ERRATA

KNOT AND LINK INVARIANTS AND MODULI SPACE OF PARABOLIC BUNDLES

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The paper [4], though correct in principle, contained some technical errors which we shall explain and rectify. The author would like to thank H. Boden who points out these errors in his review [1] and private communications.

The first error occurs on page 519 line 10, the $U(1)$ -stratum should be $\mathcal{S}(S^3 \setminus L, SU(2)) \cong \{\prod_{j=0}^{\nu(L)-1} U(1)\}/\mathbb{Z}_2$ since the center of $SU(2)$ is $\mathbb{Z}_2$ which can be used only once. Consequently, the fixing trace reducible stratum $\mathcal{S}(S^3 \setminus L, SU(2))_\alpha = \{e^{\pm\alpha_1}, e^{\pm\alpha_2}, \dots, e^{\pm\alpha_{\nu(L)}}\}$ consists of $2^{\nu(L)-1}$ isolated points for the choices of $\alpha = (\pm\alpha_1, \pm\alpha_2, \dots, \pm\alpha_{\nu(L)})$. Hence, the statements from Proposition 3.2 to Corollary 3.11, as well as of Theorem 1.2, should be corrected in the same way for $2^{\nu(L)-1}$ many different α from $(\pm\alpha_1, \pm\alpha_2, \dots, \pm\alpha_{\nu(L)})$, and their proofs can proceed as before. For instance, Proposition 3.2 should be corrected as “If $\Delta_L(e^{\pm 2i\alpha_1}, \dots, e^{\pm 2i\alpha_{\nu(L)}}) \neq 0$ and $\rho_0 \in \mathcal{S}(S^3 \setminus L, SU(2))_\alpha$, then there is a neighborhood of ρ_0 consisting of elements of $\mathcal{S}(S^3 \setminus L, SU(2))_\alpha$.” So ρ_0 is an isolated point of $\mathcal{S}(S^3 \setminus L, SU(2))_\alpha$. The sentence “The space $\mathcal{R}(S^3 \setminus L, SU(2))_\alpha$ is continuous on the parameter α .” on page 520, line 11 from bottom, should be deleted since it may create problems at some critical values of α . This sentence is not used in the proof of Proposition 3.7. In Remark 3.8 on page 521, “ Z_L is discrete” should be corrected to “ Z_L is of codimension-one”.

The second error occurs on page 528 in the second paragraph of the proof of Theorem 4.11. On page 528, line 24 “Every character in $X(S^3 \setminus K)_\mu$ is conjugate to a $SU(2)$ representation by an element in $SL_2(\mathbb{C})$.”, this claim “Every character in $X(S^3 \setminus K)_\mu$ is conjugate to a $SU(2)$ representation by an element in $SL_2(\mathbb{C})$ ” is true for finite groups but not for infinite groups since we cannot simultaneously conjugate all the elements in the image in $SU(2)$. Hence Theorem 4.11 should be corrected as “If $\mu \in U(1) \setminus \{\pm 1\}$ with $\Delta_K(\mu^2) \neq 0$ and $S^3 \setminus K$ contains no closed incompressible surface, then there is an injective map from $\mathcal{R}(S^3 \setminus K, SU(2))_\mu$ to $X(S^3 \setminus K)_\mu$. Furthermore, $\lambda_{SL_2(\mathbb{C})}(K)_\mu \geq \frac{1}{2}|\sigma_K(\mu)|$.” Every point p in $\tilde{X}(S^3 \setminus K)_\mu$ contributes by $n_p \geq 1$ and the injectivity will be enough to get the inequality. Corollaries 4.13 and 4.14 follow as before if we correct “there is no cancellation in $\mathcal{R}^*(S^3 \setminus K)_\mu$ ” by the equality hypothesis $\lambda_{SL_2(\mathbb{C})}(K)_\mu = \frac{1}{2}|\sigma_K(\mu)|$. The degree $\deg_l A'_K(l, \mu)$ in Proposition 4.10 and rest of the paper is the one in [2] without repeated factors. Thus the identification does not include the possible multiplicity from other one-dimensional components under the restriction map $X(S^3 \setminus K) \rightarrow X(T^2)$. The multiplicity m on the A -polynomial $A_m(\lambda, \mu) = (A(\lambda, \mu))^m$ will not effect the conjecture in [2, Sec. 7] for the polynomial A .

Note that the invariant $\lambda_{SL_2(\mathbb{C})}$ is well-defined under smooth perturbations (remove “holomorphic” in page 525, line 4). On page 523, line 4, the sentence should be corrected as “Unlike the $SU(2)$ case, $X(S^3 \setminus K)_\mu$ is no longer one-dimensional in general.” In Lemma 4.2(1), the proof only shows that $\{s_\pm\} \subset X(S^3 \setminus K)_{\pm 1}$ not necessary equal between these two sets.

Definition 4.7, Theorem 4.8 and its proof in [4] show that the $SL_2(\mathbb{C})$ -invariant $\lambda_{SL_2(\mathbb{C})}(K)_\mu$ is a well-defined knot invariant only for knots with no closed incompressible surface and $\Delta_K(\mu^2) \neq 0$ (see below for the restatement of [4, Definition 4.7]). A smooth perturbation (see page 525, line 8 from bottom) is necessary in case $X(S^3 \setminus K)_\mu$ has nontransverse points or higher-dimensional components. For general knots, more subtle perturbation will be involved.

Definition 4.7 of [4] (Restate Definition 4.7 of [4]). Let K be a knot in S^3 such that $S^3 \setminus K$ contains no closed incompressible surface, and let μ be an eigenvalue of $\rho(m)$ such that $\Delta_K(\mu^2) \neq 0$. Let $h : X(S^2 \setminus K)_\mu \rightarrow X(S^2 \setminus K)_\mu$ be an isotopy with compact support in the union of all zero-dimensional components of $X^*(D_+^3 \setminus K)_\mu \cap X^*(D_-^3 \setminus K)_\mu$ such that $h(X^*(D_+^3 \setminus K)_\mu) = \tilde{X}^*(D_+^3 \setminus K)_\mu$ intersects $X^*(D_-^3 \setminus K)_\mu$ transversally. Define the $SL_2(\mathbb{C})$ -knot invariant

$$\lambda_{SL_2(\mathbb{C})}(K)_\mu = \#\tilde{X}^*(D_+^3 \setminus K)_\mu \cap X^*(D_-^3 \setminus K)_\mu = \#\tilde{X}^*(S^3 \setminus K)_\mu.$$

Under the hypothesis of the above definition, $\dim_{\mathbb{C}} X(S^3 \setminus K)_\mu = 0$. By using the unique isolated reducible representation, the standard argument gives the smooth perturbation $\tilde{X}^*(D_+^3 \setminus K)_\mu$ which the intersection $\tilde{X}^*(D_+^3 \setminus K)_\mu \cap X^*(D_-^3 \setminus K)_\mu$ is a union of compact zero-dimensional components. Hence, the number $\lambda_{SL_2(\mathbb{C})}(K)_\mu$

is defined. Note that the character subvarieties $X^*(D_{\pm}^3 \setminus K)_{\mu}$ and $X^*(S^2 \setminus K)_{\mu}$ are manifolds for $\Delta_K(\mu^2) \neq 0$, the number $\lambda_{SL_2(\mathbb{C})}(K)_{\mu}$ is independent of the choice of isotopy used in the above definition by standard argument in differential topology (see [3, 5]).

References

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