SIZE EFFECT ON THE PERMEABILITY AND SHEAR INDUCED FLOW ANISOTROPY OF FRACTAL ROCK FRACTURES

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Abstract

The effect of model size on fluid flow through fractal rough fractures under shearing is investigated using a numerical simulation method. The shear behavior of rough fractures with self-affine properties was described using the analytical model, and the aperture fields with sizes

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varying from 25 to 200 mm were extracted under shear displacements up to 20 mm. Fluid flow through fractures in the directions both parallel and perpendicular to the shear directions was simulated by solving the Reynolds equation using a finite element code. The results show that fluid flow tends to converge into a few main flow channels as shear displacement increases, while the shapes of flow channels change significantly as the fracture size increases. As the model size increases, the permeability in the directions both parallel and perpendicular to the shear direction changes significantly first and then tends to move to a stable state. The size effects on the permeability in the direction parallel to the shear direction are more obvious than that in the direction perpendicular to the shear direction, due to the formation of contact ridges and connected channels perpendicular to the shear direction. The variances of the ratio between permeability in both directions become smaller as the model size increases and then this ratio tends to maintain constant after a certain size, with the value mainly ranging from 1.0 to 3.0.

Keywords: Fracture; Size Effect; Permeability; Anisotropy.

1. INTRODUCTION

The hydro-mechanical characteristics of single fractures at multiple scales strongly influence the fluid flow through fractured rock masses, the exploitation of unconventional oil and gas reservoirs and the development of nuclear waste repositories.^{1–4} Therefore, modeling of flow behavior considering the influences of geometrical and hydraulic properties of fracture continues to be an active area of scientific investigation.^{5–7} The prerequisites of modeling consist, to a large extent, in determining the complex internal topography of a fracture and its evolution under the mechanical stresses. Thus, various analytical equations, experimental tests and numerical methods have been proposed to quantify the aperture distribution and flow behavior in the fracture under normal and shear stresses.^{8–11}

Single fracture has been approximately modeled by using two smooth parallel plates that are separated by a constant distance, with the fluid flowing through it that follows the cubic law.^{12,13} However, in the nature, the surfaces of most rock fractures are rough, which results in nonuniformly distributed aperture fields in fractures. Various methods based on the techniques such as the tomography scanning and calibration of dyed fluid have been proposed to measure the roughness of fracture surface without destroying the initial fracture sample.^{14,15} Correspondingly, the parameters such as joint roughness coefficient (JRC), the root-mean-square of the fracture profile wall slope (Z_2) and fractal dimension (D_f) are defined to quantitatively characterize the surface roughness.^{16–18} Laboratory tests and numerical simulations have been performed to classify how the fracture roughness affects the fluid flow

paths and the permeability of fractures using these surface roughness parameters. The results show that the rough fracture walls generate the complex and variable aperture fields that induce the tortuous flow paths. Compared with the parallel plate model, the variable apertures can either increase or decrease the permeability, depending on the distributions of flow channels and the contact areas.^{19–21}

The rock fractures in the deep underground are generally subjected *in situ* stresses containing normal and shear stresses.^{22–25} The aperture fields and the flow paths in fractures are changed by both normal stress and shear displacement. The normal stress tends to close the fracture with more contact areas that hinder the flow through the fracture, while the shear displacement tends to open the fracture due to shear dilation and thus increases the permeability of the fracture.

The hydraulic properties of fractured rock masses are usually scale dependent, and the small laboratory samples only represent a fraction of natural fracture characteristics. Bandis *et al.*²⁶ conducted a series of direct shear experiments to analyze the size effects on the shear behavior of rock fractures. They concluded that the significant scale effects exist on both shear strength and deformation response, and the potential sources of the scale effects are mainly attributed to the geometrical and strength characteristics of fracture surface roughness. Giwelli *et al.*²⁷ studied the effect of fracture size on the closure behavior of fractures and the results show that the closure increases with the increasing fracture size. The shear induced aperture field and the flow behavior through the fracture are also scale dependent. Figure 1 shows the fracture samples of

Fig. 1 Different laboratory samples extracted from the *in situ* fracture with the flow channels indicated.

different laboratory sizes that are extracted from the *in situ* fractures.²⁸ The distribution of apertures varies with sample size, which results in different flow channels through the fractures

This study aims to investigate the influences of fracture size on the permeability and the shear induced flow anisotropy of the natural rough-walled rock fractures. A series of fractures with different surface roughness that was represented by JRC and

fractal dimension (D) were generated. Shear displacements varying from 3 to 20 mm were applied to each fracture, and aperture fields of 25–200 mm in size were extracted from the model at each shear displacement. Fluid flow through the fracture was simulated by solving the Reynolds equation. The aperture variation and flow tortuosity that were resulted from fracture surface roughness and shear displacement were estimated. Influences of fracture

Fig. 2 Fracture surfaces of different roughness generated using the modified SRA method: (a) $JRC = 2.49$, $D_f = 2.35$; (b) $JRC = 7.36$, $D_f = 2.40$; (c) $JRC = 12.38$, $D_f = 2.45$ and (d) $JRC = 17.31$, $D_f = 2.50$.

size on the flow channels, permeability and the flow anisotropy of the fracture were analyzed.

2. FRACTURE SURFACE PROPERTIES

The accuracy of quantification of the aperture field for the rough fracture depends, to a large extent, on the model of fracture surface that is usually stochastic and irregular. Fractal geometry that was developed by Mandelbrot *et al.*²⁹ has been widely used to model the irregular objects and fractional surfaces.^{30–34} The previous studies have shown that the real fracture surfaces can be described by fractal geometry obeying the self-affine scaling fractal law.^{35,36} This is of great help in many cases since it provides a numerical method to generate the fracture surface that mimic rough characteristic of natural rock fractures.

The method of self-affine fracture surface generation employed in this study is similar with the earlier works that used Fractional Brownian motion (fBm) to model the surface.^{37–39} In 2D fBm, a function $Z(x, y)$ with the independent spatial variables x and y is defined and it is continuous and single-valued. The stationary increment, $[Z(x + l_x, y + l_y) - Z(x, y)]$, over the distance $l = \sqrt{l_x^2 + l_y^2}$ displays a Gaussian distribution. The statistical self-affine fBm increment is expressed as

$$\langle Z(x + rl_x, y + rl_y) - Z(x, y) \rangle = 0, \quad (1)$$

$$\varepsilon^2(r) = r^{2H} \varepsilon^2(1), \quad (2)$$

where $\langle \cdot \rangle$ denotes the mathematical expectation, r is a constant, ε^2 is the variance and H is Hurst

exponent that is related to the 3D fractal dimension (D_f) by $D_f = 3 - H$.³⁹ The modified successive random addition (SRA) algorithm proposed by Liu *et al.*⁴⁰ is used to generate the related fBm. Different random numbers are used to generate the fracture surface. When these random numbers are fixed, the fracture surface can be obtained repeatedly.

The JRC is proposed by Barton and Choubey⁴¹ to describe the surface roughness of a rock fracture. Based on the height fields of the rough-walled fracture surface, the JRC can be calculated using the following equation that proposed by Tse and Cruden¹⁶:

$$\text{JRC} = 32.2 + 32.47 \log Z_2, \quad (3)$$

where Z_2 is the root-mean-square of the fracture profile wall slope. Figure 2 shows four generated rough fracture surfaces of 256 mm in size using modified SRA algorithm. The calculated JRC and D_f for each fracture surface are shown in Fig. 2. It indicates that a small JRC, which corresponds to a small D_f represents a smooth surface with a high spatial correlation, and a large JRC represents a rough surface with many precipitous asperities.

3. FLOW CALCULATION

3.1. Aperture Assignment

The distribution of fracture aperture defined as the separation distance between two surfaces is one of the most critical factors that determine the hydro-mechanical property of rough fractures. In this study, the two surfaces of a fracture are assumed to be matched at the initial state. The two fracture

Fig. 3 The extraction of aperture fields of different sizes for fracture with $\text{JRC} = 17.31$ at $s = 20$ mm.

surfaces are initially contacted with the apertures equal to 0. When a series of shear displacements from 3 to 20 mm are applied to the fracture, the two fracture surfaces will be shifted and the aperture field is changed with the shear displacement. The aperture distribution at each shear displacement is calculated using the following equation:

$$b(x, y) = \begin{cases} Z(x + s, y) - Z(x, y) + u & \text{if } Z(x + s, y) > Z(x, y) - u, \\ 0 & \text{if } Z(x + s, y) \leq Z(x, y) - u \end{cases} \quad (4)$$

where s is the shear displacement, u is the normal displacement generated at s , and it can be determined using the analytical model described by Indraratna *et al.*⁴² In order to investigate the influence of size scale on the flow behavior, aperture fields with different sizes $L = 25, 50, 75, 100, 150$ and 200 mm are extracted from the central part of the initial generated aperture model at each s . Figure 3 shows examples of the aperture fields with different sizes extracted from the model at $s = 20$ mm for fracture with $JRC = 17.31$.

3.2. Governing Equation

At the microscopic scale, the fluid flow through fractures is generally governed by the nonlinear Navier–Stokes equations. However, due to

Fig. 4 The aperture distribution and main flow channels for fracture with $JRC = 2.49$. The flow paths consist of the elements with the ratios of local flow rate to the total flow rate larger than 0.005.

the highly nonlinear system, exact solutions of Navier–Stokes equations are usually very difficult to achieve, especially in the case of natural rough fractures with complex geometry. For a fracture of two perfectly smooth parallel plates, the Navier–Stokes equations can be simplified to the cubic law, written as¹²

$$Q = -\frac{b^3}{12\mu}\nabla P. \quad (5)$$

However, natural fractures have anisotropic apertures formed by the two rough walls, where the cubic law is obviously over-simplified. When flow velocity is low and the fracture surface does not vary too abruptly, the local cubic law, which is also called as Reynolds equation, can be used to describe

the flow in fractures:

$$\nabla(b^3\nabla P) = 0. \quad (6)$$

This equation has been utilized to analyze the flow through geological rough fractures.^{43–46} In this method, the void spaces between the two walls of a fracture are divided into a lot of small simplified parallel-plates, and the aperture of each small plate is calculated based on the Eq. (4). The flow equation is solved using the finite element method (FEM) software COMSOL Multiphysics.³⁸ The total flux Q through the fracture equals to the integral of local fluxes through the cross-section of the fracture that can be calculated based on the simulation results. Then, equivalent permeability K of the fracture is

Fig. 5 The aperture distribution and main flow channels for fracture with JRC = 7.36. The flow paths consist of the elements with the ratios of local flow rate to the total flow rate larger than 0.005.

evaluated by

$$Q = A \frac{K}{\mu} \frac{\Delta P}{L}, \quad (7)$$

where L is the fracture size, A is the cross-sectional area. In order to investigate the flow anisotropy in the rough fracture, two different boundary conditions are considered: unidirectional flow in x -direction (parallel to the shear direction) and y -direction (perpendicular to the shear direction). For all extracted models with different sizes, a constant hydraulic gradient is maintained between the inlet and out boundaries, and other boundaries are fixed with impervious conditions.

4. EVOLUTIONS OF APERTURE FIELDS AND FLOW CHANNELS

The complex flow channels in a fracture have a close relationship with the distribution of fracture apertures. As shear displacement increases, the gradually shifted two fracture rough walls will generate a changed aperture field. Figures 4–7 show the aperture distribution and main flow paths for fractures with different L and JRC under different s , where the magnitude of aperture is indicated by the legend and the main flow paths consist of the elements with the ratios of local flow rate to the total flow rate larger than 0.005. For all fractures, with the

Fig. 6 The aperture distribution and main flow channels for fracture with JRC = 12.38. The flow paths consist of the elements with the ratios of local flow rate to the total flow rate larger than 0.005.

Fig. 7 The aperture distribution and main flow channels for fracture with JRC = 17.31. The flow paths consist of the elements with the ratios of local flow rate to the total flow rate larger than 0.005.

increasing of s , the apertures increase and the contact areas indicated by the white parts decrease, in spite of the fracture length. This is mainly caused by the shear dilations.^{45,46} Comparisons of the aperture distributions of the four fractures at three different s and three different L show that the fracture with a larger JRC generates a larger aperture field.

There exists obvious flow channels for all fractures with different s and different L . At a small s of 3 mm, a great number of narrow flow channels exist, bypassing many small pieces of contact areas. The flow paths are distributed almost evenly within the entire fracture plane. As the shear moves on, the flow starts to converge into a few main flow channels, with the dramatic increases of apertures and decreases of contact areas. As s increases to 20 mm, most of the flow concentrates into a few

major channels that mainly distributes on the partial areas of fracture plane. Comparisons of the flows of four fractures at three different s and three different L show diversity of flow patterns within the fracture planes. The main flow areas within the fracture plane tend to increase as JRC increases. The flow pattern of fracture with a small size is different with the flow pattern that directly extracted from the model with a large size, which indicates the increase of fracture size alters the effective flow channels through the model. For the fracture with JRC = 2.49, $L = 50$ mm and $s = 20$ mm, there only exists one main flow path between the inlet and outlet boundaries. The fracture with a smaller size is more likely to have a zero aperture field, which results in a totally impermeable model. As fracture size increases, more nonzero aperture areas tend to

be involved and more alternative flow paths will be available.

5. EVOLUTION OF PERMEABILITY

5.1. Size Effect on the Equivalent Permeability

The equivalent permeability of each fracture model is calculated and the variations of equivalent permeability (K_x) in direction parallel to the shear direction with L at different s for fractures of different JRC are plotted in Fig. 8. For all fractures, K_x increases by almost two orders of magnitudes as the s increases from 3 to 20 mm, which is consistent with the variances of flow channels analyzed

in Sec. 4. Comparisons of the K_x of the four fractures at the same s show that a rougher fracture has a larger permeability. The variation of K_x changes significantly when L is small since the distribution of aperture is almost independent of L for small fracture models. When L increases to a certain value, K_x would change slightly so that the variances can be almost negligible. For the fracture with $JRC = 2.49$, K_x varies by almost two orders of magnitudes as L changes from 25 to 200 mm, where a sharp decrease at $L = 50$ mm occurs. This variance tends to decrease as JRC increases. One possible explanation is that the rougher fracture generates larger apertures over the entire plane and the possibility to have an extreme low permeable area is small.

Fig. 8 Variations of K_x with L at different s for fractures of different JRC: (a) JRC = 2.49; (b) JRC = 7.36; (c) JRC = 12.38 and (d) JRC = 17.31.

Fig. 9 Variations of K_y with L at different s for fractures of different JRC: (a) JRC = 2.49; (b) JRC = 7.36; (c) JRC = 12.38 and (d) JRC = 17.31.

Figure 9 shows the variations of equivalent permeability (K_y) in direction perpendicular to the shear direction with L at different s for fractures having different JRC, following similar trends with those of K_x . However, K_y is generally larger than K_x and the size effects on K_y are less obvious than that on K_x . There may be two reasons: on the one hand, since the two surfaces of the fracture are well matched at the initial state, the upper fracture surface climbs the asperity as the shear displacement increases, which results in the collision between the adjacent fracture asperities and thus generates many stripy ridges of contact perpendicular to the shear direction. These stripy ridges of contacts tend to prevent the flow parallel to the shear direction. On the other hand, the shear-induced void channels tend to be connected along the direction perpendicular to the shear displacement, which generates

more effective flow paths perpendicular to the shear direction.

5.2. Size Effect on the Anisotropy of Permeability

Previous studies have shown that the permeability of rough fracture shows remarkable anisotropy during shear.^{47–50} In order to examine the size effect on the anisotropy of permeability, the variations of K_y/K_x with L at different s for fractures of different JRC are plotted in Fig. 10. The ratio of K_y/K_x varies remarkably at the small L . As L continues to increase, K_y/K_x tends to move to a relatively stable state with the value mainly ranging from 1.0 to 3.0. For the fractures with JRC = 2.49, K_y/K_x varies by about two orders of magnitudes as L increases from 25 to 200 mm. But the changes of K_y/K_x with

Fig. 10 Variations of K_y/K_x with L at different s for fractures of different JRC: (a) JRC = 2.49; (b) JRC = 7.36; (c) JRC = 12.38 and (d) JRC = 17.31.

L for other three fractures with larger JRC become to be less dramatic. This is most likely because of the influences of fracture roughness on the aperture fields that in turn depend on the magnitude of fracture size.

6. CONCLUSIONS

In this study, the shear flow tests were simulated on a series of fracture surfaces with self-affine properties. Aperture fields with the size varying from 25 to 200 mm were extracted under different shear displacements to investigate the size effects on the permeability and shear induced flow anisotropy of rock fractures. Based on the aperture data at each shear interval, fluid flow was simulated by

solving Reynolds equation using the FEM software COMSOL Multiphysics. Permeability in the directions both parallel and perpendicular to the shear direction was calculated. Flow channeling and shear induced flow anisotropy in fractures of different sizes under different shear displacements were estimated.

The results show that aperture fields and flow paths in rough rock fractures are influenced by the topography of fracture surface and shear condition. As fracture surface roughness and/or shear displacement increase, the apertures increase and the contact areas decrease in spite of the fracture length, which is mainly caused by the shear dilations for rough fractures. At a small shear displacement, there are a great number of narrow flow

channels that are indicated by the elements with the ratios of local flow rate to the total flow rate larger than 0.005. As the shear displacement increases, the flow tends to converge into a few main flow channels, with the dramatic increases of apertures and decreases of contact areas. The shapes of flow channels change significantly as the fracture size increases. The permeability in the directions both parallel and perpendicular to the shear direction changes significantly when fracture size is small, and then changes slightly when model size increases to a certain value. The size effects on the permeability in x -direction tend to be more obvious than that in y -direction. This is mainly caused by the formations of contact ridges and connected channels perpendicular to the shear direction during the shear. The variances of the ratio between permeability in the x -direction and that in the y -direction become smaller as the model size increases and then this ratio tends to maintain constant after a certain size, with the value mainly ranging from 1.0 to 3.0. In the future, we plan to extend our study to the motion of solute particles in rough fractures, and investigate how the factors including fracture size, roughness and shear displacement affect the solute transport behavior.

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