ERRATUM

EMPIRICAL COPULAS FOR CDO TRANCHE PRICING USING RELATIVE ENTROPY

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Loss Distributions

In Dempster *et al.* [1] empirical copulas were constructed from market CDO tranche quotes. Loss distributions for different time horizons were then generated from these implied copulas and compared to investigate the issue of stationarity of the empirical copula. It was discovered that the computation of these loss distributions contains an error, which this note addresses.

The random *number of defaults* of the portfolio up to a given time horizon T is

$$L(T, \tau) = \sum_{j=1}^n 1_{\{\tau_j < T\}},$$

where $\tau = (\tau_1, \dots, \tau_n)$ is the random vector of default times. We call the probability distribution of this random variable $\mathbf{L}$ the *loss distribution*.

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Denote by $\hat{c}$ the empirical MinRelEnt copula. To generate the loss distribution from $\hat{c}$, we need to compute

$$P(L(T, \tau) = k) = \int_{[0,1]^n} 1_{\{L(T, F(u))=k\}} \hat{c}(u) du,$$

where $F(u) = (F(u_1), \dots, F(u_n)) = (\tau_1, \dots, \tau_n)$ and F is the marginal distribution of default times assumed to be the same for all j . This can be easily computed by Monte Carlo simulation for each $k = 0, \dots, 125$ using the importance sampling method discussed in Dempster *et al.* [1].

The marginal distribution function F constructed from the iTraxx index quotes is given by

$$F(t) = 1 - \exp \left\{ - \int_0^t \lambda(s) ds \right\}.$$

The hazard rate function λ is assumed to be piecewise constant between tranches. Below is the calibration result of λ .

	5 years	7 years	10 years
Index (bps)	33.0	44.0	56.0
	0–5 years	5–7 years	7–10 years
λ	5.63×10^{-3}	1.33×10^{-2}	1.56×10^{-2}

The prior copula was chosen to be the Gaussian copula with correlation parameter $\rho = 0.4$.

Below are the correct loss distributions. Figure 1 shows the 10 year loss distribution generated from the empirical copula that was calibrated to 10 year iTraxx CDO tranche quotes. Included for comparison is the loss distribution generated from the prior Gaussian copula.

We also generate the 5 and 7 year loss distributions from the 10 year MinRelEnt copula, and compare them to the 5 year loss distribution generated from the 5 year MinRelEnt copula, and to the 7 year loss distribution generated from the 7 year MinRelEnt copula. These are shown in Figs. 2 and 3.

We can see that the 5 and 7 loss distributions implied from the 10 year MinRelEnt copula are quite different to that of the true 5 and 7 year loss distributions from the 5 and 7 year MinRelEnt copulas. In particular, we can see that the 5 year loss distribution implied from the 5 year MinRelEnt copula is clearly multi-modal, whereas the one implied from the 10 year MinRelEnt copula is uni-modal. This suggests that the market view on default dependence changes according to maturity, but in what way is not yet clear and requires further research.

(a) Implied from 10 year MinRelEnt copula

(b) Generated from prior Gaussian copula

Fig. 1. Loss distributions for the 10 year horizon.

(a) From 10 year MinRelEnt copula

(b) From 7 year MinRelEnt copula

Fig. 2. Implied loss distributions for the 7 year horizon.

(a) From 10 year MinRelEnt copula

(b) From 5 year MinRelEnt copula

Fig. 3. Implied loss distributions for the 5 year horizon.

Other Errata

In the proof of Proposition 2.1 and also in Eq. (2.6), all the $\bar{a}$'s should be $\tilde{a}$'s and *vice versa*.

In Eq. (4.3), it should be $\beta - \alpha$ instead of $\alpha - \beta$.

Reference

- [1] M. A. H. Dempster, E. A. Medova and S. W. Yang, Empirical copulas for CDO tranche pricing using relative entropy, *International Journal of Theoretical and Applied Finance* **10**(4) (2007) 679–701.