FRACTAL COMPLEXITY-BASED FEATURE EXTRACTION ALGORITHM OF COMMUNICATION SIGNALS

HUI WANG,^{*} JINGCHAO LI,^{*,†} LILI GUO,^{*} ZHENG DOU,^{*}
YUN LIN^{*,§} and RUOLIN ZHOU[‡]

^{}College of Information and Communication Engineering
Harbin Engineering University, Harbin, P. R. China*

*[†]College of Electronic and Information Engineering
Shanghai Dianji University, Shanghai, P. R. China*

*[‡]Department of Electrical and Computer Engineering
Western New England University, Springfield, MA, USA*

[§]linyun@hrbeu.edu.cn

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Abstract

How to analyze and identify the characteristics of radiation sources and estimate the threat level by means of detecting, intercepting and locating has been the central issue of electronic support in the electronic warfare, and communication signal recognition is one of the key points to solve this issue. Aiming at accurately extracting the individual characteristics of the radiation source for the increasingly complex communication electromagnetic environment, a novel feature extraction algorithm for individual characteristics of the communication radiation source based on the fractal complexity of the signal is proposed. According to the complexity of the received signal and the situation of environmental noise, use the fractal dimension characteristics of different complexity to depict the subtle characteristics of the signal to establish the

[§]Corresponding author.

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characteristic database, and then identify different broadcasting station by gray relation theory system. The simulation results demonstrate that the algorithm can achieve recognition rate of 94% even in the environment with SNR of -10dB , and this provides an important theoretical basis for the accurate identification of the subtle features of the signal at low SNR in the field of information confrontation.

Keywords: Communication Signals; Feature Extraction; Fractal Dimension; Gray Relation Theory.

1. INTRODUCTION

In the future information battlefield, various communication equipment of radiation source individuals are required to implement important tasks, e.g. offering command, control, and intelligence. Nowadays, as electronic information technology develops more and more rapidly, and the modulation pattern of communication signal becomes more and more complex, the number of communication radiation source individual equipment also increases gradually, which makes the information acquisition of communication radiation source more and more difficult,¹⁻³ and then brings great challenges for communication detection in the electronic confrontation field. In such a complex electromagnetic environment, it is difficult to meet the needs of modern battlefield by traditional means of communication detection, and using simple feature parameters,⁴⁻⁶ such as modulation parameters, carrier frequency, bandwidth, symbol rate, electric level, cannot achieve the purpose of identification of individual radiation sources. Therefore, if we can effectively extract features of the radiation source individuals, different broadcasting stations in the complex electromagnetic environment can be analyzed and recognized.^{7,8} By analyzing the effective characteristic attribute of the broadcasting station, and distinguishing the type of the broadcasting station, the type, location, threat level, interference object, and the nature of the enemy broadcasting station can be determined, and it has great application value for electronic reconnaissance, military strike and interfere with enemy signal, and meanwhile provides an important theoretical basis of command, control and so forth for military forces. Reference 9 presented a electrocardiogram (ECG) feature extraction method that consists of the morphology analysis, the fiducial point localization, and time intervals measurements by using both continuous and discrete wavelets transform (CWT and DWT, respectively).

The results were validated, and it is good method to extract signals' features under relatively high SNR environment. Reference 10 proposed two separate theoretical frameworks that can include existing multiple-parameter transforms as special cases to construct new types of transforms that is useful in signal processing and information security. A novel automatic modulation classification method based on the graph presentation of the cyclic spectrum is proposed in Ref. 11. Monte Carlo simulation results demonstrate that this proposed method can achieve much better classification accuracy than the traditional method. Reference 12 presents a methodology aimed at extracting features to obtain information that will highlight disturbances related to the field of power quality. It can classify disturbances effectively and with low computational effort. A novel method¹³ based on singular value decomposition and the main ridge slice of ambiguity function is presented for attaining a higher correct recognition rate of radar emitter signals in case of low signal-to-noise ratio. Reference 14 presented a complete framework that allows blind estimation of dynamic stream weights for audio-visual speech recognition based on coupled hidden Markov models. It significantly enhances the performance of the audio-visual automatic speech recognition system in all examined test conditions. Reference 15 proposed the method of wavelet transform combined with neural network for signal processing and feature extraction of EEG (Electroencephalogram) signal, which receive good recognition results.

Aiming at the recognition of modulation characteristics of different communication signals in the low SNR environment and the identification of subtle features of the same signal with different distribution noise, this paper proposes a new feature extraction algorithm based on multi-dimensional fractal dimension features. The main contributions of this paper are as follows: it selects the corresponding fractal feature extraction algorithm

automatically according to the complexities of different signals first, in order to improve the recognition performance and reduce the complexity of the algorithm. And then, gray relation algorithm is used to classify and identify the feature to verify the effectiveness of the proposed algorithm.

2. BASIC THEORY

Fractal theory^{16–18} is called the geometry of the nature, which is a branch of modern mathematical theory. It is complementary to the dynamic system of chaos theory. In 1975, the American mathematician who born in France, B B Mandelbrot, first proposed the concept of fractal geometry. With the continuous development of fractal theory, it pointed out that any part of the world are possible (e.g. energy, function, information, time, structure, form, etc.) in a certain process or under certain conditions, which showed a similarity to the whole. The change in spatial dimension can be continuous, also can be discrete, which expanded the people's vision.

At present, there are many definitions and measurement methods of fractal dimension. The common one-dimensional fractal dimension is Hausdorff dimension, Higuchi dimension, Petrosian dimension and Box dimension. The Hausdorff dimension^{19,20} is the most basic fractal dimension. But its computational complexity is high and its computational complexity is large, which is difficult to be realized in practical communication. The Box dimension²¹ is relatively simple, and the fractal dimension of the extracted signal can be obtained by adjusting the size of the side length of the box. So it has been widely applied. The multi-fractal dimension²² characterized the fractal dimension of the signal in multiple layers which based on the one-dimensional fractal dimension. And it is better to describe the subtle changes of the signal compared with the one-dimensional fractal dimension. However, the complexity is increasing. Therefore, how to select the corresponding fractal dimension algorithm according to the complexity of the signal, and then accurately identify the different communication signals is the key point in today's study.

2.1. Fractal Box Dimension

Fractal dimension is a tool to depict complexity characteristics of object. Box dimension is one algorithm in many calculation methods which is simple and has small amount of calculation, it can well describe the complexity characteristics of the signal.

For (X, d) is a metric space, M is the no empty tight set family of X , suppose A is a no empty tight set of X , for each positive number ε , $N(A, \varepsilon)$ represent the number of the minimum box covering A , the side length of the box is ε , so:¹⁸

$$N(A, \varepsilon) = \left\{ M : A \subset \sum_{i=1}^M N(x_i, \varepsilon) \right\}, \quad (1)$$

where $x_1, x_2, \dots, x_M$ are the different points of X , define box dimension as follows:

$$D_b = \lim_{\varepsilon \rightarrow 0} \frac{\ln N(A, \varepsilon)}{\ln(1/\varepsilon)}. \quad (2)$$

Suppose the discrete signal $x(i)$, the highest resolution is the sampling interval ε , use approximate algorithm, let minimum side length of the covering box equal to signal sampling interval ε , calculate the grid counting $N_{k\varepsilon}$ of the box with side length $k\varepsilon$ of the discrete signal $x(i)$ in turn, so

$$s_1 = \max\{x_{k(i-1)+1}, x_{k(i-1)+2}, \dots, x_{k(i-1)+k+1}\}, \quad (3)$$

$$s_2 = \min\{x_{k(i-1)+1}, x_{k(i-1)+2}, \dots, x_{k(i-1)+k+1}\}, \quad (4)$$

$$s(k\varepsilon) = \sum_{i=1}^{N_0/k} |s_1 - s_2|, \quad (5)$$

where $i = 1, 2, \dots, N_0/k$, N_0 is the number of sampling points, and $K < N_0$, $s(k\varepsilon)$ is the scale of longitudinal coordinate of the signal, so $N_{k\varepsilon}$ can be expressed as

$$N_{k\varepsilon} = s(k\varepsilon)/k\varepsilon + 1. \quad (6)$$

Choose a section with good linear degree of fitting curve $\lg k\varepsilon \sim \lg N_{k\varepsilon}$ as the free-scale area, so

$$\lg N_{k\varepsilon} = -d_B \lg k\varepsilon + b, \quad (7)$$

where $k_1 \leq k \leq k_2$, k_1, k_2 were the beginning and the end of the free-scale area respectively. In theory, using the least squares method to calculate the slope of the straight line, which is the fractal box dimensions to be calculated of discrete signal:

$$D = - \frac{(k_2 - k_1 + 1) \sum (\lg k) \cdot \lg N_{k\varepsilon} - \sum (\lg k) \cdot \sum \lg N_{k\varepsilon}}{(k_2 - k_1 + 1) \sum \lg^2 k - (\sum \lg k)^2}. \quad (8)$$

Fractal box dimension has no absolute meaning, just has relatively value. So it is valuable to use the same method to calculate different signal's box counting dimension in practical application.

According to the definition of fractal box dimension, after discussing its basic properties, the anti-noise performance of fractal box dimension is analyzed as follows:

Set signal sequences:

$$y(i) = x(i) + n(i), \quad (9)$$

where $i = 1, 2, \dots, N$, $x(i)$ is the useful signal, $n(i)$ is the additive noise.

Let N the number of box covering the whole plane, $N_i(y)$ represents the number of box covering the signal in certain signal-to-noise ratio, $N_i(x)$ represents the number of box covering useful signal, $N_i(n)$ represents the number of box covering noise, it follows:

$$N_i(y) = N_i(x) + N_i(n). \quad (10)$$

So the probability of box number covering signal wave-form point in all the box can be expressed as follows:

$$p_i(y) = \frac{N_i(y)}{N} = \frac{N_i(x) + N_i(n)}{N} = p_i(x) + \Delta_i(n), \quad (11)$$

where $\Delta_i(n) = N_i(n)/N$, because N is larger, and $N_i(n)$ is smaller, so

$$p_i(y) = p_i(x) + \Delta_i(n) = p_i(x). \quad (12)$$

We can see from the formula (12) that the noise has less effect on the fractal box dimension, it has a good anti-noise property for signal identification. Therefore, it is possible to classify signal roughly using fractal box dimension characteristics.

2.2. Higuchi Fractal Dimension

Higuchi fractal dimension is one kind of one-dimensional fractal dimension, which is defined as follows:

Suppose the time sequence is

$$x(1), x(2), \dots, x(N),$$

and the reconstructed time sequence is x_m^k :

$$x_m^k = \left\{ x(m), x(m+k), x(m+2k), \dots, x\left(m + \left[\frac{N-m}{k}\right] \cdot k\right) \right\}, \quad (13)$$

where N represents the length of the numeric sequence x , $m = 1, 2, \dots, k$ represents the initial time value of the sequence, k represents the time interval between two adjacent time series. Symbol

$[x]$ represents the integer part of x , For each reconstructed time series x_m^k , calculate the average length of the sequence $L_m(k)$, then:

$$L_m(k) = \frac{\sum_{i=1}^{\lfloor (N-m)/k \rfloor} |x(m+ik) - x(m + (i-1) \cdot k)| \cdot (N-1)}{\left[\frac{N-m}{k}\right] \cdot k}, \quad (14)$$

where N is the length of numeric sequence x , $\frac{(N-1)}{\left[\frac{N-m}{k}\right] \cdot k}$ is the normalization factor. For all, calculate the average length of the signal, where, hence, to every k value, the total average length of the discrete time signal sequence can be calculated as

$$L(k) = \sum_{m=1}^k L_m(k). \quad (15)$$

At this time, the total average length $L(k)$ of the discrete time signal sequence is proportional to the scale k , that is

$$L(k) \propto k^{-D}. \quad (16)$$

Take logarithm on both sides, then the following can be obtained:

$$\ln(L(k)) \propto D \cdot \ln\left(\frac{1}{k}\right). \quad (17)$$

The least squares method is used to fit the curve $\ln(\frac{1}{k}) \cdot \ln(L(k))$, and the slope D of the fitting curve is the Higuchi fractal dimension.

2.3. Petrosian Fractal Dimension

Singular spectrum entropy²¹ is a partial analysis of subtle features of signal from the perspective of singularity. Suppose the receiver simultaneously collected information from different channels with the number of L . Let $X_t = \{x_t^1, x_t^2, \dots, x_t^L\}$ represent the received information sequence. For the signal in one of the channels, the signal after sampling is $\{x_i, i = 1, 2, \dots, N\}$, and the sampling number is N .

Suppose the waveform signal consists of a series of points $\{y_1, y_2, \dots, y_N\}$, binary the sequence first, and set the binary matrix z_i , then:

$$z_i = \begin{cases} 1, & y_i > \text{mean}(y), \\ -1, & y_i \leq \text{mean}(y), \end{cases} \quad i = 1, 2, \dots, N, \quad (18)$$

where $i = 1, 2, \dots, N$ represent the points' number of the signal.

So the Petrosian fractal dimension can be defined as

$$D = \frac{\log_{10}^N}{\log_{10}^N + \log_{10} \left(\frac{N}{N+0.4N_{\Delta}} \right)}, \quad (19)$$

where N_{Δ} is the total changes' number of adjacent symbol z_i :

$$N_{\Delta} = \sum_{i=1}^{N-2} \left| \frac{z_{i+1} - z_i}{2} \right|. \quad (20)$$

From the definition of Petrosian fractal dimension, it can be known that, Petrosian fractal dimension is a relatively simple definition of the fractal dimension, compared with the other fractal dimension, the calculated method is easier.

2.4. Katz Fractal Dimension

Suppose the waveform signal consists of a series of points (x_i, y_i) , the length of signal is N . Then Katz fractal dimension is given by

$$D = \frac{\log(N)}{\log(N) + \log\left(\frac{d}{L}\right)}, \quad (21)$$

where define L as the length of the waveform, and L can be described as follows:

$$L = \sum_{i=0}^{N-2} \sqrt{(y_{i+1} - y_i)^2 + (x_{i+1} - x_i)^2}. \quad (22)$$

Define d as the maximum distance from the initial point (x_1, y_1) to the other points, then the d can be calculated as

$$d = \max \left(\sqrt{(x_i - x_1)^2 + (y_i - y_1)^2} \right). \quad (23)$$

2.5. Sevcik Fractal Dimension

Similarly, suppose the waveform signal consist of a series of points (x_i, y_i) , the length of the signal is N .

Normalize the signal first, then

$$x_i^* = \frac{x_i - x_{\min}}{x_{\max} - x_{\min}}, \quad y_i^* = \frac{y_i - y_{\min}}{y_{\max} - y_{\min}}. \quad (24)$$

So the Sevcik fractal dimension D can be calculated as

$$D = 1 + \frac{\ln(L) + \ln(2)}{\ln[2 \times (N - 1)]}, \quad (25)$$

where L is the length of the waveform, and L can be calculated as:

$$L = \sum_{i=0}^{N-2} \sqrt{(y_{i+1}^* - y_i^*)^2 + (x_{i+1}^* - x_i^*)^2}. \quad (26)$$

2.6. Multi-Fractal Dimension

Multi-fractal dimension is used to describe signal characteristics of different levels, and it is also used to discuss the probability distribution of parameters. It divides the research object (set its linear degree equal to 1) into N small areas. Suppose the line degree of areas i is ε_i , and the density distribution function P_i of area i can be described by different scaling index α_i as

$$P_i = \varepsilon_i^{\alpha_i}, \quad i = 1, 2, \dots, N_i. \quad (27)$$

Non-integer α_i can commonly be called as singular index, and its value is relevant to its region.

In order to get a series of distribution characteristics of the subset, define the function $X_q(\varepsilon)$ as follows, which is the weighting summation of probability in each area.

$$X_q(\varepsilon) = \sum_{i=1}^N P_i^q. \quad (28)$$

Then, further define the generalized fractal dimension D_q as

$$\begin{aligned} D_q &= \frac{1}{q-1} \lim_{\varepsilon \rightarrow 0} \frac{\ln X_q(\varepsilon)}{\ln \varepsilon} \\ &= \frac{1}{q-1} \lim_{\varepsilon \rightarrow 0} \frac{\ln \left(\sum_{i=1}^N P_i^q \right)}{\ln \varepsilon}. \end{aligned} \quad (29)$$

$X_q(\varepsilon)$ reflects the different effects of P_i . The formula shows that, when $q \gg 1$, large probability area plays the leading role in the summation formula $\sum_{i=1}^N P_i^q$, and at this time, $X_q(\varepsilon)$ and D_q reflect the properties of big probability area. Conversely, when $q \ll 1$, $X_q(\varepsilon)$ and D_q reflect the properties of small probability areas (sparse area). In this way, different q values reflect the properties of different probability characteristic areas. Through the weighting summation processing, it divides the signal into many areas with different singular degree, and therefore the signal's internal structure is hierarchical.

When $q = 0, 1, 2$, D_q is defined as capacity dimension (box dimension) D_0 , information dimension D_1 and correlation dimension D_2 , respectively.

Through the phase space reconstruction of signal, then calculate the value of P_i , so different probability characteristics of the signal can be extracted, and multi-level feature results can be obtained.

In summary, six commonly used fractal dimension algorithms were defined, and the waveform features can be extracted. The calculation method of

Higuchi fractal dimension, Petrosian fractal dimension, Katz fractal dimension and Sevcik fractal dimension is simpler to some extent. But the feature extracted is not precise, so it is suitable for rough features extraction. Compared with other fractal dimension algorithms, fractal box dimensions can extract the waveform features more precisely. So it is widely applied in many fields. And the computational complexity is also increasing. Multi-fractal dimension can describe the characteristics of the signal from different perspective, the computational complexity also increased. It has better application for the subtle features' extraction of the same signal in different noise environments. Therefore, this paper chooses the corresponding fractal dimension feature extraction algorithm based on the prior analysis of the signal and noise complexity, and makes an optimal choice in the complexity and effectiveness of the algorithm.

3. SYSTEM IDENTIFICATION MODEL

Subtle feature extraction is the key link in the process of radiation source identification. The quality of the extracted features directly affects the recognition rate and the complexity of the subsequent classifier design, so the subtle feature extraction occupies a pivotal position in the communication signals' identification system. Aiming at the problem that the signals' features are difficult to be extracted in complex electromagnetic environment, this paper proposes a multi-dimensional fractal dimension based signal feature extraction algorithm, which characterizes the modulation complexity of communication signals from different geometrical scales, and then achieves the purpose of different complexity signals' recognition. According to the type and complexity of the signals, one-dimensional fractal dimension can be first chosen to extract the rough features of different modulation signals. For the signals with poor separation degree, multi-fractal dimension algorithm was used to extract the detailed features of the signals. For the same signal with different noise in the low SNR environment, it is difficult to characterize the features using the simple fractal dimension. So phase space reconstruction is directly used, then calculate the multi-fractal dimension of the signals to realize the recognition of noise's subtle feature. Finally, gray relation algorithm is used to realize the accurate classification of the extracted features. So the

Fig. 1 Multi-fractal dimension-based recognition system model of communication signals.

effectiveness of the proposed algorithm can be verified. The flow chart of the identification system can be shown in Fig. 1.

The subtle features of individual signals can be seen as a time-varying random function. Generally speaking, for the pure internal noise, the individual nuances from different signals can be distinguished by waveforms. This is a process to extract the signals' features from global to local, macro to micro. Fractal theory is similar with it in nature, it is a general term for self-similar structures with no characteristic length but a certain meaning. It has a fine structure and a certain self-similarity in statistics, in which the fractal box dimension reflects the geometric scale of the signal sequence, and the information dimension can reflect the information of the signal sequence on the distribution density. The subtle features of the communication signals can be extracted by using the noise characteristic of the radiation source, which is mainly reflected in the amplitude, frequency, and phase of the internal noise. The characteristics of the noise include their geometric and distributed information. Therefore, it is possible to use the fractal dimension to describe the complexity of the radio signal, so it is possible to extract the subtle features of the radio station. Furthermore, using the extended multi-fractal dimension to extract the

features of noise from different levels, and describe the waveform points from different probability, it can improve the number of differentiated noise categories and to more accurately identify the nuances of different types of radiation sources.

4. BASIC STEPS OF PROPOSED ALGORITHM

According to the analysis of system block diagram, the corresponding fractal dimension characteristics can be selected according to the signals' complexity and different SNR environment to extract the subtle features of the signals. Specific implementation steps can be described as follows.

4.1. Feature Extraction of Different Modulation Signals with the Same Noise

For different modulated signals with the same white noise, the feature is more obvious than the signals with different noise distribution. In general, it is not necessary to use multi-fractal dimension directly to extract the features which has higher complexity. Therefore, for different communication modulation signals in the relatively higher SNR environment, roughly classify the signals first and then classify the signals in detail. It has less amount of calculation. For the signals that cannot be accurately identified in low SNR, multi-fractal dimension algorithm can be used to calculate, the specific steps are as follows:

- (1) Firstly, using one-dimensional fractal dimension algorithm that introduced in Secs. 2.1–2.5 to roughly extract the signals' features first.
- (2) Secondly, for the signals with poor separation degree, multi-fractal dimension can be used to carry out the secondary classification, in order to realize the subtle features' extraction.

4.2. Feature Extraction of the Same Modulation Signals with Different Noise

For the same communication signal with different noise, the characteristics of the signal are not obvious. Therefore, it is necessary to reconstruct the phase space of the signal, and then use multi-fractal dimension to extract the subtle features. Compared with the algorithm that deal with different signals with the same noise, the complexity of this algorithm is more, but the effect of classification result

is also greatly improved, the specific implementation steps are as follows:

- (1) First of all, preprocess the received unknown communication signal, and discretizing the signal first:

Assuming that the received individual signal is s , the discrete signal sequence after preprocessing is $\{s(i)\}$, where $i = 1, 2, \dots, N_0$ is the number of sampling points, N_0 represent the length of the signal sequence,

- (2) Reconstruct the discretized signal sequence: First, define the following characteristic parameters of the preprocessed discrete communication signal sequence $\{s(i)\}$, $i = 1, 2, \dots, N_0$:

Definition: $n = \log_2^{N_0}$ represents the number of different vectors of reconstructed signals;

Definition: $t(j) = 2^j$ represents the number of discrete signal's points in each reconstructed signal, where, $j = 1, 2, \dots, n$ denotes the value of different vectors' number of reconstructed signal;

Defining the numeric sequences:

$$T(j) = \frac{N_0}{t(j)} = \frac{N_0}{2^j}, \quad (30)$$

where $j = 1, 2, \dots, n$;

The definition of reconstructed signal sequence $S(j)$ is:

$$S(j) = s(T(j) * (t(j) - 1) + T_0(j)), \quad (31)$$

where $T_0(j) = [1 : T(j)]$, $j = 1, 2, \dots, n$;

- (3) The multi-fractal dimension of the reconstructed vector is chosen, and the fractal dimension of the signal is extracted.

Multi-fractal dimension can be regarded as a union of fractal subsets with different dimensions. Suppose the research object is divided into M small areas, and the line degree of the i th area is ε_i , the density distribution function of the i th region is P_i , then the scaling exponent α_i of different region i can be described as

$$P_i = \varepsilon_i^{\alpha_i}, \quad i = 1, 2, \dots, N_i. \quad (32)$$

The non-integer α_i is called singular exponent, it represents the fractal dimension of a region. Since a signal can be divided into many different small regions, a variable $f(\alpha)$ can be composed of a series of different variables α_i , and $f(\alpha)$ is the multi-fractal dimension of the signal.

Define the function $X_q(\varepsilon)$ as the weighted sum of various regions' probability, ε is the line degree of the area, q is the power of density distribution function P_i , that is

$$X_q(\varepsilon) = \sum_{i=1}^N P_i^q. \quad (33)$$

The definition of generalized fractal dimension D_q is

$$\begin{aligned} D_q &= \frac{1}{q-1} \lim_{\varepsilon \rightarrow 0} \frac{\ln X_q(\varepsilon)}{\ln \varepsilon} \\ &= \frac{1}{q-1} \lim_{\varepsilon \rightarrow 0} \frac{\ln \left(\sum_{i=1}^N P_i^q \right)}{\ln \varepsilon}. \end{aligned} \quad (34)$$

Thus, each of the reconstructed signals $S(j)$ in step (2) is summed together, $S(j)$ represents the j th reconstructed signal, that is

$$\begin{aligned} S_J &= \sum S(j) \\ &= \sum s(T(j) * (t(j) - 1) + T_0(j)) \\ &= \sum_{T_0(j)=1}^{T(j)} s(T(j) * (t(j) - 1) + T_0(j)), \end{aligned} \quad (35)$$

where $J = 1, 2, \dots, J_0$, $j = 1, 2, \dots, n$, S_J is the sum of J th reconstructed signals, J_0 is the number of reconstructed signals.

And then sum the entire discrete signal sequence, the result is S , that is

$$S = \sum_{i=1}^{N_0} s(i), \quad (36)$$

where $i = 1, 2, \dots, N_0$, $s(i)$ is the i th sampling point value of discrete signal sequence, then the J th probability measure degree P_J can be defined as

$$P_J = \frac{S_J}{S}, \quad (37)$$

where $J = 1, 2, \dots, J_0$, take P_J into the multi-fractal dimension formula to calculate D_q , then the multi-fractal dimension of the signal can be obtained.

4.3. Realization of Gray Relational Classifier

Gray relation theory was used to calculate the relation degree of the extracted unknown signal's multi-fractal dimension with the known modulation signal's features in the database. It is judged that the modulation type of the unknown signal is the

modulation type of the signal in the database with the highest relation degree. Hence, identification of individual signals can be realized, the specific steps are as follows:

Set the q value form $-q_0$ to q_0 , then $2q_0 + 1$ number of multi-fractal dimension can be obtained. Every q value has $n = \log_2^{N_0}$ number of characteristic points. For one individual signal, the feature vector is composed of $N = (2q_0 + 1) * n = (2q_0 + 1) * \log_2^{N_0}$ number of feature points. So multi-fractal feature sequence F_0 of unknown signal can be constructed. Gray relation theory is used to calculate the characteristic sequence of the unknown signal with the feature sequence F_i in the database. Suppose $\gamma(F_0, F_i)$ to be the relation degree of the two sequences and there are k kinds of different emitter templates, that is the types of individual signals are $i = 1, 2, \dots, k$, so the constituted characteristic matrix can be expressed as

$$\begin{aligned} F_0 &= (D_{-q_0}^0(1), D_{-q_0}^0(2), \dots, D_{-q_0}^0(n), \\ &\quad D_{-q_0+1}^0(1), \dots, D_{-q_0+1}^0(n), \dots, D_{q_0}^0(1), \\ &\quad D_{q_0}^0(2), \dots, D_{q_0}^0(n)) \\ F_1 &= (D_{-q_0}^1(1), D_{-q_0}^1(2), \dots, D_{-q_0}^1(n), \\ &\quad D_{-q_0+1}^1(1), \dots, D_{-q_0+1}^1(n), \dots, D_{q_0}^1(1), \\ &\quad D_{q_0}^1(2), \dots, D_{q_0}^1(n)) \\ &\vdots \\ F_i &= (D_{-q_0}^i(1), D_{-q_0}^i(2), \dots, D_{-q_0}^i(n), \\ &\quad D_{-q_0+1}^i(1), \dots, D_{-q_0+1}^i(n), \dots, D_{q_0}^i(1), \\ &\quad D_{q_0}^i(2), \dots, D_{q_0}^i(n)) \\ &\vdots \\ F_k &= (D_{-q_0}^k(1), D_{-q_0}^k(2), \dots, D_{-q_0}^k(n), \\ &\quad D_{-q_0+1}^k(1), \dots, D_{-q_0+1}^k(n), \dots, D_{q_0}^k(1), \\ &\quad D_{q_0}^k(2), \dots, D_{q_0}^k(n)), \end{aligned} \quad (38)$$

where $i = 1, 2, \dots, k$ represents the number of the modulation type.

So the calculated method of relation coefficient $\gamma(F_0, F_i)$ is defined as

$$\begin{aligned} \gamma(F_0, F_i) &= \frac{\min_i \min_N |F_0(N) - F_i(N)| + \rho \max_i \max_N |F_0(N) - F_i(N)|}{|F_0(N) - F_i(N)|} \\ &\quad + \rho \max_i \max_N |F_0(N) - F_i(N)| \end{aligned} \quad (39)$$

$N = 1, 2, \dots, (2q_0 + 1) * \log_2^{N_0}$ represents N th characteristic of every signal's vector, ρ is the resolution coefficient, and $\rho \in (0, 1)$, here it sets as 0.5. Hence gray relation degree $\gamma_0(F_0, F_i)$ of unknown signal F_0 's multi-fractal dimension with the individual template radiation source in the database is defined as

$$\gamma_0(F_0, F_i) = \frac{1}{k} \sum_{i=1}^k \gamma(F_0, F_i). \quad (40)$$

The gray relation degree is the correlation value of the unknown signal with the signal in the known database, through the calculated results to judge the type of the unknown signals, thus to achieve the classification of individual signal.

5. SIMULATION RESULTS AND ANALYSIS

Experiment 1. The classification of different modulation signals under the same noise environment

Take six different modulation communication signals AM, FM, PM, ASK, FSK and PSK with the same white noise added as examples to get the simulation. In the SNR range of -10 dB– 20 dB, the fractal box dimension, Higuchi fractal dimension, Petrosian fractal dimension, Katz fractal dimension and Sevcik fractal dimension values are calculated and the characteristic curves are drawn. The simulation results are shown in Figs. 2–6. While, in the figures, x -coordinate represents SNR, and y -coordinate represents different fractal dimensions.

From the simulation results in Figs. 2–6, it can be seen that one-dimensional fractal dimension is difficult to classify accurately for FM and PM signals, ASK, FSK and PSK signals. If the signals can be classified by one-dimensional fractal dimension, we will use this algorithm first due to its simple and easy calculation. If the signals are difficult to use one-dimensional fractal dimension to classify, multi-fractal dimension should be used although it has relatively higher calculated complexity. The multi-fractal dimension characteristics of the different signals under the three SNR environment of -10 dB, 0 dB and 10 dB are shown in Figs. 7–9. While, x -coordinate Ine represents $\ln \varepsilon$, and y -coordinate $\ln x_q$ represents $\ln X_q(\varepsilon)$ in equation (34).

Experiment 2. Classification of the same signal under different SNR environments

Fig. 2 Fractal box dimension-based features.

Fig. 3 Higuchi dimension-based features.

Fig. 4 Petrosian dimension-based features.

Fig. 5 Katz dimension-based features.

Fig. 6 Sevcik dimension-based features.

Take FSK signals with different distributions noise as examples — uniformly distributed noise, Rayleigh distributed noise, exponential distributed noise, beta distributed noise, and normal distributed noise. It is difficult to characterize the subtle features of the noise by one-dimensional fractal dimension. Therefore, phase space reconstruction is directly carried out on the signals, and then multi-fractal dimension is used to extract the noise's subtle features, and finally to extract the communication stations' "fingerprint" features. Simulation results are shown in Fig. 10. The figures on left represents the waveforms of FSK signals with different distributed noise, x -coordinate represents the

Fig. 7 Multi-fractal dimension-based features of different modulation signals under the SNR of 10 dB.

Fig. 8 Multi-fractal dimension-based features of different modulation signals under the SNR of 0 dB.

number of the signals' points, and y -coordinate represents the amplitudes of signals. The figures on the right represents the multi-fractal dimensions of the signals. And while, x -coordinate $\ln \varepsilon$, and y -coordinate $\ln X_q(\varepsilon)$ in Eq. (34).

From the simulation we can see that, fractal dimension curves of FSK signals with different distributed noise are different, so gray relation classifier can be used to calculate the correlation degree of each reconstructed sequence, which can achieve good recognition effect. A Monte Carlo simulation experiment is used to generate 1000 random

Fig. 9 Multi-fractal dimension-based features of different modulation signals under the SNR of -10 dB.

Fig. 10 (a) FSK signal + Uniform distributed noise, (b) FSK signal + Rayleigh distributed noise, (c) FSK signal + Exponential distributed noise, (d) FSK signal + Beta distributed noise, (e) FSK signal + Normal distributed noise, Multi-fractal dimension curves of FSK signals + different distribution noise sequences.

samples with different distribution to investigate the recognition effect of multi-fractal dimension of different distributed noise sequences. Simulation results are shown in Table 1.

Information entropy is an important index to measure the uncertainty of signal's distribution and signal's complexity. It is often used to quantitatively describe the information contained in the signal, so

Table 1 Recognition Rate of Different Feature Extraction Methods Under Different SNR.

SNR(dB)	-20	-10	0	10	20
Recognition rate of multi-fractal dimension-based features (%)	90.6	94	98	100	100
Recognition rate of entropy-based features (%)	80.8	83.2	85.4	94.6	97.6

it also can be used to detect and extract the complexity of noise. Nowadays, information entropy has been widely used in many fields. Reference 23 used information entropy and probabilistic neural network together to identify the target in the sea battlefield, which receive good recognition result. In this experiment, the information entropy features and multi-fractal features of FSK signals with different distributed noise are extracted, respectively. The recognition results under different SNR environment are compared in Table 1, and the recognition time are shown in Table 2.

Simulation results show that multi-fractal curves of different distribution sequences are different, so gray relation theory can be used to distinguish them, which has a good recognition effect. From the comparison of Table 1, it can be seen that the recognition results of multi-fractal-based features of FSK signal with different distributed noise sequence is better than entropy-based feature. Even the signal has been completely submerged by noise at low SNR, it still has a relatively high recognition rate. However, comparing the recognition time, the entropy based feature extraction algorithm is slightly better than the multi-fractal-based feature extraction algorithm, that is, in the calculation complexity, the entropy-based algorithm is simpler than the multi-fractal-based algorithm.

Table 2 Simulation Time of Different Feature Extraction Methods.

Recognition Methods	Time of Recognition Processes (ms)
Multi-fractal-based feature extraction method	14.2
Entropy-based feature extraction method	10.2

6. CONCLUSIONS

Fractal dimension is an important tool to describe the complexity of signals. Aiming at the problem that the signals' subtle features are difficult to extract in low SNR environment, this paper proposes a new feature extraction algorithm based on multi-fractal dimension and one-dimensional fractal dimension algorithm according to the complexity of the signal. For the signals with different noise, it is difficult to perform efficient feature extraction algorithm to get the subtle features in low SNR. Therefore, a multi-fractal dimension feature extraction algorithm based on phase space reconstruction is proposed. FSK signals with different distributed noise were used as the signals to be identified to verify the effectiveness of proposed algorithm. Simulation results show that, although the feature extraction algorithm based on multi-fractal dimension has a higher computational complexity than the one-dimensional feature extraction algorithm, it has better performance on the subtle features' extraction. Even in the SNR of -10 dB, it still has a high recognition rate.

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