sigma)$). After these data are determined, the initial condition is adjusted (see Fig. 1(b)) and corrections in the monodromy matrix at the switching times are introduced from the saltation matrices.²⁷ The monodromy matrix is given by

$$M_m(T, 0) = \hat{\phi}_2 S \hat{\phi}_1, \quad (2.4)$$

where the solution matrices are $\hat{\phi}_1 = e^{A_1(t_\Sigma)}$ and $\hat{\phi}_2 = e^{A_2(T-t_\Sigma)}$ and S is the saltation matrix that relates the disturbance in the switching event as $\delta x_{\Sigma+} = S\delta x_{\Sigma-}$ (see Fig. 1(b)), which is computed as¹⁴

$$S = I + \frac{(f_+(x(t_\Sigma), t_\Sigma) - f_-(x(t_\Sigma), t_\Sigma))n^T}{n^T f_-(x(t_\Sigma), t_\Sigma) + \frac{\partial h(x(t_\Sigma), t_\Sigma)}{\partial t}}, \quad (2.5)$$

where I is the identity matrix and n is the normal vector to the switching surface. Using the monodromy matrix and the Floquet theory, it is possible to study the stability of periodic orbits in a PWL system. Particularly, considering the period-1 orbit, it will be stable if all the eigenvalues of the monodromy matrix are inside the unit circle, i.e. the maximum absolute value of eigenvalues is lower than 1 ($\text{MVP} < 1$). In a general form, for a system with $n - 1$ switching events per period, the monodromy matrix is computed as

$$M_m = \hat{\phi}_n S_{n-1} \hat{\phi}_{n-1}, \dots, \hat{\phi}_2 S_1 \hat{\phi}_1. \quad (2.6)$$

2.2. Matrix norm

The matrix norm is an induced norm by the vector norm that maps the space of the matrices into $\mathcal{R}^+ \cup \{0\}$, and it is defined by^{28,29}

$$\|A\|_i = \sup_{\forall x \neq 0} \frac{\|Ax\|}{\|x\|} = \sup_{\|x\|=1} \|Ax\| = \sup_{\|x\| \leq 1} \|Ax\|, \quad (2.7)$$

where the subscript i indicates what vector norm is used. After some computations and using the norm L^2 (more commonly known as 2-norm or Euclidean norm), the matrix norm is expressed as³⁰

$$\|A\|_2 = \sqrt{\rho(A^T A)}, \quad (2.8)$$

where $\rho(A^T A)$ is the spectral radius, i.e. the maximum eigenvalue λ that makes $A^T A - \lambda I$ singular, which is the norm that is used to analyze the behavior of the saltation matrices.

3. Power Converters: Stability of the Period-1 Orbit

In this section, the period-1 orbit of the current mode controlled boost and boost-flyback converters are analyzed using the monodromy matrix. Fundamental theory about electronic devices, magnetically coupled inductors, circuits and power electronics can be found in Refs. 19 and 31.

3.1. Boost converter

The boost power converter analyzed in this paper is depicted in Fig. 2. In the circuit diagram V_{in} is the input voltage and V_{ref} is a scaled reference voltage. L is the inductor, C is the capacitor, R is the load resistance. D and S_D resemble the diode

Fig. 2. Diagram of the boost power converter peak-current-mode-controlled.

and the MOSFET. r_L , r_s , r_D and r_C are the internal resistances of the inductor, MOSFET, diode and capacitor, respectively. R_1 and R_2 constitute a voltage divider. As the converter is peak current-mode controlled,³² r_a and C_a with the operational amplifier form the PI controller, and a compensation ramp is added to the output of the PI controller to obtain i_{ref} and to avoid subharmonic behaviors. The inductor current i_L multiplied by M ($M i_L$) is compared with i_{ref} to handle the flip-flop. For a complete description of the boost converter and the operation mode, the reader is referred to Refs. 20 and 33. The system has three state variables which are $X(t) = [i_L \ V_c \ V_a]^T$, where i_L is the inductor current, V_C is the voltage in capacitor C and V_a is the voltage in capacitor C_a . The system can operate in three topologies depending on the switch and diode positions. In a compact form, the dynamical equations are written as

$$\dot{X} = \begin{cases} A_1 X(t) + B_1 V_{\text{in}} S_D = 1, & D = 0 \\ A_2 X(t) + B_1 V_{\text{in}} S_D = 0, & D = 1 \\ A_3 X(t) + B_2 V_{\text{in}} S_D = 0, & D = 0. \end{cases} \quad (3.1)$$

The matrices A_1 , A_2 and A_3 as well as vectors B_1 and B_2 are given by

$$\begin{aligned} A_1 &= \begin{bmatrix} -\frac{r_L + r_s}{L} & 0 & 0 \\ 0 & -\frac{1}{T_m(1+k_C)} & 0 \\ 0 & \frac{g}{T_a(1+k_C)} & 0 \end{bmatrix}, & B_1 &= \begin{bmatrix} \frac{V_{\text{in}}}{L} \\ 0 \\ -\frac{gk_D V_{\text{ref}}}{T_a V_{\text{in}}} \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -\frac{r_L + r_D}{L} - \frac{r_C}{L(1+k_C)} & -\frac{1}{L(1+k_C)} & 0 \\ \frac{1}{C(1+k_C)} & -\frac{1}{T_m(1+k_C)} & 0 \\ \frac{gr_C}{T_a(1+k_C)} & \frac{g}{T_a(1+k_C)} & 0 \end{bmatrix}, & B_1 &= \begin{bmatrix} 0 \\ 0 \\ -\frac{gk_D V_{\text{ref}}}{T_a V_{\text{in}}} \end{bmatrix}, \\ A_3 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{T_m(1+k_C)} & 0 \\ 0 & \frac{g}{T_a(1+k_C)} & 0 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0 \\ 0 \\ -\frac{gk_D V_{\text{ref}}}{T_a V_{\text{in}}} \end{bmatrix}, \end{aligned} \quad (3.2)$$

with $g = r_a/R_1$, $T_m = RC$, $T_a = r_a C_a$, $k_c = r_c/R$ and $k_D = (R_1 + R_2)/R_2$. The voltage loop output V_k is given by

$$V_k(t) = V_{\text{ref}}(1 + gk_D) - \left(\frac{g}{1 + k_C} \right) V_C(t) - V_a(t), \quad (3.3)$$

where V_{ref} is the reference voltage. The reference current is

$$i_{\text{ref}} = V_k(t) - m_a \bmod \left(\frac{t}{T} \right), \quad (3.4)$$

and the switching function is

$$h_B(x(t), t) = k_0 - Mi_L - \frac{g}{1 + k_c} V_c - V_a - m_a \bmod \left(\frac{t}{T} \right), \quad (3.5)$$

where V_{ref} is obtained from the desired output voltage (20 V) and the voltage divider formed by R_1 and R_2 . In this way, $V_{\text{ref}} = 20 R_2 / (R_1 + R_2)$ and $k_0 = V_{\text{ref}}(1 + gk_D)$. For all of the computations the parameter values are³⁴: $L = 165 \mu\text{H}$, $C = 150 \mu\text{F}$, $r_L = 0.04 \Omega$, $r_C = 0.03 \Omega$, $r_D = 0.01 \Omega$, $r_S = 0.055 \Omega$, $T = 60 \mu\text{s}$, $R_1 = 47 \text{k}\Omega$, $R_2 = 6.8 \text{k}\Omega$, $V_{\text{ref}} = 2.5 \text{V}$, $r_a = 4.7 \text{k}\Omega$, $C_a = 2000 \text{nF}$, $M = 0.3$ and $m_a = 0.445$. The monodromy matrix in continuous conduction mode is obtained as follows¹⁴:

$$M_m = \hat{\phi}(A_2, t = DT..T) \cdot S_{1,2} \cdot \hat{\phi}(A_1, t = 0..DT), \quad (3.6)$$

where the time instant $t = DT$ corresponds to the time when the MOSFET switches from on (closed) to off (opened), as illustrated in Fig. 3. Figure 4 shows the evolution

Fig. 3. Behavior of the inductor current i_L . The figure shows the instance when i_L meets i_{ref} and the switch changes the position.

Fig. 4. Evolution of the MVP as V_{in} varies (boost converter case).

of the largest absolute value of the eigenvalues (MVP) of the monodromy matrix as the input voltage V_{in} varies. For $V_{in} < 5.84$ V, the period-1 orbit is unstable, and for $V_{in} \in (5.84$ V 10 V), it is stable.

3.2. Boost-flyback converter

This converter integrates two basic structures, as shown in Fig. 5. It has a pair of output capacitors C_1 and C_2 , load resistance R , two diodes D_1 and D_2 , and the MOSFET S_D . The main feature of this converter is that it has two coupled inductors, L_p and L_s , with a coupled inductance M_L . The output of the converter is obtained as $V_{out} = V_{C_1} + V_{C_2}$; the parasitic resistances are r_p , r_s , r_{DS} , r_{D_1} , r_{D_2} , r_{C_1} and r_{C_2} . Similarly to the boost converter case, the control parameters are R_1 , R_2 , C_a and r_a , which define a PI external control loop. The state variables are $X = [i_p \ i_s \ V_{C_1} \ V_{C_2} \ V_a]^T$, and they correspond to the primary and secondary inductor currents and the voltages on the capacitors C_1 , C_2 and C_a , respectively. The system of equations that depend on the position of the switching devices are written as follows:

$$\dot{X} = \begin{cases} A_1 X + B_1 V_{in} & \text{if } S_D = 0, D_1 = 0, D_2 = 0, \\ A_2 X + B_2 V_{in} & \text{if } S_D = 0, D_1 = 1, D_2 = 0, \\ A_3 X + B_3 V_{in} & \text{if } S_D = 0, D_1 = 0, D_2 = 1, \\ A_4 X + B_4 V_{in} & \text{if } S_D = 0, D_1 = 1, D_2 = 1, \\ A_5 X + B_5 V_{in} & \text{if } S_D = 1, D_1 = 0, D_2 = 0, \\ A_6 X + B_6 V_{in} & \text{if } S_D = 1, D_1 = 0, D_2 = 1. \end{cases} \tag{3.7}$$

Fig. 5. Diagram of the boost-flyback converter peak-current-mode-controlled.

However, when the system works in a particular period-1 solution, the converter evolves in four topologies in a period. At the beginning of period, while $S_D = 1$ (i.e. the MOSFET is closed) the converter evolves through ϕ_6 and ϕ_5 . After at $t = DT$, $S_D = 0$ (the MOSFET opens) and the system evolves according to the dynamic defined by ϕ_4 and ϕ_3 . In [Appendix A](#), a brief summary related to obtaining these equations is presented. The matrices A_3 , B_3 , A_4 , B_4 , A_5 , B_5 , A_6 and B_6 are given by:

$$A_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & A_{3_{22}} & \frac{k_{a_2}}{L_s} & -\frac{k_{b_2}}{L_s} & 0 \\ 0 & -\frac{k_{a_2}}{C_1} & -\frac{k_c}{T_{C_1}} & -\frac{k_c}{T_{C_1}} & 0 \\ 0 & \frac{k_{b_2}}{C_2} & -\frac{k_c}{T_{C_2}} & -\frac{k_c}{T_{C_2}} & 0 \\ 0 & A_{3_{52}} & \frac{k_{c_2}g}{T_a} & \frac{k_{c_2}g}{T_a} & 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\frac{k_D g V_{\text{ref}}}{T_a V_{\text{in}}} \end{bmatrix}, \quad (3.8)$$

where $A_{3_{22}} = -\frac{r_s + r_{D_2} + r_{C_2} k_{b_2}}{L_s}$, $A_{3_{52}} = \frac{g(r_{C_2} k_{b_2} - r_{C_1} k_{a_2})}{T_a}$.

$$A_4 = \begin{bmatrix} A_{4_{11}} & A_{4_{12}} & A_{4_{13}} & A_{4_{14}} & 0 \\ A_{4_{21}} & A_{4_{22}} & A_{4_{23}} & A_{4_{24}} & 0 \\ \frac{k_{a_1}}{C_1} & -\frac{k_{a_2}}{C_1} & -\frac{k_c}{T_{C_1}} & -\frac{k_c}{T_{C_1}} & 0 \\ -\frac{k_{b_1}}{C_2} & \frac{k_{b_2}}{C_2} & -\frac{k_c}{T_{C_2}} & -\frac{k_c}{T_{C_2}} & 0 \\ A_{4_{51}} & A_{4_{52}} & \frac{k_{c_2}g}{T_a} & \frac{k_{c_2}g}{T_a} & 0 \end{bmatrix}, \quad B_4 = \begin{bmatrix} \frac{L_s}{n} \\ -\frac{M_L}{n} \\ 0 \\ 0 \\ -\frac{k_D g V_{\text{ref}}}{T_a V_{\text{in}}} \end{bmatrix}, \quad (3.9)$$

where $A_{4_{11}} = -\frac{L_s(r_p + r_{D_1} + r_{C_1} k_{a_1}) + r_{C_2} k_{b_1} M_L}{m}$, $A_{4_{12}} = \frac{M_L(r_s + r_{D_2} + r_{C_2} k_{b_2}) + r_{C_1} k_{a_2} L_s}{m}$,
 $A_{4_{13}} = -\frac{k_{a_1} L_s + k_{a_2} M_L}{m}$, $A_{4_{14}} = \frac{k_{b_1} L_s + k_{b_2} M_L}{m}$, $A_{4_{21}} = \frac{M(r_p + r_{D_1} + r_{C_1} k_{a_1}) + r_{C_2} k_{b_1} L_p}{m}$,
 $A_{4_{22}} = -\frac{L_p(r_s + r_{D_2} + r_{C_2} k_{b_2}) + r_{C_1} k_{a_2} M_L}{m}$, $A_{4_{23}} = \frac{k_{a_1} M_L + k_{a_2} L_p}{m}$, $A_{4_{24}} = \frac{k_{b_1} M_L + k_{b_2} L_p}{m}$,
 $A_{4_{51}} = \frac{(r_{C_1} k_{a_1} - r_{C_2} k_{b_1})g}{T_a}$, $A_{4_{52}} = \frac{(r_{C_2} k_{b_2} - r_{C_1} k_{a_2})g}{T_a}$.

$$A_5 = \begin{bmatrix} -\frac{(r_p + r_{DS(\text{on})})}{L_p} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{k_c}{T_{C_1}} & -\frac{k_c}{T_{C_1}} & 0 \\ 0 & 0 & -\frac{k_c}{T_{C_2}} & -\frac{k_c}{T_{C_2}} & 0 \\ 0 & 0 & \frac{k_{c_2}g}{T_a} & \frac{k_{c_2}g}{T_a} & 0 \end{bmatrix}, \quad B_5 = \begin{bmatrix} \frac{1}{L_p} \\ 0 \\ 0 \\ 0 \\ -\frac{k_D g V_{\text{ref}}}{T_a V_{\text{in}}} \end{bmatrix}, \quad (3.10)$$

$$A_6 = \begin{bmatrix} A_{611} & A_{612} & -\frac{k_{a2}M_L}{n} & \frac{k_{b2}M_L}{n} & 0 \\ A_{621} & A_{622} & \frac{k_{a2}L_p}{n} & -\frac{k_{b2}L_p}{n} & 0 \\ 0 & -\frac{k_{a2}}{C_1} & -\frac{k_c}{T_{C_1}} & -\frac{k_c}{T_{C_1}} & 0 \\ 0 & \frac{k_{b2}}{C_2} & -\frac{k_c}{T_{C_2}} & -\frac{k_c}{T_{C_2}} & 0 \\ 0 & A_{652} & \frac{k_{Cg}}{T_a} & \frac{k_{Cg}}{T_a} & 0 \end{bmatrix}, \quad B_6 = \begin{bmatrix} \frac{L_s}{n} \\ -\frac{M_L}{n} \\ 0 \\ 0 \\ -\frac{k_D g V_{\text{ref}}}{T_a V_{\text{in}}} \end{bmatrix}, \quad (3.11)$$

where $A_{611} = -\frac{L_s(r_p+r_{DS(\text{on})})}{m}$, $A_{612} = \frac{M_L(r_s+r_{D_2}+r_{C_2}k_{b2})}{m}$, $A_{621} = \frac{M_L(r_p+r_{DS(\text{on})})}{m}$, $A_{622} = -\frac{L_p(r_s+r_{D_2}+r_{C_2}k_{b2})}{m}$, $A_{652} = \frac{g(r_{C_2}k_{b2}-r_{C_1}k_{a2})}{T_a}$. The constant n is defined as $n = L_p L_s - M^2$. The output of the outer voltage loop is

$$V_k = (1 + gk_D)V_{\text{ref}} - gk_c(V_{C_1}(t) + V_{C_2}(t)) - V_a(t). \quad (3.12)$$

The current reference is

$$i_{\text{ref}} = V_k - m_a \bmod\left(\frac{t}{T}\right), \quad (3.13)$$

and the switching function is

$$h_{\text{BF}}(x(t), t) = k_0 - Mi_p - gk_c(V_{C_1} + V_{C_2}) - V_a - m_a \bmod\left(\frac{t}{T}\right), \quad (3.14)$$

where $k_0 = V_{\text{ref}}(1 + gk_D)$. The monodromy matrix is obtained as follows³⁵:

$$M = \hat{\phi}(A_3, t = t_2, \dots, T) \cdot S_3 \cdot \hat{\phi}(A_4, t = DT, \dots, t_2) \\ \cdot S_2 \hat{\phi}(A_5, t = t_1, \dots, DT) \cdot S_1 \cdot \hat{\phi}(A_6, t = 0, \dots, t_1) \quad (3.15)$$

Fig. 6. Behavior of the inductor currents i_p and i_s (boost-flyback converter case).

Fig. 7. Evolution of the MVP as V_{in} varies (boost-flyback converter).

where the switching time instants are $t = t_1$, $t = DT$ and $t = t_2$, as they are shown in Fig. 6. The parameters used are $L_p = 129.2 \mu\text{H}$, $L_s = 484.9 \mu\text{H}$, $C_1 = C_2 = 220 \mu\text{F}$, $r_p = 0.0368 \Omega$, $r_s = 0.1307 \Omega$, $r_{DS} = 0.04 \Omega$, $r_{C_1} = r_{C_2} = 30 \text{ m}\Omega$, $r_{D_1} = r_{D_2} = 10 \text{ m}\Omega$, $T = 50 \mu\text{s}$, $V_{\text{ref}} = 100 \text{ V}$, $R_1 = 1 \text{ M}\Omega$, $R_2 = 20 \text{ K}\Omega$, $r_a = 390 \text{ K}\Omega$, $C_a = 15 \text{ nF}$, $M = 0.196$, $m_a = 0.392/T$. Figure 7 shows the evolution of the MVP. The period-1 orbit loses stability near 17.7 V .

4. A New Method to Enhance the Stability of the Period-1 Orbit

When the voltage gain increases above a limit value, it is necessary to add a compensation ramp. For a boost power converter, this limit is close to 2 ($V_{\text{out}} > 2V_{\text{in}}$), and for the analyzed boost-flyback converter, it is close to 5.6 ($V_{\text{out}} > 5.6V_{\text{in}}$). However, as the value of the slope increases, over-compensation occurs and deteriorates the response of the dynamical system. To guarantee the stability of the period-1 orbit as the voltage gain increases, it proposes changing the norm of the saltation matrix by re-tuning the parameters of the controller based on their original values. This approach affects the switching expression since the saltation matrix includes information from both the matrices and switching function. In the boost (B) and the boost-flyback (BF) converters, the switching functions $h_{(\cdot)}(x(t))$ are given by:

$$\begin{aligned}
 h_B(x(t), t) &= k_0 - Mi_L - \frac{g}{1 + k_c} V_c - V_a - m_a \bmod \left(\frac{t}{T} \right), \\
 h_{\text{BF}}(x(t), t) &= k_0 - Mi_p - gk_c(V_{C_1} + V_{C_2}) - V_a - m_a \bmod \left(\frac{t}{T} \right).
 \end{aligned} \tag{4.1}$$

The switching function $h_{(\cdot)}(x(t), t)$ affects the saltation matrix from both its time derivative and its normal vector n . As the slope compensation increases, the time derivative also increases and the norm of the saltation matrix decreases. This is the

same reason behind adding a high period-small amplitude-function to the compensation ramp. The normal vector $n_{(.)}$ for every converter is obtained as $n_{(.)} = \partial h / \partial X$.

$$\begin{aligned} n_B &= \frac{\partial h_B}{\partial X} = \left[-M \quad -\frac{g}{1+k_c} \quad -1 \right]^T, \\ n_{BF} &= \frac{\partial h_{BF}}{\partial X} = [-M \quad 0 \quad -gk_c \quad -gk_c \quad -1]^T. \end{aligned} \tag{4.2}$$

4.1. Boost converter case

The norm of the monodromy matrix of the boost converter as V_{in} varies is depicted in Fig. 8(a). The norm close to where the system loses stability is approximately 7.3. Since the norm of the saltation matrix is similar to the norm of the monodromy matrix (Fig. 8(b)), the relation between the norms of the normal vector ($\|n_B\|$) and the saltation matrix ($\|S\|$) is studied.

The norm of the normal vector n_B in terms of the parameters used in this paper is 1.0041, and the saltation matrix norm close to where the system loses stability is 7.3. Note that the norm of normal vector only depends on the controller parameters. Since the maximum value of the monodromy matrix was close to 14, it proposes using an optimization algorithm such that the norm of the normal vector given by

$$n_B = \left[-M + \alpha \quad -\frac{g}{1+k_c} + \beta \quad -1 + \gamma \right]^T, \tag{4.3}$$

be equal to 1.0041/2, reducing the norm by 50%. Remember that $M = 0.3$ and $g/(1+k_c) = 0.099$. The constants α , β and γ must be determined through tuning. The **sqp** minimization method was implemented using the **fmincon** function provided by Matlab[®], and the values $\alpha = 0.1570$, $\beta = 0.0523$ and $\gamma = 0.5233$ were

Fig. 8. Matrix norms versus input voltage (boost converter case).

Fig. 9. Evolution of the MVP for the boost converter, as V_{in} varies, after applying the new approach.

obtained. Using these values for the constants, the values of the controller were computed and determined to be $r_a = 2.35 \text{ k}\Omega$, $C_a = 4 \mu\text{F}$ and $M = 0.15$. The MVPA was again computed using these new values, and the result is depicted in Fig. 9. The figure shows that the period-1 orbit is stable for all of the ranges of the input voltage considered in the computations. Matlab R2017a was used and eight decimals were considered in all computations and routines.

4.2. Boost-flyback converter case

Since the system has three commutations, it is necessary to compute the matrix norm of these three matrices, which are shown in Figs. 10(a)–10(c). In Fig. 10(d), the behavior of the monodromy matrix norm as V_{in} varies is displayed.

The saltation matrix that should be targeted is S_2 since it is the only matrix that was affected by varying V_{in} . Therefore, if the norm of $\|S_2\|$ is reduced, the spectral radius of M_m will decrease and the stability range will be extended. The value of the monodromy matrix norm when the period-1 orbit loses stability ($V_{in} = 17.7$) is approximately 6.4 (see Figs. 7 and 10(d)) and the saltation matrix norm is approximately 35 for the same value of V_{in} . As the objective is for the system to work successfully for $V_{in} \in (12, 18)\text{V}$, the goal is obtained reducing the value of the norm from 55 to 34, which implies a reduction of approximately 40%. To do this, the value of the norm of the vector $\|n_{BF}\|$ is reduced using the constants α , β and γ as follows:

$$n_{BF} = [-M + \alpha \quad 0 \quad -gk_c + \beta \quad -gk_c + \beta \quad -1 + \gamma]^T. \quad (4.4)$$

Therefore, the vector norm $\|n_{BF}\|$ was reduced by 45% to attain the stability of the period-1 orbit. As previously described, the minimization function **fmincon** was used to obtain the values of $\alpha = 0.0761$, $\beta = 0.2959$, and $\gamma = 0.3884$. With these values, the parameters for the outer voltage and the current input gain M , were again computed. The new values for the elements were $r_a = 100 \text{ k}\Omega$, $C_a = 39 \text{ nF}$ and $M = 0.12$. Figure 11 shows the evolution of the maximum absolute value of the

Fig. 10. Matrix norms (boost-flyback converter case).

Fig. 11. Evolution of the MVP for the boost-flyback converter, as V_{in} varies, after applying the new approach.

eigenvalues as the input voltage varies. The period-1 orbit is now stable for the entire range of V_{in} .

5. Experimental Results

After obtaining successful results on a theoretical basis, the authors design and assemble an experimental set-up to prove the validity of the proposed controller. It is very complicated to control the boost-flyback converter since it is a 5-dimensional dynamical system, it has two diodes and one MOSFET, and its period-1 orbit evolves switching among four topologies. It is due to this complexity that this system was chosen to validate our approach. The results were obtained using two Schottky diodes C3D10060A (D_1 and D_2). The controlled switch (S) is a MOSFET IRFP260N with the driver TC4427. The values of the experimental parameters are the same as described in Sec. 3.2, resulting in a converter of 100 Watts of power. Figure 12 plots the peak current reference ($I_c = V_k - V_r$), a ratio M of the primary inductor current

(a) Period-1 orbit for $V_{in} = 18$ V

(b) Chaotic solution for $V_{in} = 12$ V

(c) Period-1 orbit for $V_{in} = 18$ V and using the proposed approach

(d) Period-1 orbit for $V_{in} = 12$ V and using the proposed approach

Fig. 12. Boost-flyback converter experimental results.

(Mi_p) and a ratio ($1/10$) of the secondary inductor current ($i_s/10$). The experimental results were obtained using two values for the input voltage, $V_{in} = 18\text{ V}$ and $V_{in} = 12\text{ V}$. For $V_{in} = 18\text{ V}$ and without using the tuning approach, the converter obtains a period-1 solution for $V_{in} = 18\text{ V}$, as is shown in Fig. 12(a), but for $V_{in} = 12\text{ V}$, the period-1 orbit loses stability (see Fig. 12(b)). In the second case, the proposed method is used and the controller is tuned according to the values computed after the reduction of the matrix norm. Figure 12(c) presents the results for $V_{in} = 18\text{ V}$, which are stable. Figure 12(d) presents that period-1 orbit remains stable for $V_{in} = 12\text{ V}$ when the controller is tuned.

6. Conclusions

Control techniques for switched systems that are based on the analysis and change of the saltation matrix have emerged in recent years. Using this technique and information provided by the matrix norm, a new strategy for widening the stability range of the period-1 orbit was developed. This methodology first uses a set of values from the control parameters, and then, based on the matrix norm, it computes a new set of parameters so that the final controller is a better tuned version of the initial controller.

Although the application, computations and experiments were performed for power converters, the theory and methodology presented in this paper are broad enough that they could be applied to any other discontinuity system.

The specific advantages of this approach must be analyzed. For example, in the case of power converters, this approach is comparatively better than only increasing the slope value of the compensation ramp. Re-tuning the parameters using the matrix norm improves the performance of the system compared to only increasing the slope of the compensation ramp.

The tuning of the controller is carried out offline. The new controller is designed and implemented using the parameters α , β and γ obtained after the tuning, and it remains unchanged.

It is important to note that the final tuning of the controller does not require much more effort than is currently used because analysis of the stability is already a forced task in control design. One only needs to compute the norms and minimize a function.

Finally, it is important to note that all results presented in this paper only include behavior analysis for linear loads.

Acknowledgments

This work was supported by Universidad Nacional de Colombia, projects 31492 and COLCIENCIAS under Contracts FP44842-052-2016 and National Ph.D. Programs 6172-2013, and Young Researchers Program 645-2014.

Appendix A. Equation System for the Boost-Flyback Converter

In this section is briefly explained how the ODEs for the boost-flyback converter are found using Kirchhoff's law. First, the Kirchhoff's voltage law to compute V_p and V_s (voltage across the inductors L_p and L_s) is applied. When $D_2 = 1$, V_p and V_s can be written as

$$V_p = V_{in} - V_{S_D}, \quad (\text{A.1})$$

$$V_s = -V_{D_2} - V_{C_2} - r_{C_2} C_2 \frac{dV_{C_2}}{dt}, \quad (\text{A.2})$$

where V_{D_2} is the voltage across the diode D_2 and V_{S_D} is the voltage across the MOSFET. V_{S_D} depends on the switch position. For $S_D = 1$, it can be expressed as $V_{S_D} = r_{DS(on)} i_p$; when $S_D = 0$, it can be expressed as $V_{S_D} = V_{D_1} + V_{C_1} + r_{C_1} C_1 \frac{dV_{C_1}}{dt}$. After, using Kirchhoff's current law to compute i_{D_1} and i_{D_2} (currents flowing through diodes), it obtains

$$i_{D_1} = C_1 \frac{dV_{C_1}}{dt} + \frac{V_{C_1}}{R} + \frac{r_{C_1} C_1}{R} \frac{dV_{C_1}}{dt} + \frac{V_{C_2}}{R} + \frac{r_{C_2} C_2}{R} \frac{dV_{C_2}}{dt}, \quad (\text{A.3})$$

$$i_{D_2} = C_2 \frac{dV_{C_2}}{dt} + \frac{V_{C_1}}{R} + \frac{r_{C_1} C_1}{R} \frac{dV_{C_1}}{dt} + \frac{V_{C_2}}{R} + \frac{r_{C_2} C_2}{R} \frac{dV_{C_2}}{dt}. \quad (\text{A.4})$$

From (A.3) and (A.4) and replacing $T_{C_1} = RC_1$, $T_{C_2} = RC_2$, $k_{a_1} = (1 + r_{C_2}/R)/(1 + r_{C_1}/R + r_{C_2}/R)$, $k_{a_2} = (r_{C_2}/R)/(1 + r_{C_1}/R + r_{C_2}/R)$, $k_{b_1} = (r_{C_1}/R)/(1 + r_{C_1}/R + r_{C_2}/R)$, $k_{b_2} = (1 + r_{C_1}/R)/(1 + r_{C_1}/R + r_{C_2}/R)$ and $k_c = (1)/(1 + r_{C_1}/R + r_{C_2}/R)$, it can obtain

$$\frac{dV_{C_1}}{dt} = \frac{k_{a_1}}{C_1} i_{D_1} - \frac{k_{a_2}}{C_1} i_{D_2} - \frac{k_c}{T_{C_1}} (V_{C_1} + V_{C_2}), \quad (\text{A.5})$$

$$\frac{dV_{C_2}}{dt} = -\frac{k_{b_1}}{C_2} i_{D_1} + \frac{k_{b_2}}{C_2} i_{D_2} - \frac{k_c}{T_{C_2}} (V_{C_1} + V_{C_2}). \quad (\text{A.6})$$

The state variable V_a is found from operational amplifier as

$$\frac{V_{out}}{R_1} - V_{ref} \frac{(R_1 + R_2)}{R_2} = C_a \frac{dV_a}{dt}. \quad (\text{A.7})$$

Replacing $g = \frac{r_a}{R_1}$, $k_D = \frac{R_1 + R_2}{R_2}$ and $T_a = r_a C_a$, it is obtained

$$T_a \frac{dV_a}{dt} = gV_{out} - gk_D V_{ref}, \quad (\text{A.8})$$

where V_{out} is

$$V_{out} = V_{C_1} + r_{C_1} C_1 \frac{dV_{C_1}}{dt} + V_{C_2} + r_{C_2} C_2 \frac{dV_{C_2}}{dt}. \quad (\text{A.9})$$

Finally, the equation system defined in (3.8)–(3.11) are found from (A.1), (A.2), (A.5), (A.6) and (A.8) and replacing the variables V_p , V_s , V_{S_D} , V_{D_2} and i_{D_1} depending

on the switch and diodes positions, as follows:

- **Equation (3.8)** if $S_D = 0$, $D_1 = 0$, $D_2 = 1$.

$$\begin{aligned} & * \frac{di_p}{dt} = 0 \\ & * V_s = r_s i_s + L_s \frac{di_s}{dt} \\ & * V_{D_2} = r_{D_2} i_{D_2} \\ & * i_{D_1} = i_p = 0 \text{ and } i_{D_2} = i_s \end{aligned}$$

- **Equation (3.9)** if $S_D = 0$, $D_1 = 1$, $D_2 = 1$.

$$\begin{aligned} & * V_p = r_p i_p + L_p \frac{di_p}{dt} + M_L \frac{di_s}{dt} \\ & * V_s = r_s i_s + L_s \frac{di_s}{dt} + M_L \frac{di_p}{dt} \\ & * V_{S_D} = r_{D_1} i_{D_1} + V_{C_1} + r_{C_1} C_1 \frac{dV_{C_1}}{dt} \\ & * V_{D_2} = r_{D_2} i_{D_2} \\ & * i_{D_1} = i_p \text{ and } i_{D_2} = i_s \end{aligned}$$

- **Equation (3.10)** if $S_D = 1$, $D_1 = 0$, $D_2 = 0$.

$$\begin{aligned} & * V_p = r_p i_p + L_p \frac{di_p}{dt} \\ & * V_{S_D} = r_{DS(\text{on})} i_p \\ & * \frac{di_s}{dt} = 0, i_{D_1} = 0 \text{ and } i_s = i_{D_2} = 0 \end{aligned}$$

- **Equation (3.11)** if $S_D = 1$, $D_1 = 0$, $D_2 = 1$.

$$\begin{aligned} & * V_p = r_p i_p + L_p \frac{di_p}{dt} + M_L \frac{di_s}{dt} \\ & * V_s = r_s i_s + L_s \frac{di_s}{dt} + M_L \frac{di_p}{dt} \\ & * V_{S_D} = r_{DS(\text{on})} i_p \\ & * V_{D_2} = r_{D_2} i_{D_2} \\ & * i_{D_1} = 0 \text{ and } i_s = i_{D_2} \end{aligned}$$

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