ERRATA

[INT. J. OF MOD. PHYS. E, Vol. 3, No. 1 (March, 1994) 1-38]

COLOR TRANSPARENCY: FACTS AND FANCY*

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Received 30 April 1992

Revised 30 June 1992

This is a review of the principal motivations behind, and the status of, color transparency physics, with particular emphasis on the effects of a large coherence length at high energies.

“Ich habe bemerkt” sagte Herr K., “daß wir viele abschrecken von unserer Lehre dadurch, daß wir auf alles eine Antwort wissen. Könnten wir nicht im Interesse der Propaganda eine Liste der Fragen aufstellen, die uns ganz ungelöst erscheinen?”

Bertolt Brecht.

Geschichten vom Herrn Keuner.

1. Introduction

Recently, there was much discussion on Color Transparency in the high-energy nuclear interactions. Regarding the classical aspects, Color Transparency is precisely the QCD counterpart of the QED Chudakov effect: weak ionization of medium by the Bethe-Heitler-produced ultrarelativistic e^+e^- pairs at short distances from the production vertex.¹ When (anti)quarks of the colorless hadronic state sit on top of each other, colored gluons decouple from such a small-sized state, resulting in a small interaction cross-section and a weak nuclear attenuation,^{2,3} hence the term “Color Transparency”.^{4,5} The quantum aspects do differ significantly, though.

The seminal contributions to the field are due to Zamolodchikov, Kopeliovich and Lapidus,² Bertsch, Brodsky, Goldhaber and Gunion,³ Mueller⁴ and Brodsky.⁵

*Extended version of lectures delivered at Institut für Kernphysik, Forschungszentrum Jülich GmbH., 5170 Jülich, Germany, 4-6 February 1992.

The subject has been greatly enlivened by early evidence for Color Transparency from the AGS ($p, p'p$) experiment.⁶ However, the AGS effect disappeared at higher energy rather than becoming stronger, which prompted much speculation and remains unexplained as yet.^{7–12}

Nonetheless, as we understand the phenomenon now, Color Transparency is as fundamental a property of QCD as the gauge theory. It is very closely related to, and has its roots in, the 1969 Gribov theory of inelastic shadowing in nuclear cross-sections,¹³ and in the large coherence length in high-energy diffractive interactions, introduced in 1953 by Landau, Pomeranchuk and Feinberg.^{14–17} As such, Color Transparency deserves much study, and the future high-energy, high-intensity electron facilities may well become the dedicated Color Transparency machines. The constraint of a large coherence length, imperative for manifestations of Color Transparency, requires energies above 20–30 GeV.

In the past, following Mueller⁴ and Brodsky,⁵ the major interest was focused on the hard scattering processes, expected to produce ejectiles of small transverse size. However, the onset of a shrinkage of the ejectile in hard scattering is elusive up to very large momentum transfer squared, and may well be preceded by an expansion of the ejectile.¹⁸ Even if the small-sized ejectile is produced at the production vertex, its further attenuation is controlled by Gribov's inelastic shadowing via soft, diffractive intranuclear rescatterings. This justifies and makes indispensable searches for Color Transparency signal in the soft, diffractive, scattering of hadrons off nuclei, the point I would like to emphasize in this review.

In fact, Kopeliovich and Zakharov have deduced^{9,19} the early evidence for Color Transparency from the 1982²⁰ and 1987²¹ Serpukhov data on the charge-exchange of 40 GeV pions on nuclei. Moreover, in the late 60s and 70s there was much discussion of a mysterious weak attenuation of diffractively produced states in nuclear matter (see Ref. 22 and the reviews Refs. 23 and 24). This weak attenuation can be partially attributed to Color Transparency.

The major objective of this review is to formulate answers to three principal questions:

- How one can quantify the property of Color Transparency,
- What are the necessary conditions for the observation of Color Transparency,
- How one can produce small-sized hadronic states.

In this task, I shall follow the above advice from Brecht's hero, Herr Keuner: I shall first present what is well understood in the Color Transparency physics, and will not hide the less clear issues, which require more work and/or are not yet understood. I shall put major emphasis on a large coherence length in the high-energy scattering as one of the pillars of the Color Transparency physics. Large coherence length invalidates semiclassical evaluations of the Color Transparency effects the recent literature is abundant with. I shall demonstrate that Color Transparency has already been tested to great detail in deep inelastic scattering at large $1/x$.^{25,26} Regarding the theoretical status of hadronic scattering off nuclei, and ($e, e'p$) or

$(e, e'\pi)$ reactions on nuclei, the onset of the Color Transparency effects is not yet fully understood.^{18,27} My personal opinion is that diffractive reactions on nuclei could be as good, and perhaps better, a testing field of the Color Transparency ideas as the hard scattering processes. Moreover, understanding diffractive interactions is just necessary for quantitative interpretation of the hard scattering data.

2. Exponential Attenuation and Coherence Length

The exponential decay law of radioactive substances with the lifetime τ ,

$$N(t) = N(0) \exp(-t/\tau), \quad (1)$$

and the exponential attenuation of a beam of particles in a medium,

$$N(d) = N(0) \exp(-d/l), \quad (2)$$

are two fundamental laws of physics. Here d is a thickness of the absorber and l is the mean free path

$$l = \frac{1}{n\sigma}, \quad (3)$$

where n is the density of scatterers and σ is the total cross-section.

The exponential attenuation law (2) is derived under the implicit, and seldom mentioned, assumption of *uncorrelated consecutive interactions*. In a rescattering of the atomic and nuclear particles, which is followed by excitation, this amounts to the short relaxation (de-excitation) time τ_R , i.e., to the coherence length $l_c = v\tau_R \ll l$. In high-energy interactions, by virtue of the Lorentz dilation, $l_c = \gamma c\tau_R$ can become very large, and in particular, much larger than the size of nuclei R_A . This observation dates back to the 1953–54 papers by Landau, Pomeranchuk and Feinberg,^{14–17} who discovered a novel, coherent diffraction dissociation mechanism of hadro- and photoproduction of nuclei in the regime of $l_c > R_A$.

The relativistic growth of the coherence length (coherence zone, formation length l_f , formation zone) leads to a few celebrated phenomena:

- (i) The Landau-Pomeranchuk phenomenon — suppression of the Bremsstrahlung and e^+e^- pair creation in the multiple scattering of ultrarelativistic electrons in dense amorphous media.²⁸
- (ii) Transparency of nuclear matter to the high-energy secondary particles and freezing of the intranuclear cascading in high-energy nuclear collisions (hadronic Landau-Pomeranchuk effect) (Refs. 29 and 30; for the review see Ref. 24).
- (iii) The Chudakov effect — weak ionization by e^+e^- pairs near the production vertex.¹
- (iv) Coherent diffractive production on nuclei at high energies.^{15–17}
- (v) Nonexponential attenuation of the ultrarelativistic positronium atoms in dense medium.^{31,32}
- (vi) Color transparency in QCD, which is a subject of this review.

3. Hadron-Hadron Scattering in QCD

We start with the Gedanken experiment of scattering of (b) Υ on (a) Ω_{bbb} baryon. The bottomonium atom has a small size, $r_\Upsilon \sim 0.15f$. Hence, gluons with the wavelength $\lambda > r_\Upsilon$ decouple from the color-neutral bottomonium. Since the momentum of gluons $k \gtrsim 1/r_\Upsilon \gg \Lambda_{\text{QCD}}$, one can treat $\Upsilon\Omega_{bbb}$ scattering perturbatively.

Fig. 1. Subsample of the two-gluon exchange diagram for ab scattering amplitude.

The color-singlet states in the t -channel first appear in the two-gluon exchange approximation³³ (Fig. 1) and give the scattering amplitude

$$A(s, \mathbf{q}) = isN_a N_b \frac{8\alpha_s^2}{27} \int \frac{d^2\kappa V^{(a)}(\mathbf{q}, \kappa) V^{(b)}(\mathbf{q}, \kappa)}{[\kappa^2 + \mu_G^2][(\mathbf{q} - \kappa)^2 + \mu_G^2]}, \quad (4)$$

where $N_{a,b}$ are the numbers of constituents in hadrons a and b and

$$\begin{aligned} V^{(i)}(\mathbf{q}, \kappa) &= \Phi_1^{(i)}(\mathbf{q}) - \Phi_2^{(i)}(\kappa, \mathbf{q} - \kappa) \\ &= \langle i | \exp[i\mathbf{q}\mathbf{r}_1] - \exp[\kappa\mathbf{r}_1 + i(\mathbf{q} - \kappa)\mathbf{r}_2] | i \rangle. \end{aligned} \quad (5)$$

The corresponding total cross-section equals^{33,34}

$$\sigma_{\text{tot}} = N_a N_b \frac{8\alpha_s^2}{27} \int \frac{d^2\kappa V^{(a)}(0, \kappa) V^{(b)}(0, \kappa)}{(\kappa^2 + \mu_G^2)^2}. \quad (6)$$

In Eqs. (4) and (6), the effective mass of gluons μ_G is included so as not to have the color forces propagating beyond the confinement radius R_c . However, since at small κ

$$V^{(i)}(0, \kappa) \propto \kappa^2, \quad (7)$$

the total cross-section (6) is infrared finite even with $\mu_G = 0$. The behavior (7) is a manifestation of color gauge invariance: soft gluons with the wavelength $\lambda_g \gtrsim R_h$ decouple from the colorless hadrons. Therefore, for heavy quark states with small size, $R_h \sim 1/m_q \alpha_s$, the virtuality of the exchanged gluons is large, $|\kappa^2| \gtrsim 1/R_h^2 \sim m_q^2 \alpha_s^2$, the corresponding strong coupling α_s is small, and the perturbative 2-gluon exchange is a good approximation. This holds even if only one of the hadrons has a small size.

In the above derivation, I have neglected the longitudinal momentum of the exchanged gluons. It can easily be estimated from $(p_a + \kappa)^2 = M_{b\bar{b}}^2$:

$$\kappa_L \approx \frac{M_{b\bar{b}}^2 - M_Y^2}{2E} \sim \frac{\Delta E_Y}{\gamma}, \quad (8)$$

where ΔE_Y is the typical excitation energy in the bottomonium. Neglecting κ_L is justified if $R_{a,b}\kappa_L \ll 1$, i.e., when

$$l_c = \frac{\gamma}{\Delta E_Y} \gg R_{a,b}. \quad (9)$$

4. Color Transparency in QCD

For the sake of simplicity, let the projectile b be the meson $|M\rangle = |q\bar{q}\rangle$. Making use of Eq. (5), the cross-section (6) can be cast in a simple quantum-mechanical form^{2,3}:

$$\sigma_{\text{tot}}(ab) = \int dz d^2\rho \sigma(\rho) |\Psi_b(z, \rho)|^2, \quad (10)$$

where $\sigma(\rho)$ is the total cross-section for a $q\bar{q}$ pair of the transverse size ρ :

$$\sigma(\rho) = \frac{16\alpha_S^2}{9} \int \frac{d^2\kappa V^{(a)}(0, \kappa) [1 - \exp(-i\kappa \cdot \rho)]}{(\kappa^2 + \mu_G^2)^2}. \quad (11)$$

The quantum mechanical representation (10) has a few salient features:

- (i) It emphasizes the transverse separation of quarks ρ being a frozen, diagonal parameter of the scattering matrix. The freezing condition

$$l_c = \gamma \frac{R_Y}{v} \approx \gamma \frac{1}{\Delta E_Y} \gg R_b, \quad (12)$$

where v is a velocity of bound quarks in the bottomonium, coincides with (9).

- (ii) The cross-section (11) depends only on the target and is universal for all projectiles of the same size ρ .
- (iii) At $\rho \ll R_h \sim R_c$ one can expand the factor $[1 - \exp(i\kappa \cdot \rho)]$ in (11) and^{2,3}

$$\sigma(\rho) \propto \rho^2, \quad \rho \ll R_h \sim R_c, \quad (13)$$

which is a direct consequence of the color gauge invariance. The property (13) is often referred to as the color transparency: colorless particles of small size have a small interaction cross-section and formula (11) quantifies the color transparency property.

- (iv) In the opposite limit of $\rho \gg R_c$, the cross-section (11) saturates,²⁵

$$\sigma_\rho \sim R_c^2, \quad \rho \gg R_h \sim R_c \quad (14)$$

since the color forces cannot propagate beyond the confinement radius R_c .

Therefore, the behavior of $\sigma(\rho)$ shown in Fig. 2 is protected against higher order QCD corrections.

Let the target be a nucleon. Then $V^{(a)}(0, \kappa) = 1 - F_{ch}(3\kappa^2)$. The Gedanken ΥN scattering experiment probes $\sigma(\rho)$ at $\rho \sim r_\Upsilon$. The J/Ψ -nucleon scattering probes the same universal $\sigma(\rho)$ at $\rho \sim R_{J/\Psi} \sim 0.45f$ with the result $\sigma_{\text{tot}}((J/\Psi)N) \approx 5$ mb and similarly $\sigma_{\text{tot}}(\Psi'N) \approx 12$ mb.³⁵ With a good educated guess, one can also be quantitative for the light hadrons. Specifically, the πN total cross-section receives leading contribution from $\rho \sim 0.7f$. With a realistic pion wave function one finds $\sigma_{\text{tot}}(\pi N) \approx 25$ mb.^{25,26,33,34} In Fig. 2, I indicate what range of ρ is probed in different processes.

Fig. 2. Universal $\sigma(\rho)$ versus ρ for the nucleon target. The effective mass of a gluon is $\mu_G \approx m_\pi$. Different processes probe this universal cross-section at different ρ .

It is instructive to elaborate on color transparency property in terms of the mass-eigenstates $|i\rangle$. The mass-eigenstates contain all ρ with the expansion coefficient

$$\Psi_i(\rho) = \langle \rho | i \rangle, \quad (15)$$

which is precisely the conventional wave function (hereafter I suppress the longitudinal size z , as all interactions are diagonal in z), and the ρ -eigenstates expand as

$$|\rho\rangle = \sum_i C(\rho)_i |i\rangle = \sum_i \Psi_i(\rho)^* |i\rangle. \quad (16)$$

Let $\hat{\sigma}$ be the cross-section operator: $\sigma_{ii} = \langle i|\hat{\sigma}|i\rangle = \sigma_{\text{tot}}(ia)$, and the off-diagonal matrix elements describe the forward diffraction excitation $ia \rightarrow ka$:

$$\left. \frac{d\sigma_D(ia \rightarrow ka)}{dt} \right|_{t=0} = \frac{\sigma_{ik}^2}{16\pi}. \quad (17)$$

The quantum mechanical expansion for $\sigma(\rho)$ reads

$$\langle \rho|\hat{\sigma}|\rho\rangle = \sum_{i,k} \langle \rho|k\rangle \langle k|\hat{\sigma}|i\rangle \langle i|\rho\rangle = \sum_{i,k} C(\rho)_k^* C(\rho)_i \sigma_{ki}. \quad (18)$$

The conventional wisdom, supported by the experiment, is that $\sigma_{ik} \ll \sigma_{ii}, \sigma_{kk}$, and it is tempting to neglect the off-diagonal σ_{ik} in (18). Then

$$\langle \rho|\hat{\sigma}|\rho\rangle \approx \sum_i C(\rho)_i^* C(\rho)_i \sigma_{\text{tot}}(ia) \sim \sigma_{\text{tot}}(\Upsilon a) \sum_i |C(\rho)_i|^2 = \sigma_{\text{tot}}(\Upsilon a) \quad (19)$$

would not depend on the size ρ . This makes it obvious that color transparency comes from strong cancellations between the diagonal and off-diagonal transitions, and any manifestation of color transparency is only possible when a large number of inelastic transitions contributes, as discussed below.

5. Color Transparency and Hadron-Nuclei Cross-Sections

Consider now the Υ -nucleus scattering. If the coherence length $l_c \gg R_A$, then for a $q\bar{q}$ pair with fixed ρ the nuclear cross-section is given by the standard Glauber's formula³⁶:

$$\sigma_A(\rho) = 2 \int d^2\mathbf{b} \left\{ 1 - \exp \left[-\frac{1}{2} \sigma(\rho) T(\mathbf{b}) \right] \right\}. \quad (20)$$

Here $T(\mathbf{b})$ is the optical thickness of a nucleus:

$$T(\mathbf{b}) = \int_{-\infty}^{+\infty} dz n_A(z, \mathbf{b}), \quad (21)$$

so that $\exp[-\sigma(\rho)T(\mathbf{b})/2] = \exp(-d/2l)$ is precisely the exponential attenuation amplitude. Following (10), we have^{2,37} (hereafter we suppress the longitudinal variable in the wave function)

$$\sigma_{\text{tot}}(\Upsilon A) = \int d^2\rho |\Psi_\Upsilon(\rho)|^2 \sigma_A(\rho). \quad (22)$$

Glauber's formula (20) sums up all the multiple elastic rescattering diagrams of Fig. 3 (left schematic). At high energies, the fixed- ρ states undergo only elastic rescattering. In terms of the mass-eigenstates, besides the elastic rescatterings one has to include all diffraction excitation transitions of Fig. 3 (right schematic), alias

Gribov inelastic shadowing.¹³ Formula (22) sums up in a compact form all the elastic + inelastic shadowing diagrams. Indeed, compare the double scattering term in the ρ -basis and the mass-eigenstate basis:

$$\langle i | \hat{\sigma}^2 | i \rangle = \int d^2 \rho |\Psi_i(\rho)|^2 \sigma(\rho)^2 = \sum_k \sigma_{ik} \sigma_{ki}. \quad (23)$$

The significance of the large coherence length, $l_c \gg R_A$, is evident from the following: In the inelastic transition $i \rightarrow k$ there is a finite longitudinal momentum transfer κ_L (see Eq. (8)), the elastic and inelastic double scattering amplitudes will differ by the phase factor $\exp(i\kappa_L \delta z)$, where $\delta z \sim R_A$ is a separation of scatterers, and the condition $l_c \gg R_A$ corresponds precisely to $\kappa_L \delta z \ll 1$, i.e., to the coherent interference of the elastic and inelastic scattering, essential for the color transparency property (recall Eqs. (18) and (19)).

Fig. 3. Hadron-nucleus multiple scattering diagrams: (a) elastic rescatterings, (b) Gribov's inelastic rescatterings.

For Υ -nucleon interaction we can write^{2,3}

$$\sigma(\rho) = \frac{\rho^2}{r_Y^2} \sigma_{\text{tot}}(\Upsilon N). \quad (24)$$

Assuming, for the sake of simplicity, the Gaussian wave function,

$$|\Psi_Y|^2 = \frac{1}{\pi r_Y^2} \exp\left(-\frac{\rho^2}{r_Y^2}\right), \quad (25)$$

we find a dramatic change from the *exponential* to the *inverse thickness* attenuation law²:

$$\left\langle \exp\left[-\frac{1}{2}\sigma(\rho)T(\mathbf{b})\right] \right\rangle_Y = \frac{1}{1 + \frac{1}{2}\sigma_{\text{tot}}(\Upsilon N)T(\mathbf{b})}. \quad (26)$$

Up to the normalization factor, the inverse-thickness law is found irrespective of the detailed form of the projectile meson's wave function.

The prediction (26) is cute, but difficult to test experimentally. One cannot use the ρ^2 law (24) for nucleons, pions or kaons (see Fig. 2). In this case²²:

$$\begin{aligned} & \left\langle \exp \left[-\frac{1}{2} \sigma(\rho) T(\mathbf{b}) \right] \right\rangle_{\pi} \\ &= \exp \left[-\frac{1}{2} \langle \sigma(\rho) \rangle_{\pi} T(\mathbf{b}) \right] \left\langle \exp \left[-\frac{1}{2} (\sigma(\rho) - \langle \sigma(\rho) \rangle_{\pi}) T(\mathbf{b}) \right] \right\rangle_{\pi} \\ &= \exp \left[-\frac{1}{2} \sigma_{\text{tot}}(\pi N) T(\mathbf{b}) \right] \left\{ 1 + \frac{1}{8} [\langle \sigma(\rho)^2 \rangle_{\pi} - \langle \sigma(\rho) \rangle_{\pi}^2] T(\mathbf{b})^2 + \dots \right\}, \quad (27) \end{aligned}$$

where $\sigma_{\text{tot}}(\pi N) = \langle \sigma(\rho) \rangle_{\pi}$. Notice that

- (i) Gribov's inelastic shadowing correction in curly braces in Eq. (28) already makes the attenuation nonexponential.
- (ii) Since $\langle \exp(x) \rangle > \exp(\langle x \rangle)$, Gribov's inelastic shadowing always reduces the nuclear opacity in the elastic scattering channel.
- (iii) The leading term of the inelastic shadowing is directly related to the inclusive forward diffraction dissociation cross-section and, in this respect, is model-independent:

$$\begin{aligned} \left. \frac{d\sigma_D}{dt} \right|_{t=0} &= \int dM_X^2 \frac{d\sigma(\pi N \rightarrow XN)}{dM_X^2 dt} \Big|_{t=0} \\ &= \frac{1}{16\pi} \sum_{k \neq i} \sigma_{ik}^2 = \frac{1}{16\pi} [\langle \sigma(\rho)^2 \rangle_{\pi} - \langle \sigma(\rho) \rangle_{\pi}^2]. \quad (28) \end{aligned}$$

Inelastic shadowing is numerically quite significant and lowers the nA , πA and $K_L A$ total cross-sections by $\sim 5-7\%$, $\sim 10\%$ and $\sim 15\%$, respectively.³⁸ As heavier and heavier states can be excited diffractively at higher energy, the inelastic shadowing correction rises with energy,^{13,39} and is crucial for the quantitative description of the energy dependence of the nA and $K_L A$ total cross-sections^{40,41} (see Fig. 4). The higher order shadowing corrections to the hadron-nuclei total cross-sections are more sensitive to the detailed form of $\sigma(\rho)$, but numerically they are small.³⁸

6. Nonexponential Attenuation of Ultrarelativistic Positronium

A remarkable problem with the macroscopic coherence length l_c is attenuation of ultrarelativistic positronium in medium.^{31,32} In this case

$$l_c \approx \gamma \frac{a_B}{v}, \quad (29)$$

where a_B is the Bohr radius and $v \approx \alpha_{em} \approx 1/137$. For the beam of 10 GeV positronium $l_c \sim 0.03$ cm! For a comparison, with $\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons}) \approx 6 \cdot 10^{-19}$ cm², the mean free path in graphite would have been $l_{2C} \approx 0.1$ μ m.

Fig. 4. Compilations of the experimental data on (left) nA (Ref. 40) and (right) $K_L A$ (Ref. 41) total cross-sections versus energy. The dashed curve is the Glauber calculation with only elastic rescatterings, the solid curve includes Gribov's inelastic shadowing and gives much better description of the energy dependence of $\sigma_{tot}(nA)$ and $\sigma_{tot}(K_L A)$.

Since the size of positronium is about twice the atomic size, the electron-atom interaction is strong only inside the inner shells. Hence, only the positron or only the electron gets the transverse kick when flying by a separate atom, and one can use the sudden impact approximation. The nonexponential attenuation of positronium, which I am going to derive following Lyuboshitz and Podgoretzky,³² emphasizes the large coherence length property, rather than the charge-neutrality of the positronium.

Let $\Psi_0(\rho)$ be the ground state wave function. If the coherence length l_c is much larger than the multiple-Coulomb scattering length l and the target thickness d , then the transverse size ρ remains frozen. Hence, downstream of the target we are left with the wave function

$$\Psi(\rho) = \Psi_0(\rho) \exp \left[-\frac{i}{2} \mathbf{Q} \cdot \rho \right], \quad (30)$$

where

$$\mathbf{Q} = \sum \mathbf{q}_i^{(+)} - \sum \mathbf{q}_i^{(-)} \quad (31)$$

is the total momentum transfer to the positronium. The probability to find the positronium in the ground state downstream of the target is given by

$$w_0 = \left| \int d^3\rho \Psi_0(\rho)^* \Psi(\rho) \right|^2 = \Phi_1(Q)^2 = \left[\frac{1}{1 + Q^2 a_B^2 / 16} \right]^4. \quad (32)$$

In the “large” positronium approximation, $\mathbf{q}_i^{(+)}$ and $\mathbf{q}_j^{(-)}$ are uncorrelated, for the thick target Q will have the Gaussian distribution with $\langle Q^2 \rangle = 2Q_0^2 d/l \gg 1/a_B^2$, so that

$$\langle w_0 \rangle = \int d^2Q \frac{1}{\pi \langle Q^2 \rangle} \exp \left(- \frac{Q^2}{\langle Q^2 \rangle} \right) \Phi_1(Q)^2 = \frac{8}{3} \frac{1}{a_B^2 Q_0^2} \frac{l}{d} \quad (33)$$

and we have obtained the inverse thickness law for the survival probability. The nonexponential attenuation was observed experimentally in the ingenious experiment performed at the Serpukhov accelerator in the positronium beam formed in the decay $\pi^0 \rightarrow \gamma(e^+e^-)_{\text{atom}}$ (Ref. 42).

Numerically, the above “large” positronium approximation is rather crude, and corrections to the frozen approximation become large for thick targets. The problem of corrections was treated by B. G. Zakharov in much detail using the path integral technique (Ref. 43, see also Ref. 44).

7. Color Transparency in Deep Inelastic Scattering and Bjorken Scaling

The quantity measured in the deep inelastic scattering of muons (electrons) on nucleons (nuclei) is the total photoabsorption cross-section $\sigma_{\text{tot}}(\gamma^* N)$, related to the structure function $F_2(x, Q^2)$ by

$$\sigma_{\text{tot}}(\gamma^* N, x, Q^2) = \frac{4\pi\alpha_{em}}{Q^2} F_2(x, Q^2). \quad (34)$$

Here x is the Bjorken scaling variable, $x = Q^2/2(pq)$, p and q are the target nucleon's and the virtual photon's 4-momenta, $q = (\nu, 0, 0, \sqrt{\nu^2 + Q^2})$, $Q^2 = -q^2$ and $2(pq) = 2m_N\nu$. The structure function $F_2(x, Q^2)$ is approximately Q^2 -independent (the Bjorken scaling). The (logarithmic) scaling violations are calculable in QCD.

QCD is a theory of interacting quarks and gluons. Higher order QCD corrections to the Compton scattering on valence quarks at high Q^2 can be treated as radiative generation of the glue and quark-antiquark sea in nucleons. The lowest order perturbative QCD diagram for DIS on the radiatively generated sea is shown in Fig. 5. At $x \ll 1$, the same perturbative QCD diagram can be treated as scattering on a nucleon of the quark-antiquark pairs that the virtual photon transforms into at a large distance upstream from the target (see Eqs. (8) and (9)):

$$l_c \approx \frac{2\nu}{Q^2 + M^2} \sim 1/m_N x \ll R_N. \quad (35)$$

Fig. 5. Deep inelastic scattering on the radiatively generated sea, or photoabsorption mediated by conversion of photons into $q\bar{q}$ pairs.

Here I assumed that excited masses $M^2 \lesssim Q^2$, which can be justified by a direct QCD calculation of the mass spectrum.⁴⁵

The $q\bar{q}$ interaction cross-section is the same universal cross-section (11) shown in Fig. 2. We only have to find the wave function of the $q\bar{q}$ fluctuations of photons. This problem was solved in Ref. 25. Namely, for the (T) transverse and (L) longitudinal photons,

$$|\Psi_T(z, \rho)|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_1^{N_f} Z_f^2 \{ [z^2 + (1-z)^2] \varepsilon^2 K_1(\varepsilon\rho)^2 + m_f^2 K_0(\varepsilon\rho)^2 \}, \quad (36)$$

$$|\Psi_L(z, \rho)|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_1^{N_f} Z_f^2 4Q^2 z^2 (1-z)^2 K_0(\varepsilon\rho)^2 \quad (37)$$

are squares of wave functions of the $q\bar{q}$ fluctuations of the (T) transverse and (L) longitudinal photon, $K_\nu(x)$ is the Bessel function, and

$$\varepsilon^2 = z(1-z)Q^2 + m_f^2. \quad (38)$$

Then,

$$\sigma_{T,L}(\gamma^* N) = \int_0^1 dz \int d^2\rho |\Psi_{T,L}(z, \rho)|^2 \sigma(\rho). \quad (39)$$

Notice that only the wave functions (36) and (37) depend on the quark mass m_f and the Sudakov variable z -fraction of photon's momentum carried by a quark of the $q\bar{q}$ pair, $0 < z < 1$, whereas $\sigma(\rho)$ does not depend on the projectile. The wave functions (36) and (37) describe, in fact, the ultrarelativistically boosted vacuum polarization charge distribution. Notice that $\Psi_L(z, \rho)^2 \propto Q^2$, so that at small Q^2 we have $R = \sigma_L/\sigma_T \propto Q^2$ and longitudinal photons decouple in the limit of $Q^2 \rightarrow 0$, as the gauge invariance dictates.

Let us examine the transverse cross-section (39) in more detail. After the z integration we find

$$\sigma_T(\gamma^* N) \propto \frac{1}{Q^2} \int_{Q^{-2}}^{m_q^{-2}} \frac{d\rho^2}{\rho^4} \sigma(\rho). \quad (40)$$

With the color transparency property, $\sigma(\rho) \propto \rho^2$, we find from (40) the scaling, $\propto 1/Q^2$, photoabsorption cross-section with a $\log(1 + Q^2/m_q^2)$ scaling violating factor. More accurate considerations, with allowance for the running QCD coupling, show that at small ρ

$$\sigma(\rho) \propto \alpha_S(1/\rho^2) \log \left[\frac{1}{\alpha_S(1/\rho^2)} \right] \rho^2, \quad (41)$$

and the standard $\log[1/\alpha_S(Q^2)]^2$ scaling violation factor emerges from (40).

The corollary of Eq. (40) is that at large $1/x$ the structure function receives contributions from

$$\frac{1}{Q^2} < \rho^2 < \frac{1}{m_q^2} \quad (42)$$

and the scaling violations probe the universal $\sigma(\rho)$ at $\rho^2 \sim 1/Q^2$. The small- x structure function predicted²⁵ by Eq. (39) agrees well with the EMC/NMC data,⁴⁷ shown in Fig. 6. This agreement shows that the color transparency property of $\sigma(\rho)$ has already been probed in the deep inelastic scattering in a very broad range of ρ . Had $\sigma(\rho)$ been constant at small ρ , we would have run into the Gribov-Bjorken disaster^{48,49}: $\sigma(\gamma^* N) \sim \text{const}$, which is a case in the naive applications of the vector meson dominance model.

The charm excitation cross-section probes the universal $\sigma(\rho)$ at $\rho^2 \lesssim 1/m_c^2$ (see Fig. 2), with the result^{26,45} $\sigma(\gamma N \rightarrow c\bar{c}X) \approx 1.1\mu b$, in good agreement with experiment.⁵⁰ When applied to the light quarks with the effective mass $m_{u,d} \sim m_\pi$, Eq. (39) gives^{26,45} the total photoabsorption cross-section $\sigma_{\text{tot}}(\gamma N) \approx 100\mu b$, which agrees with experiment⁵¹ and shows that our educated guess for the large- ρ behavior of $\sigma(\rho)$ was sensible.

Fig. 6. Structure function of μD scattering at small x predicted by Eq. (39) versus the EMC data.⁴⁷

8. Color Transparency and Nuclear Shadowing in Deep Inelastic Scattering

We start with deep inelastic scattering on deuterium. The standard Glauber's formula for the hadron-deuteron scattering reads⁵²:

$$\sigma_D = 2\sigma_N - \sigma_N^2 \left\langle \frac{1}{4\pi r_D^2} \right\rangle. \quad (43)$$

Here r_D is the internucleon distance in deuterium and the average is taken with the deuterium wave function. Since in the deep inelastic scattering $\sigma_N \propto 1/Q^2$, one would naively expect the relative contribution of the shadowing term to vanish $\propto 1/Q^2$. For a while this was a prevailing opinion among theorists and experimentalists alike, and it is wrong.

Correct calculation of the shadowing goes as follows: Eq. (43) holds for the $q\bar{q}$ pair of fixed size ρ with $\sigma_N \rightarrow \sigma(\rho)$. Then, one has to average $\sigma_D(\rho)$ over ρ with the wave functions (36) and (37). For the single scattering contributions (see Eqs. (39) and (40)), here I concentrate on the shadowing term:

$$\int dz d^2\rho |\Psi_T(z, \rho)|^2 \sigma(\rho)^2 \propto \frac{1}{Q^2} \int_{Q^{-2}}^{m_q^{-2}} \frac{d\rho^2}{\rho^4} \sigma(\rho)^2 \propto \frac{1}{Q^2} \int_{Q^{-2}}^{m_q^{-2}} d\rho^2. \quad (44)$$

Notice that the cross-section of the diffraction dissociation of virtual photons $\gamma^* N \rightarrow XN$ is also proportional to the moment (44) (see Eq. (28)).^{25,26,45}

A few important conclusions can be drawn from Eq. (44)²⁵:

- (i) The shadowing term scales $\propto 1/Q^2$ — like the total cross-section, the nuclear shadowing is a scaling phenomenon, and an essential use was made of color transparency to arrive at this conclusion.
- (ii) Shadowing is dominated by large, hadronic size $q\bar{q}$ pairs. Again, this is a direct consequence of color transparency.
- (iii) Shadowing scales exactly, whereas the single scattering term rises with Q^2 by virtue of the scaling violations. Henceforth, at moderately large $1/x$ and moderately large Q^2 , the nuclear shadowing should slowly, $\propto 1/\log(Q^2)$, decrease with Q^2 . This prediction also derives from color transparency.
- (iv) Shadowing is $\propto 1/m_q^2$ and is negligible in the charm structure function of nuclei.
- (v) Diffraction excitation rate of heavy flavors decreases $\propto 1/m_q^2$. For the light quarks, diffraction excitation amounts to 10–15%, and for the charmed quarks to $\sim 1\%$, of the total excitation of a given flavor.^{26,45}

Calculation of the shadowing for heavy nuclei is a straightforward application of Eq. (22) with the wave functions (36) and (37). A comparison of theoretical predictions for μCa scattering^{25,46} with the recent NMC data⁵³ is shown in Fig. 7. Shadowing is sensitive to the large- ρ behavior of $\sigma(\rho)$ and the agreement with the data indicates that our educated guess for $\sigma(\rho)$ in this region is correct.

Fig. 7. QCD predictions⁴⁶ for nuclear shadowing versus the NMC data.⁵³

I come back to a shadowing in deuterium, which has customarily and illegitimately been neglected in all the μD data analysis (for instance, see Refs. 54 and 55). I would like to emphasize that there exists a model-independent relationship between shadowing in He and D ^{25,46,56}:

$$1 - \frac{F_2^D}{F_2^N} \approx \frac{1}{4} \left(1 - \frac{F_2^{He}}{F_2^N} \right). \quad (45)$$

Therefore, the shadowing effect in D should be $\approx \frac{1}{3}$ of the shadowing effect in the He/D ratio, shown in Fig. 7. There is some (2–3%) uncertainty in this ratio, related to the EMC effect in He (Ref. 46). Still, the NMC data⁵² in He/D give convincing direct experimental evidence for ~ 2 –3% shadowing effect in μD scattering at $x \lesssim 0.01$, which should have been properly included in the NMC determinations of the neutron structure function.^{54,55}

The error in determinations of the neutron structure function $F_2^n(x, Q^2)$ introduced by neglecting the shadowing exceeds, by a huge factor, the experimental error bars. Correct determination of $F_2^n(x, Q^2)$ has a strong impact, for instance, on the Gottfried sum rule integral⁵⁷

$$I_{GSR} = \int_0^1 \frac{dx}{x} [F_2^p(x, Q^2) - F_2^n(x, Q^2)] \quad (46)$$

and leads^{26,46,58,59} to $I_{GSR} \approx 0.16\text{--}0.17$ compared to the incorrect NMC determination of $I_{GSR}(NMC) = 0.24 \pm 0.017$ (Ref. 54).

One more comment on the nuclear shadowing is in order. In the ν -fold scattering diagrams of Fig. 8, the $q\bar{q}$ pair can be attributed to none of the ν nucleons of the nucleus. This corresponds precisely to the parton fusion model of the nuclear shadowing formulated in 1975.⁶⁰ Above we have explicitly proven that the shadowing is dominated by the large, hadronic, size $q\bar{q}$ pairs, so that fusing partons have small transverse momenta $k_{\perp} \sim m$, not $k_{\perp} \propto \sqrt{Q^2}$, again as predicted in Ref. 60.

Fig. 8. The ν -fold scattering contribution to photoabsorption on nuclei: one quark-antiquark loop is shared by all ν participating nucleons.

9. Color Transparency in Photoproduction of J/Ψ and Ψ' on Nuclei: The Frozen Size Limit

Photoproduction $\gamma N \rightarrow J/\Psi(\Psi')N$ can be viewed as the photoproduction of $c\bar{c}$ pairs which recombine into J/Ψ or Ψ' . Once the pair is produced, the recombination, or the J/Ψ formation, time in the comoving frame is given by $\tau_f \approx R_{J/\Psi}/v \approx 1/\Delta E_{c\bar{c}}$, and in the laboratory frame

$$l_f = \gamma \frac{1}{\Delta E_{c\bar{c}}} \approx 0.2 \left(\frac{\nu}{1 \text{ GeV}} \right) f. \quad (47)$$

This formation length already exceeds the nucleus size at a few tens of GeV. Still another length scale in the problem is the coherence length (35) in $c\bar{c}$ pair production:

$$l_c = \frac{2\nu}{Q^2 + (2m_c)^2} \approx 0.04 \left(\frac{\nu}{1 \text{ GeV}} \right) f. \quad (48)$$

Notice that $l_f \gg l_c$.

There are three interesting cases: (a) $l_c > R_A$, (b) $l_c \sim R_A$, $l_f > R_A$, (c) $l_f \sim R_A$. A very rich pattern of shadowing and antishadowing in the coherent and quasi-elastic J/Ψ and Ψ' photoproduction emerges. The relevant technique is the coupled-channel reformulation^{22,37} of Gribov's theory of inelastic shadowing.¹³

9.1. The conventional wisdom

To define the reference frame, let me cite the standard predictions from the Vector

Dominance Model (VDM) calculations (Refs. 61 and 62; for the review see Ref. 63) and from the classical treatment of CT,⁶⁴ which are part of the conventional wisdom.

Applying the two-channel Glauber model, one finds the transparency factor $\text{Tr}(V) = d\sigma(\gamma A \rightarrow V A^*)/A d\sigma(\gamma N \rightarrow V N)$ in the low energy ($l_c \lesssim R_A$) and high energy, ($l_c \gtrsim R_A$), limits:

$$\begin{aligned} \text{Tr}_{\text{VDM}}(l_c \leq R_A) &= \frac{1}{A} \int d^2\mathbf{b} \int_{-\infty}^{+\infty} dz n_A(\mathbf{b}, z) \exp \left[-\sigma_{\text{tot}}(VN) \int_z^{+\infty} dz' n_A(\mathbf{b}, z') \right] \\ &= \frac{1}{A} \int d^2\mathbf{b} \left\{ \frac{1 - \exp[-\sigma_{\text{tot}}(VN)T(b)]}{\sigma_{\text{tot}}(VN)} \right\} \\ &\approx 1 - \frac{1}{2A} \sigma_{\text{tot}}(VN) \int d^2\mathbf{b} T(b)^2. \end{aligned} \quad (49)$$

$$\begin{aligned} \text{Tr}_{\text{VDM}}(l_c \gg R_A) &= \frac{1}{A} \int d^2\mathbf{b} T(b) \exp[-\sigma_{\text{tot}}(VN)T(b)] \\ &\approx 1 - \frac{1}{A} \sigma_{\text{tot}}(VN) \int d^2\mathbf{b} T(b)^2. \end{aligned} \quad (50)$$

Notice that in VDM the shadowing measures the charmonium-nucleon cross-section.

The often-applied classical treatment of CT leads to the following predictions:

- (i) The produced $c\bar{c}$ -pair has a size $1/m_c \ll R_{J/\Psi}$ (see Eqs. (36)–(38)), hence nuclear attenuation of J/Ψ , Ψ' should be much weaker than expected from the VDM calculations (Eq. (50)) with the free nucleon cross-section.⁶⁵
- (ii) At moderate energies, when $l_f \sim R_A$, $c\bar{c}$ -pairs have enough time to expand inside the nucleus. Since $\sigma_{\text{tot}}(\Psi' N) \approx 2.5\sigma_{\text{tot}}(J/\Psi N)$, attenuation of Ψ' should be stronger⁶⁴ than attenuation of J/Ψ . (See Eq. (50).)
- (iii) The higher the energy ν , the slower is an expansion of the $c\bar{c}$ -pairs and the weaker is the nuclear attenuation.⁶⁴
- (iv) In the quasi-elastic production there is always some attenuation and $\text{Tr} = A_{\text{eff}}/A < 1$ (Ref. 64).

Both the VDM and classical considerations have an intuitive appeal, but none of these predictions is born out by the quantum mechanical treatment,^{35,66} as is to be presented below.

9.2. High energy limit: $l_c > R_A$

In this limit, one can apply the frozen- ρ approximation directly, as $\kappa_{ik} R_A \ll 1$ in both the photon-to-charmonium and charmonium-to-charmonium transition vertices (see Sec. 5). The forward $\gamma N \rightarrow V N$ amplitude is given by the off-diagonal matrix element

$$\sigma_{\gamma V}^N = \langle V | \sigma(\rho) | \gamma \rangle. \quad (51)$$

The forward (excluding the coherent peak region) quasi-elastic production cross-section on a nuclear target equals^{22,35}

$$16\pi \frac{d\sigma(\gamma A \rightarrow VA^*)}{dt} \Big|_{t=0} = \int d^2\mathbf{b} T(b) \left| \left\langle V \left| \sigma(\rho) \exp \left[-\frac{1}{2} \sigma(\rho) T(b) \right] \right| \gamma \right\rangle \right|^2 \quad (52)$$

and

$$\text{Tr}(V) = 1 - \frac{1}{A} \frac{\langle V | \sigma(\rho)^2 | \gamma \rangle}{\langle V | \sigma(\rho) | \gamma \rangle} \int d^2\mathbf{b} T(b)^2. \quad (53)$$

Equation (53) is quite different from the VDM formula (50).

Quarks in the charmonium are nonrelativistic and^{25,35,67}

$$|\gamma\rangle \propto m_c K_0(m_c \rho). \quad (54)$$

The wave function (54) confirms the suggestion by Brodsky and Mueller that $c\bar{c}$ pairs have a small size $\rho \sim 1/m_c$ (Ref. 65). This, however, is only a part of the story. Numerical evaluations of the shadowing term in the transparency Tr do crucially depend on the CT-dictated shape of $\sigma(\rho)$. The overlap integrals in Eqs. (51)–(53) are controlled by the product $\sigma(\rho)K_0(m_c\rho)$, shown in Fig. 9. CT makes this product peak at $\rho \approx \rho_{\text{in}} = 0.25 \text{ fm} > 1/m_c$. Shown also in Fig. 9 are WF's of J/Ψ and Ψ' , integrated over the longitudinal coordinate (neither $\sigma(\rho)$, nor the wave function (54) depend on the longitudinal coordinate).

Fig. 9. The impact parameter structure of the overlap integrals for the J/Ψ and Ψ' photoproduction amplitudes.

Regarding J/Ψ photoproduction, there is a certain numerical conspiracy: $\sigma(\rho)|\gamma\rangle$ is dominated by a fairly large $\rho \sim \rho_{\text{in}}$, although ρ_{in} is still smaller than $R_{J/\Psi}$ (see Fig. 9). However, by virtue of CT, the matrix element of $\sigma(\rho)^2$ in the shadowing term is controlled by still larger $\rho \sim R_{J/\Psi}$. By this numerical conspiracy one finds⁶⁶

$$\frac{\langle J/\Psi | \sigma(\rho)^2 | \gamma \rangle}{\langle J/\Psi | \sigma(\rho) | \gamma \rangle} \sim \sigma_{\text{tot}}(J/\Psi N) = \langle J/\Psi | \sigma(\rho) | J/\Psi \rangle, \quad (55)$$

so that $\text{Tr}(J/\Psi)$ is marginally similar to the one given by VDM formula (Eq. (50)). In this instance CT predicts strong shadowing.

The situation is exactly reversed for Ψ' : Since $\sigma_{\text{tot}}(\Psi' N) \approx 2.5\sigma_{\text{tot}}(J/\Psi N)$ (Ref. 35), the VDM formula Eq. (50) predicts $\text{Tr}(\Psi') < \text{Tr}(J/\Psi)$. However, the wave function of $|\Psi'\rangle$ has a node (Fig. 9). The resulting cancellations reduce drastically the overlap integral in Eq. (53), so that $d\sigma(\gamma N \rightarrow \Psi' N)/d\sigma(\gamma N \rightarrow J/\Psi N) = 0.16$ (Ref. 35), in good agreement with the measured value⁶⁸ $0.20 \pm 0.05(\text{stat}) \pm 0.07(\text{syst})$. CT makes these cancellations still stronger in the second moment, so that $\langle \Psi' | \sigma(\rho)^2 | \gamma \rangle < 0$ and $\text{Tr}(\Psi') > 1$! Both the Ψ' suppression on nucleons and nuclear enhancement (antishadowing) are the direct consequence of CT.

Fig. 10. The multiple scattering diagrams of photoproduction of charmonium on a nucleus.

9.3. Intermediate energy: $l_c \lesssim R_A$, $l_f > R_A$ (Ref. 31)

Consider the typical multiple-scattering contribution of Fig. 11 to the photoproduction amplitude. In the $\gamma N \rightarrow V N$ transition, the longitudinal momentum transfer equals $\kappa_L = M_V^2/2\nu = 1/l_c$, in the $V_i N \rightarrow V_j N$ transitions $\kappa_{ij} \sim 1/l_f$. In charmonium photoproduction we have the strong inequality $l_f \gg l_c$, and κ_{ij} can safely be neglected, whereas κ_L becomes universal for all the $\gamma \rightarrow V$ transitions.⁶⁶ This preserves a description of photoproduction on nuclei in the frozen- ρ representation.

A bit of the Glauber formalism should be introduced. Let $\Gamma_\rho(\mathbf{b})$ be the profile function for interaction of $c\bar{c}$ pair of size ρ with the nucleon. Here $\mathbf{b}$ is the impact parameter. The normalization is such that $\sigma(\rho) = 2 \int d^2\mathbf{b} \Gamma_\rho(\mathbf{b})$. Let $\mathbf{r}_i = (z_i, \mathbf{b}_i)$ be the positions of nucleons. Then the profile function for a transition $\gamma \rightarrow V$ can

be written as:

$$\begin{aligned} \langle V | \Gamma_A | \gamma \rangle &= \left\langle V \left| \sum_{i=1}^A \Gamma_\rho(\mathbf{b} - \mathbf{b}_i) \exp(i\kappa_L z_i) \prod_{j \neq i} [1 - \Gamma_\rho(\mathbf{b} - \mathbf{b}_j)] \right| \gamma \right\rangle \\ &= \sum_i A_i \exp(i\kappa_L z_i). \end{aligned} \quad (56)$$

One has to calculate the matrix element of (56) between the ground state $|in\rangle$ and excited state $|f\rangle$ of the target nucleus, sum up the corresponding differential cross-sections, and apply the closure to sum over all final states, excluding the coherent channel $|f\rangle = |in\rangle$. In the resulting sum

$$\sum_{i,j} A_i A_j^* \exp \left[i \frac{(z_i - z_j)}{l_c} \right], \quad (57)$$

the terms with $i = j$ correspond to an incoherent sum of differential cross-sections of scattering on separate nucleons and give precisely a half of the shadowing in Eq. (53). If $l_c \ll R_A$, the contribution of terms with $i \neq j$ vanishes upon integration over the nucleons positions.

Fig. 11. QCD prediction by Benhar *et al.*,⁶⁶ (Eq. (58), solid line) and semiclassical prediction of Farrar *et al.*⁶⁴ (dotted line) for the ratio of transparency in quasi-elastic J/Ψ photoproduction on tin and carbon as a function of energy versus the NMC data for the 200 and 280 GeV primary muons.⁶⁹

At $l_c \sim R_A$, quasi-elastic $\gamma \rightarrow V$ transitions on different nucleons at the same impact parameter are coherent. The z integrations in (57) result in a simple formula, predicting a rise of the shadowing by the factor 2 going from $l_c \ll R_A$ to $l_c \gg R_A$

(Ref. 66):

$$\text{Tr} = 1 - \frac{1 + F_{ch}(\kappa_L)^2}{2A} \frac{\langle V|\sigma(\rho)^2|\gamma\rangle}{\langle V|\sigma(\rho)|\gamma\rangle} \int d^2\mathbf{b} T(b)^2. \quad (58)$$

Derivation of the energy dependence (58) was possible only because of the strong inequality $l_f \gg l_c$. This prediction of a rise in J/Ψ shadowing with energy is in excellent agreement with the recent NMC data⁶⁹ (see Fig. 11).

In the semiclassical treatment,⁶⁴ the opposite energy dependence is predicted: at lower energies shadowing is stronger, as the $c\bar{c}$ system has time to expand inside the nucleus. Equation (57) shows clearly the importance of the quantal interference effects, ignored in Ref. 64. Unlike in Ref. 64, the VDM treats the quantal interference correctly. VDM fails because of the illegitimate reduction of the coupled many-channel problem to the two-channel, $\gamma + V$, problem. The perils of such a reduction were well understood long ago,^{22,24,70,71} in the context of hadronic diffractive production on nuclei (see Sec. 14 below).

10. Color Transparency in $\gamma \rightarrow J/\Psi, \Psi'$: Beyond the Frozen-Size Approximation

In the low-energy range of $l_f \lesssim R_A$, the interaction in, and expansion of, the $c\bar{c}$ system cannot be neglected. By virtue of CT (6), the interaction of the $c\bar{c}$ system with the nuclear medium is the absorbing harmonic oscillator potential

$$V_{\text{opt}}(\rho, t) = -\frac{i}{2} \gamma c \sigma(\rho) n_A[\mathbf{r}(t)]. \quad (59)$$

The harmonic oscillator model,

$$V(\rho) = \frac{1}{4} m_c \omega^2 \rho^2, \quad (60)$$

with $\omega = (M_{\Psi'} - M_{J/\Psi})/2$, is a good approximation to spectroscopy of the low-lying levels of the charmonium. Then, the net effect of the nuclear attenuation will be a nuclear-density dependent shift of the oscillator frequency³⁵

$$\omega^2 \rightarrow \omega^2 - i n_A(\mathbf{r}) \gamma c \omega \sigma_{\text{tot}}(J/\Psi N). \quad (61)$$

The intrinsic motion in the charmonium can be neglected, i.e., taken as frozen, if $w \ll \gamma n_A \sigma_{\text{tot}}(J/\Psi N)$, i.e., $l_c \gg l$.

Since $l_f \lesssim R_A$ implies $l_c \ll R_A$, then $c\bar{c}$ is produced incoherently on a single bound nucleon, so that the low-energy counterpart of Eq. (52) is³⁵

$$\text{Tr} = \frac{1}{A |\langle f|\sigma(\rho)|\gamma\rangle|^2} \int d^2\mathbf{b} \int dz n_A(\rho, z) \langle f|\hat{U}\sigma(\rho)|\gamma\rangle^2. \quad (62)$$

The evolution operator for the harmonic oscillator is known in an explicit form⁷²:

$$\langle y|\hat{U}|x\rangle = \left[\frac{m\omega}{2\pi i \sin(\omega t)} \right]^{1/2} \exp \left\{ \frac{m\omega}{2 \sin(\omega t)} [(y^2 + x^2) \cos(\omega t) - 2xy] \right\}. \quad (63)$$

For propagation of $c\bar{c}$, one can apply the straight path approximation, with z playing a role of time t . One can approximate the variable density nucleus by thin slices of constant density and apply the convolution formula:

$$\hat{U}(t_{n+1}) = \hat{U}(t_{n+1} - t_n) \otimes \hat{U}(t_n). \quad (64)$$

The principle finding of Kopeliovich and Zakharov³⁵ is that expansion of charmonium has a weak effect on the shadowing of J/Ψ , whereas for Ψ' the transparency Tr rises rapidly with energy and exceeds unity for all nuclei (Fig. 12).

The case of very low energy, $l_f \ll R_A$, is the only case, when the semiclassical probabilistic treatment of the nuclear shadowing in the two-channel approximation is applicable (for a review see Ref. 24). The transparency is given by the VDM formula (49) and measures the charmonium-nucleon cross-section. In Fig. 11, the curves for J/Ψ and Ψ' start from the Glauber model point. The SLAC determination⁷³ of the J/Ψ nucleon total cross-section $\sigma_{\text{tot}}(J/\Psi N) = 3.5 \pm 0.8$ mb from the Ta/Be quasi-elastic cross-section ratio at $\nu = 20$ GeV can be regarded as a reliable one. QCD calculations predict³⁵ $\sigma_{\text{tot}}(J/\Psi N) \approx 5$ mb.

Fig. 12. The energy dependence of the transparency in photoproduction of J/Ψ and Ψ' predicted by Kopeliovich and Zakharov.²⁷ The rise of shadowing towards high energy, given by Eq. (58), is not included (approximation of $F_{ch}(\kappa_L)^2 = 0$). At very low energy the solid curves should start from the crosses, which indicate the low-energy VDM predictions (Eq. (50)).

11. Color Transparency in $(e, e'\pi)$, $(e, e'p)$ Reactions on Nuclei: The Transverse Size of the Ejectile

The deep inelastic scattering and photoproduction of charmonium on nuclei were exactly soluble cases. By virtue of large Q^2 and/or large mass of the charmed quark one had a handle on the size of the hadronic states. Besides, for the same reason, these reactions were perturbatively tractable.

The initial motivation behind hard $(e, e'p)$ $(e, e'\pi)$ and $(p, p'p)$ reactions on the bound nucleons was that, by virtue of the large momentum transfer, the ejectile will emerge with the small transverse size^{4,5}

$$\rho^2 \sim \frac{1}{q^2}. \quad (65)$$

A constraint of the large coherence length $\nu \geq m_N R_A \Delta E_N$ requires very large q^2 :

$$q^2 \sim 2m_N \nu \gtrsim (5-10)A^{1/3}(\text{GeV}/c)^2. \quad (66)$$

The issue at stake is what q^2 is big enough for the onset of the asymptotic shrinkage (65).

Isgur and Llewellyn Smith⁷⁴ noticed long ago that the hard scattering regime in the form factors is elusive up to very large q^2 . *A priori*, it is not obvious whether the shrinking law (65) holds in the soft scattering domain. In this section, I shall concentrate on evaluations of ρ^2 in the electromagnetic scattering, following Ref. 18 (a similar analysis is being carried out by G. A. Miller⁷⁵).

I start with the nonrelativistic form factor calculation:

$$F_{em}(\mathbf{q}) = \int d^2\rho \int dz |\Psi(z, \rho)|^2 \exp(-iqz) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Psi^*(\mathbf{k} + \mathbf{q}) \Psi(\mathbf{k}). \quad (67)$$

As a measure of the ejectile's size I calculate the moment $\langle \rho^2 \rangle$, defined as

$$\begin{aligned} \langle \rho^2 \rangle &= \frac{1}{F_{em}(\mathbf{q})} \int d^2\rho \rho^2 \int dz |\Psi(z, \rho)|^2 \exp(-iqz) \\ &= \frac{1}{F_{em}(\mathbf{q})} \int \frac{d^3\mathbf{k}}{(2\pi)^3} [\partial_{\mathbf{k}_\perp} \Psi^*(\mathbf{k} + \mathbf{q})] [\partial_{\mathbf{k}_\perp} \Psi(\mathbf{k})]. \end{aligned} \quad (68)$$

With the simplest Gaussian Ansatz,

$$F_{em}(\mathbf{q}) = \int d^2\rho \exp\left(\frac{-\rho^2}{R^2}\right) \int dz \exp\left(\frac{-z^2}{R^2}\right) \exp(-iqz) \quad (69)$$

and the ejectile does not shrink at large q^2 !

However, the one-gluon exchange, Coulomb, interaction between the quarks leads to the slowly decreasing $\Psi(\mathbf{k})$ (Ref. 76). Specifically, the Schrödinger equation reads

$$\Psi(\mathbf{q}) = \frac{1}{\varepsilon - \mathbf{q}^2/2m} \int \frac{d^3\mathbf{k}}{(2\pi)^3} V(\mathbf{q} - \mathbf{k}) \Psi(\mathbf{k}). \quad (70)$$

If the short-range Coulomb interaction can be treated as weak perturbation, then the dominant part of $\Psi(\mathbf{k})$ is a steep function of k^2 , whereas $V(\mathbf{q} - \mathbf{k}) \propto 1/(\mathbf{q} - \mathbf{k})^2$ is a smooth function, so that at large q^2

$$\Psi(\mathbf{q}) \sim \frac{2m}{q^2} V(\mathbf{q}) \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Psi(\mathbf{k}). \quad (71)$$

With the Coulomb tail (71), the form factor will receive two identical, leading contributions, when one of the wave functions in Eqs. (67) and (68) is in the Gaussian-like regime and the second in the Coulomb regime. Then, in (67) one can neglect $\mathbf{k}$ compared to $\mathbf{q}$ in $\Psi(\mathbf{k} + \mathbf{q})$, factor out $\Psi(\mathbf{q})$, and will find $F_{em}(\mathbf{q}) \propto V(\mathbf{q}) \propto 1/(q^2)^2$. The $\partial_{\mathbf{k}_\perp}$ differentiations in (68) lead to the extra factor $\sim R^2 \mathbf{k}^2/q^2$ in the integrand, and we indeed find $\langle \rho^2 \rangle \propto 1/q^2$.

In the relativistic case $q^2 \gg m^2$, it is imperative to use light-cone wave functions. In the one hand, the *laboratory* frame kinematics corresponds to the ejectile proton receiving large *longitudinal* momentum transfer (the Fermi momentum can safely be neglected). On the other hand, the form factor can be written in terms of the light cone wave functions only provided the momentum transfer is purely *transversal*, $q = (0, \mathbf{q}, 0)$ (the Breit frame). Let the momentum transfer $\mathbf{q}$ be along the y -axis. The Lorentz transformation between the Breit frame and the laboratory frame leaves intact ρ_x — the transverse size along the normal to the scattering plane. Hence, one can safely calculate $\langle \rho_x^2 \rangle = \langle \partial_{k_x}^2 \rangle$ using the light cone formalism, and use azimuthal symmetry in the laboratory frame to reconstruct $\langle \rho^2 \rangle$. Hereafter, I shall concentrate on the $q\bar{q}$ state — the pion.

The light cone formulae for the form factor and $\langle \partial_{k_x}^2 \rangle$ read:

$$F_{em}(\mathbf{q}) = \int \frac{d^2\mathbf{k}}{2\pi} \frac{dx}{x(1-x)} \Psi^*(M_f^2) \Psi(M_{in}^2), \quad (72)$$

$$\langle \rho_x^2 \rangle = \frac{1}{F_{em}(\mathbf{q})} \int \frac{d^2\mathbf{k}}{2\pi} \frac{dx}{x(1-x)} \Psi'^*(M_f^2) \Psi'(M_{in}^2) \left[\frac{2k_x}{x(1-x)} \right]^2 \quad (73)$$

where

$$M_{in}^2 = \frac{m^2 + \mathbf{k}^2}{x(1-x)}, \quad (74)$$

$$M_f^2 = \frac{m^2 + \mathbf{k}^2}{x(1-x)} - \frac{2\mathbf{k}\mathbf{q}}{x} + \frac{(1-x)\mathbf{q}^2}{x} \quad (75)$$

With the Gaussian ansatz, $\Psi(M^2) = \Psi_0 \exp(-\frac{1}{2}R^2 M^2)$, the form factor is dominated by

$$1-x \sim 2\sqrt{m_q^2/q^2}, \quad (76)$$

i.e. the struck quark carries nearly 100% of the pion momentum. Correspondingly, at large q^2 the ejectile expands:

$$\langle \rho_x^2 \rangle_{\text{Gaussian}} \sim \left\langle \frac{R^2}{1-x} \right\rangle \propto \sqrt{\frac{q^2}{m_q^2}}. \quad (77)$$

Although this result is a peculiar property of the Gaussian ansatz, it shows that one cannot expect significant shrinkage of the ejectile unless the wave function deviates from the Gaussian one. By virtue of the one-gluon exchange in the realistic wave functions of hadrons, the Gaussian-like steeply falling $\Psi(M^2)$ at moderately large M^2 is followed by the “hard” QCD component, slowly decreasing as inverse powers of M^2 at very large M^2 (as the first power for pions). The further consideration repeats that of the Coulomb tail in the nonrelativistic case: $\Psi^*(M_f^2)$ in the Coulomb regime of large $M_f^2 \sim (1-x)q^2$ and $\Psi(M_{in}^2)$ in the Gaussian regime of small M_{in}^2 (and vice versa, $M_{in}^2 \leftrightarrow M_f^2$). In this regime, $\Psi'(M_f^2) \sim \Psi(M_f^2)/M_f^2 \propto \Psi(M_f^2)/q^2$ and $\Psi'(M_{in}^2) \sim R^2 \Psi(M_{in}^2)$.

Notice that the factor $(1-x)$ in $M_f^2 \sim (1-x)q^2$, is responsible for the elusive onset of the QCD asymptotics.⁷⁴ One approaches the hard scattering regime with $(1-x)$ following the law (76), but once in the hard scattering regime, the dominant contribution to the form factor will come from finite, approximately q^2 -independent $(1-x)$.

In Fig. 13, I present results from a toy model of the pion, which incorporates the principal features of the QCD wave function:

$$\Psi(M^2) \propto \exp\left(-\frac{1}{2}R^2M^2\right) + \alpha \frac{1}{1+R^2M^2/2} \quad (78)$$

I adjust R^2 to $\langle R_{ch}^2 \rangle = 6/m_\rho^2$. In the problem of interest, it is natural to use the effective mass of quarks m_q somewhere in between the spectroscopic constituent mass of ~ 300 MeV and the current mass ~ 10 MeV. In Fig. 13, I plot $q^2 F_{em}(q)$ and compare it with expectations for the monopole form factor $F_1(q) = 1/(1+q^2/m_\rho^2)$, which reproduces the experimental data up to $q^2 \sim 10$ (GeV/c)² (Ref. 77). Still another reference point is the perturbative QCD asymptotic $q^2 F_{em}(q) = 8\pi\alpha_S f_\pi^2 \sim 0.1$ (GeV/c)² (Ref. 76). The principal findings are (Fig. 13):

- (i) The purely Gaussian form factor, $\alpha = 0$, $R = 1.90$ (GeV/c)⁻¹, follows closely the monopole form factor up to $q^2 \sim 10$ (GeV/c)² and gives an expanding ejectile.
- (ii) The form factor with $\alpha = 0.1$, $R = 2.23$ (GeV/c)⁻¹ is a good approximation to the monopole form factor and exceeds the asymptotic QCD prediction by a factor of approximately 5–10. The ejectile’s size stays approximately constant up to $q^2 \sim 20$ –30 (GeV/c)², and then decreases rapidly with q^2 .
- (iii) The form factor with $\alpha = 0.01$, $R = 1.95$ (GeV/c)⁻¹ follows the monopole form factor up to $q^2 \sim 10$ (GeV/c)² and has an asymptotic normalization

close to the QCD prediction. The ejectile first expands up to a saturation at $q^2 \sim 30\text{--}40 \text{ (GeV/c)}^2$, and starts shrinking steeply beyond $\sim 100 \text{ (GeV/c)}^2$.

- (iv) The situation with the more conservative choice of $m = 300 \text{ MeV/c}^2$ is similar. With $\alpha = 0.01$, $R = 2.72 \text{ (GeV/c)}^{-1}$, the asymptotic normalization is again close to that expected from QCD. The transverse size stays approximately constant up to $q^2 \sim 10 \text{ (GeV/c)}^2$ and then starts decreasing. However, the form factor starts decreasing too rapidly, in conflict with the experimentally observed monopole behavior⁷⁷ up to $q^2 \sim 10 \text{ (GeV/c)}^2$.

The QCD asymptotics should be even more elusive in electron-proton scattering.^{74,78} The weak point of the above analysis is a saturation of the pion wave function by the $q\bar{q}$ Fock states. It is quite possible that mesonic cloud effects dominate in $\langle R_{ch}^2 \rangle$, which is the low-energy parameter, whereas the asymptotics of the form factor are controlled by the $q\bar{q}$ or qqq core with the size smaller than R_{ch} (I am grateful for J. Speth⁷⁹ for this observation).

Fig. 13. Top: $q^2 F_{em}(q)$. Bottom: $\rho_\pi(q)^2/\rho_\pi(0)^2$ versus q^2 . (a) the Gaussian wave function, $m = 150 \text{ (MeV/c)}^2$; (b) the Gaussian + Coulomb wave function, $m = 150 \text{ (MeV/c)}^2$, $\alpha = 0.1$; (c) the Gaussian + Coulomb wave function, $m = 150 \text{ (MeV/c)}^2$, $\alpha = 0.01$; (d) the Gaussian + Coulomb wave function, $m = 300 \text{ (MeV/c)}^2$, $\alpha = 0.01$; (e) the monopole form factor.

12. Color Transparency in $(p, p'p)$ Reactions on Nuclei

With certain reservations about the poorly known mechanism of hard elastic scattering, the conclusions of Sec. 11 are relevant to large-angle proton–proton scattering too: the ejectile state could well have large transverse size up to very large momentum transfer. The theoretical analysis of pp scattering is further compounded by the so-called Landshoff amplitude⁸⁰ (Fig. 14), which is the asymptotically leading one and is known to be dominated by large-size configurations in the proton.⁸ Besides, an important message from the above analysis of photoproduction of J/Ψ and Ψ' is that, even with the relatively large-size ejectile, quantum interference of coherent transitions between the nucleon and radial excitations of the nucleon could produce anomalously strong and/or anomalously weak nuclear attenuation in the specific exclusive $p, p^*, \dots$ production experiments.

Fig. 14. The Landshoff mechanism of the large-angle pp elastic scattering.

The experimental data on the transparency

$$Tr = d\sigma(pA \rightarrow ppX)/Ad\sigma(pp \rightarrow pp)$$

in $pA \rightarrow ppX$ reaction studied in the AGS experiment, versus the incident momentum, are shown in Fig. 15. The rising transparency fits the original expectations,^{4,5} but a drop to the Glauber model prediction at higher energies remains as yet unexplained.

In the energy range of the experiment,⁶ the frozen size approximation does not hold, as $l_c \sim R_A$. The most detailed analysis of the energy dependence and ejectile's expansion problems is due to Kopeliovich and Zakharov,⁹ who have applied the path integral formalism. For the three-quark state too, with the harmonic oscillator model of the nucleon and $\sigma(\rho) \propto \rho^2$, the interaction with the nuclear medium reduces to the oscillator frequency shift, as described briefly in Sec. 10, Eq. (61). The fundamental difficulty is with the size of the ejectile at the production vertex. For a pointlike ejectile, the nuclear transparency rises monotonically with energy (Fig. 15).

Fig. 15. The Kopeliovich-Zakharov predictions of nuclear transparency in quasi-elastic ($p, p'p$) scattering on aluminum as a function of the incident momentum⁹ versus AGS data.⁶ The two solid curves describe the transparency for a pointlike ejectile wave function at the production vertex and differ by the harmonic oscillator parameter ω . The dashed curve corresponds to the Ralston-Pire wave function⁸ with admixture of the Landshoff mechanism.

Ralston and Pire suggested that the hard-scattering and the Landshoff amplitudes have a rapidly energy-dependent relative phase,⁸ which produces a non-monotonic energy dependence of the pp scattering cross-section. Then, the experimentally observed bump in Tr is attributed to nuclear distortion or interference of the two mechanisms. Kopeliovich and Zakharov⁹ have studied this possibility too, and found that the Ralston-Pire amplitude does not explain a drop of the transparency to the Glauber model value. The problem is compounded by poor knowledge of the initial wave function and the necessity of a correct treatment of ejectile's expansion. The analysis of Ref. 9 uses the technique³⁵ outlined in Sec. 10. However, because of the saturation property (Fig. 2), the $\sigma(\rho) \propto \rho^2$ approximation is not very good for the light-quark states.

The alternative to the path integral approach⁹ is the direct coupled-channel approach, in which the relative-phase generating longitudinal momentum transfer (see Secs. 5 and 9) is explicitly taken into account.^{23,39} Here one can compute the cross-section matrix $\langle i|\hat{\sigma}|k\rangle$, using a more realistic $\sigma(\rho)$ and more realistic quark model wave functions of the nucleon and nucleon resonances. Jennings and Miller have made an initial step in this direction, still in the ρ^2 approximation, but their treatment of the coupled-channel problem and of the multiple-scattering theory is not quite correct¹⁰: after a series of questionable approximations, their final result is the single-channel description of attenuation with a complex cross-section. In the coupled-channel approach too, understanding $\langle i|\hat{\sigma}|k\rangle$ and the initial wave

function requires a *quantitative* knowledge of nucleon structure, and this cannot be circumvented even if a small-sized ejectile is procured.

There exist some problems with the AGS data analysis, since in the energy range of the AGS experiment the pp cross-section varies steeply with energy, and very careful determination of the Doppler shift of the effective energy by the Fermi motion is required. The experimental setup⁹ did not allow an unambiguous selection of the two-body scattering: the effects of the quasi-elastic rescattering of the projectile and the ejectile could have been mistaken for the Doppler shift.⁸¹ Indeed, the acoplanarity momentum distribution in the same data⁹ gives too broad a Fermi momentum distribution, with broad tails beyond $k \gtrsim \langle p_F \rangle$ and $\langle p_F \rangle$ exceeding by $\approx 20\%$ the standard values.⁸² The rescattering corrections reproduce⁸¹ the observed broadening of the effective Fermi momentum distribution, which could contribute to a rise of the transparency, shown in Fig. 14, but makes it even more difficult to explain a drop of the transparency at higher energies.

Brodsky and Teramond⁷ suggested still another mechanism of the soft interaction dominated pp scattering at 90° in the AGS energy range. They observe that $p_{\text{lab}} \sim 12$ (GeV/c) corresponds to $E_{\text{cm}} \sim 5$ GeV, which is the charm production threshold and the region of large spin correlation A_{NN} . Brodsky and Teramond attribute normal nuclear attenuation to pp scattering at $p_{\text{lab}} \sim 12$ (GeV/c) being dominated by an s -channel, hidden charm, $B = 2$ resonance, which couples to fully-interacting large-scale structure of the proton.

13. Filtering Small-Size Ejectile: The Sudakov Form Factor

The celebrated Sudakov form factor⁸³ could provide shrinkage of the ejectile even with the dominant Landshoff mechanism.⁸⁰ I shall present here the “back-of-an-envelope derivation”, which illustrates the major physics behind the Sudakov effects. I urge readers to consult Ref. 84 for more rigorous considerations and further references. I conclude that the Sudakov form factor effects are weak at attainable values of q^2 (Refs. 18 and 84).

Asking for the elastic scattering one asks for no-radiation of gluons. For the quark-antiquark system of size ρ , hit by the electron with the momentum transfer q , the mean number of the would-be radiated gluons N_g , which would have taken a fraction x_g of the hadron's momentum $x_{\text{min}} < x_g < 1$, equals⁴⁶

$$N_g(\rho, q^2, x_{\text{min}}) \approx \frac{16}{3} \int_{x_{\text{min}}}^1 \frac{dx}{x} \int_0^{q^2} \frac{d^2\mathbf{k}}{2\pi} \frac{\alpha_S(\mathbf{k}^2)}{2\pi} \frac{1 - \exp(-i\mathbf{k}\rho)}{\mathbf{k}^2} \quad (79)$$

where $\alpha_S(q^2)$ is the running QCD coupling. With the Gaussian-like wave function, $x_{\text{min}} \sim \sqrt{m^2/q^2}$ (see Eq. (76)) and the probability of no-radiation, alias the Sudakov form factor, can be estimated as

$$F_S(\rho, q, x_{\text{min}}) \approx \exp[-N_g(\rho, q^2)] \approx \exp \left\{ -\frac{8}{27} \log \left(\frac{q^2}{m^2} \right) \log \left[\frac{\alpha_S(1/\rho^2)}{\alpha_S(q^2)} \right] \right\}. \quad (80)$$

Here, the exponent written for three active flavors, $\log(q^2/m^2)$ is, in fact, $2\log(1/x_{\min})$.

The Sudakov form factor (80) is a very slow function of the size ρ ; still, it gives a preference to a contribution from the small- ρ configurations. Numerical estimations by Botts⁸⁴ indicate that in the Landshoff elastic pp scattering the Sudakov effects become significant only beyond $q^2 \sim 10^2\text{--}10^4$ (GeV/c)². Still another reason why the Sudakov form factor cannot play the dominant role in the shrinkage of the ejectile system is as follows¹⁸: One has to sandwich (80) between the initial and final-state wave functions in the form factor formula (72), written in the mixed (x, ρ) representation:

$$F(q) = \int_0^1 dx \int d^2\rho \exp[-i(1-x)\rho q] |\Psi(x, \rho)|^2 F_S(\rho, q, x_{\min}). \quad (81)$$

If the struck quark has the momentum fraction x , then one asks for no-radiation of gluons with the momentum fraction $x_g > x_{\min} \sim 1-x$, so that in (80) $\log(q^2/m^2) \rightarrow 2\log(1/(1-x))$. In the hard scattering regime, to a leading approximation, $1-x$ stays finite and independent of q^2 , see Sec. 11 (for reviews see Refs. 76 and 78).

14. Color Transparency and Color Opacity in Diffractive Hadron-Nucleus Scattering

The major conclusion of Sec. 11 was that hard scattering does not necessarily produce a small-sized ejectile, unless the momentum transfer squared is very large. Furthermore, from Sec. 9 we have learned that, by virtue of the quantum interference effects, even a relatively small size of the projectile does not necessarily imply weak nuclear attenuation. Cross-sections of the hard scattering in nuclei are prone to large Doppler shift corrections, and accurate separation of the two-body processes is very difficult experimentally. Is it not a prejudice, then, that only hard scattering processes are suitable for the color transparency studies? In fact, in retrospect, color transparency effects seem to have already been observed 20-odd years ago in the diffractive scattering off nuclei! Perhaps this review should have rather started with this section.

The conventional description of diffraction dissociation on nuclei is based on the two-channel approximation, treating the off-diagonal $hN \rightarrow h^*N$ transitions to the first, lowest, order. For the coherent diffraction excitation $hA \rightarrow h^*A$, one easily finds the transition profile function:

$$\Gamma_A^{(12)}(\mathbf{b}) = 2\Sigma(hN \rightarrow h^*N)F_L(\mathbf{b}, \kappa_L) \times \left\{ \frac{\exp[-\frac{1}{2}\sigma_1 T(\mathbf{b})] - \exp[-\frac{1}{2}\sigma_2 T(\mathbf{b})]}{\sigma_1 - \sigma_2} \right\}. \quad (82)$$

The form factor $F_L(\mathbf{b}, \kappa_L)$ is defined by

$$\int dz \exp[i\kappa_L z] n_A(\mathbf{b}, z) = T(\mathbf{b})F_L(\mathbf{b}, \kappa_L), \quad (83)$$

where $\kappa_L = (m_2^2 - m_1^2)/2\nu$ is the familiar longitudinal momentum transfer (8). Similar treatment of the quasi-elastic, or incoherent, diffraction dissociation $hA \rightarrow h^*A^*$, when one sums over all excitations and breakup of a nucleus, gives

$$\left. \frac{d\sigma(hA \rightarrow h^*A^*)}{dt} \right|_{t=0} = \left. \frac{d\sigma(hN \rightarrow h^*N)}{dt} \right|_{t=0} \int d^2\mathbf{b} \times \left\{ \frac{\exp[-\sigma_1 T(\mathbf{b})] - \exp[-\sigma_2 T(\mathbf{b})]}{\sigma_1 - \sigma_2} \right\}. \quad (84)$$

Equations (83) and (84) were the basis of suggestions to measure $\sigma_{\text{tot}}(h^*N) = \sigma_2$ for resonances and multiparticle states in the coherent and incoherent diffraction dissociation on nuclei.^{61,85} The semiclassical formula (84) is correct only at $l_c \ll R_A$, although it is widely and erroneously applied to the high-energy data too. In the high-energy limit of $l_c \gg R_A$, more involved analysis gives^{22,23,86,87}:

$$\left. \frac{d\sigma(hA \rightarrow h^*A^*)}{dt} \right|_{t=0} = \left. \frac{d\sigma(hN \rightarrow h^*N)}{dt} \right|_{t=0} \int d^2\mathbf{b} T(\mathbf{b}) \times \left\{ \frac{\sigma_1 \exp[-\sigma_1 T(\mathbf{b})/2] - \sigma_2 \exp[-\sigma_2 T(\mathbf{b})/2]}{\sigma_1 - \sigma_2} \right\}^2. \quad (85)$$

More than a decade of active experiments followed, all with the confusing and counterintuitive result that σ_2 for resonances and multiparticle states is much smaller than $\sigma_{\text{tot}}(hN)$. The typical results of the high-energy experiments^{88,89} are shown in Fig. 15. The data⁹⁰ on $nA \rightarrow p\pi^-A$ at $E = 12\text{--}14$ GeV give rather $\sigma_2 \approx 40$ mb. Different values of σ_2 for the same $p\pi^-\pi^+$ system come out from the coherent and incoherent dissociation.^{91,92} For the 5π system σ_2 is smaller than for the 3π system, for the 3π system $\sigma_2 \lesssim \sigma_{\text{tot}}(\pi N)$ and depends strongly on the spin and parity of the produced system.⁹³

The subject was abandoned before the coherent dynamical theory of the diffraction dissociation had emerged. Still, by the late '70s it was well understood that the perturbative two-channel approximation is too crude, that the formulae (83) and (85) do not have any applicability domain and the diffraction dissociation does not measure the unstable particle cross-sections^{22,70,71,94} (for the review see Ref. 24). An extensive analysis of J/Ψ and Ψ' photoproduction in Sec. 9 is an excellent illustration of this statement, cf. Eqs. (50) and (53). Still, it is convenient to characterize the nuclear attenuation in terms of the parameter σ_2 .

Here, I just comment on a diffractive production of the first radial excitation of the pion $\pi(1300)$, which resembles the Ψ' photoproduction for the presence of a node in the wave function of $\pi(1300)$. The major difference is that, in the photoproduction case, the radius of $|\gamma\rangle_{c\bar{c}}$ was much smaller than the charmonium size; in $\pi \rightarrow \pi(1300)$ the initial pion has a large size and the overlap integral $\langle \pi^* | \sigma(\rho) | \pi \rangle$ is rather dominated by ρ above the node. Therefore, the overall nuclear filtering is

dominated by $\sigma(\rho)$ at $\rho \gtrsim R_\pi$. Besides, suppression of large ρ by nuclear filtering effectively lowers the overlap integral. Both effects amount to $\sigma_2 > \sigma_{\text{tot}}(\pi N)$, i.e., the same mechanism which has led to enhancement $\text{Tr}(\gamma \rightarrow \Psi') > 1$ leads to the nuclear opacity in $\pi \rightarrow \pi^*$. Indeed, the Kölbig–Margolis parametrization of the 15 GeV $\pi A \rightarrow 3\pi A$ data in the $A_1 - \pi^*$ mass range gives $\sigma(0^-) = 49^{+9}_{-7}$ mb, compared to $\sigma(1^+) = 20^{+1.8}_{-1.5}$ mb (Ref. 93). Similar analysis⁹⁵ of the 40 GeV data from the Bologna–Dubna–Milano experiment⁹⁶ gives $\sigma_2(0^-) = 32^{+6}_{-5}$ mb, compared to $\sigma_2(1^+) = 11 \pm 1$ mb. In both cases, the combined mass spectrum is dominated by 1^+ production. The wave function of the Roper $N^*(1470)$ resonance should have a similar radial node, but an accurate partial wave analysis of the diffractively excited $N\pi$, $N\pi\pi$ systems is lacking.

Fig. 16. Cross-section σ_2 for $p\pi^-$ and $p\pi^-\pi^+$ systems determined in the Kölbig–Margolis approximation from (left) diffraction dissociation on deuterons⁸⁴ and (right) from the diffractive dissociation $n \rightarrow p\pi^-$ on nuclei.⁸⁵

The QCD approach to diffractive transitions allows one to combine color transparency ideas with the quark model spectroscopy of resonances, and a fresh look at diffraction dissociation physics is called for. Other soft processes can be treated

similarly (for a recent review see Ref. 97). For instance, in the quasi-elastic charge exchange of pions of nuclei,^{9,19}

$$\begin{aligned} \frac{d\sigma(\pi^+ A \rightarrow \pi^0 A')}{dt} &= \frac{1}{8\pi} \frac{Z}{A} \int d^2\mathbf{b} \\ &\times \text{Tr} \left\{ \left\langle f_{\text{cex}}(\mathbf{q}, \rho')^+ \exp \left[-\frac{1}{2} \sigma(\rho') T(\mathbf{b}) \right] \right\rangle_{\rho'} \right. \\ &\times \left. \left\langle f_{\text{cex}}(\mathbf{q}, \rho) \exp \left[-\frac{1}{2} \sigma(\rho) T(\mathbf{b}) \right] \right\rangle_{\rho} \right\}. \end{aligned} \quad (86)$$

In the coincidence experiment $\pi^+ A \rightarrow \pi^0 p A'$, the integrand of (87) will contain the proton absorption factor. The trace in (87) refers to the spin-dependence of the charge exchange amplitude:

$$f_{\text{cex}}(\mathbf{q}, \rho) = C[1 + i\beta q\sigma \cdot \mathbf{n}] \exp \left[\frac{i}{2} \mathbf{q} \cdot \rho - \frac{1}{2} \lambda q^2 \right]. \quad (87)$$

Fig. 17. Nuclear transparency in pion charge exchange on carbon at 40 (GeV/c).^{20,21} The solid curve is computed with $\sigma(\rho) \propto \rho^2$, the dashed curve is the Glauber model calculation with $\sigma(\rho) = \sigma_{\text{tot}}(\pi N)$.^{9,19}

Since $T(b) \sim R_A n_A(0) \sim A^{1/3}/(60 \text{ mb})$, the ρ -dependent absorption factor $\exp[-\sigma(\rho)T(b)/2]$ varies significantly at $\rho \sim R_\pi$ and modifies strongly the q^2 -dependence of the differential cross-section (87) already at small q^2 . Compared with the standard Glauber model formula in which $\sigma(\rho) = \sigma_{\text{tot}}(\pi N)$, formula (87)

leads to a much more rapid rise of the nuclear transparency with $|t| = \mathbf{q}^2$ (Fig. 17), which overshoots unity at $q^2 \sim 0.4 \text{ (GeV/c)}^2$. The bump at $q^2 \approx 0.6 \text{ (GeV/c)}^2$ is due to a dip in the charge exchange on free nucleons and a contribution of the higher order quasi-elastic rescattering of pions in a nucleus (for more details see Refs. 9 and 19).

Similar considerations of the color transparency effects in the antiproton charge exchange $\bar{p}p \rightarrow \bar{\Lambda}\Lambda$ on nuclei are contained in Ref. 98. In Ref. 99 it was argued that the Regge rise of the diffraction slope in the coherent K_S -regeneration on nuclei $K_L A \rightarrow K_S A$ enhances a contribution of the large- ρ , i.e., $\rho \gtrsim R_K$; components of the kaon wave function to the regeneration amplitude. In interaction with nuclear targets, this results in an attenuation stronger than estimated with the free KN total cross-section, i.e., in the color opacity. The experimental data¹⁰⁰ on K_S -regeneration on nuclei give evidence for precisely such an enhanced nuclear opacity. If the $K_L N \rightarrow K_S N$ regeneration amplitude had been structureless and ρ -independent, then inelastic shadowing corrections would have rather predicted reduced nuclear opacity (see Eq. (28)).

15. Conclusions

The answers to the three principal questions raised in the Introduction are:

- The Color Transparency property is quantified by the universal cross-section $\sigma(\rho)$ (Fig. 2), and wave functions of the incident and final particle (hadron, photon). The overlap integrals for specific (semi)exclusive hard scattering processes are not necessarily dominated by small ρ , though, and accurate quantum mechanical analysis is imperative.
- Gribov's inelastic shadowing with strong cancellations of the diagonal and the off-diagonal transitions is a necessary condition for the manifestation of Color Transparency, and requires a large coherence length $l_c > R_A$.
- Hard scattering filters small-sized configurations of interacting hadrons, but the onset of the asymptotic shrinkage may be elusive up to very large momentum transfer. However, small global size of an ejectile at the production vertex is not a sufficient condition for weak attenuation in the specific exclusive channels.

The best process to test color transparency and to measure $\sigma(\rho)$ is a diffractive deep inelastic scattering on hydrogen and nuclei. In the diffractive nuclear reactions with hadronic ejectiles, $\sigma(\rho)$ of Fig. 2 may lead to both color transparency and color opacity depending on the particular reaction. Semiclassical considerations suggest vanishing nuclear attenuation and $\text{Tr} = 1$ in hard scattering processes, when the emerging ejectile has small transverse size $\rho^2 \sim 1/q^2 \ll R_h^2$. The principal message from the correct quantum-mechanical treatment of Gribov's inelastic shadowing is that this semiclassical expectation is not necessarily realised in specific exclusive or semiexclusive reactions. Nuclear transparency can exceed unity, $\text{Tr} > 1$, and can be larger for the exclusive final states, which have larger free nucleon cross-section.

Even if a small size of the ejectile is procured at the production vertex, the further nuclear attenuation is controlled by the soft diffractive interactions, and we simply cannot proceed further without first understanding inelastic shadowing in diffractive processes. Regarding all reaction with pions and nucleons as ejectiles, the complexity of hard scattering versus soft diffractive scattering is just higher, as we understand the initial wave function better in diffractive rather than hard scattering.

A signal of the relevance of quantum-mechanical effects in charmonium propagation in nuclei has also been provided by the FNAL E772 data for forward J/Ψ and Ψ' production in pA collisions¹⁰¹: despite the very different free-nucleon cross-sections, J/Ψ and Ψ' have strong and identical nuclear attenuation. With only the exception of quasi-elastic production at low energies, $f_f \lesssim R_A$, neither photoproduction nor hadroproduction on nuclei measure the charmonium-nucleon cross-section.

Acknowledgments

This review originates from lectures delivered at the Institut für Kernphysik, KFA, Jülich. I am grateful to Prof. J. Speth for his hospitality at IKP-KFA and for illuminating discussions on the form factor. Thanks are due to G. A. Miller for the hospitality at INT, University of Washington, during the program "Nucleons and Nucleon Structure" in July–August 1991, and for discussions on the ejectile's size. I am indebted to V. Barone, M. Genovese, B. Z. Kopeliovich, E. Predazzi, B. G. Zakharov and V. R. Zoller for the pleasure of collaborative work and fruitful discussions.

Note Added (24 April 1994)

The major development in the subject of color transparency during the two years since this review was completed is as follows:

i) The technique of Sec. 9 was extended^{102,103} to be the virtual photoproduction of vector mesons on nuclei. Because of the shrinkage of wave functions (36,37) with Q^2 , nuclear shadowing was predicted to decrease with Q^2 . This prediction was confirmed by the Fermilab E665 experiment¹⁰⁴ on exclusive production of the ρ^0 mesons in deep inelastic scattering on nuclei. The E665 result is the first decisive test of color transparency in exclusive scattering.

ii) The detailed theory of the onset of color transparency in $A(e, e'p)$ scattering was developed in Ref. 105. The coherency constraints discussed in Sec. 9, were shown to lead to a very slow onset of color transparency in $A(e, e'p)$ reaction. This prediction is in excellent agreement with the SLAC NE18 experiment,¹⁰⁶ which did not find any signal of color transparency at $Q^2 \lesssim 7 \text{ GeV}^2$.

The detailed review of this new development is contained in Ref. 107.

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