ERRATUM

ON THE SUPERSTRING HAMILTONIAN IN THE FRIEDMANN SPACE-TIME

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The following are corrections to errors in the published paper:

- (1) On p. 845, in line 21, printed as “anti-de Sitter” should read “de Sitter”
- (2) On p. 845, in line 22, printed as “ $\Lambda = \{18/[175\zeta(3) - \frac{1}{2}]\}^{1/3} A_r \kappa^{-2}$ ” should read “ $\Lambda = \{18/[337\zeta(3) + \frac{1}{2}]\}^{1/3} A_r^{-1} \kappa^{-2}$ ”
- (3) On p. 845, in line 23, printed as “singularity-free and horizon-free” should read “singularity-free”
- (4) On p. 845, in line 23, printed as “stable” should read “which, however, is unstable”
- (5) On p. 845, in line 25, printed as “ $f \approx (28.6 - 369)$ ” should read “ $f \approx (19.8 - 82.0)$ ”
- (6) On p. 845, in line 27, printed as “anti-de Sitter” should read “de Sitter”
- (7) On p. 845, in line 27, printed as “Nordström” should read “potential”
- (8) On p. 845, lines 29–31 should read as “of non-interacting gravitons at zero temperature”
- (9) On p. 847, in line 11, printed as “...high-curvature anti-” should read “...high-curvature”

*Temporary address.

(10) On p. 847, Eq. (10) should read as

$$L^{(4)} = \frac{1}{2}\kappa^4 A_r^3 \left\{ \left[-\frac{75}{8}\zeta(3) + \frac{1}{32} \right] \mathcal{B}^2 + \left[\frac{178}{9}\zeta(3) - \frac{1}{288} \right] \mathcal{B}R^2 \right. \\ \left. - \frac{87}{3}\zeta(3)\mathcal{C}R + \frac{79}{4}\zeta(3)\mathcal{D} - \left[\frac{65}{24}\zeta(3) + \frac{1}{1,152} \right] R^4 \right\}, \quad (10)$$

(11) On p. 849, in Eq. (23), printed as “ $\dots + C_0 \dot{a}^8 C_2 \dot{a}^4 \ddot{a}^2 + \dots$ ” should read “ $\dots + C_0 \dot{a}^8 + C_2 \dot{a}^4 \ddot{a}^2 + \dots$ ”

(12) On p. 849, Eqs. (30)–(40) should read

$$\lambda'_0 = 228\zeta(3) + \frac{15}{16}, \quad (30)$$

$$\alpha'_0 = 1,608\zeta(3) + \frac{15}{4}, \quad (31)$$

$$b_0 = 2,974\zeta(3) + \frac{25}{4}, \quad (32)$$

$$b'_2 = 546\zeta(3) + \frac{3}{8}, \quad (33)$$

$$k_0 = \frac{1}{5}[10,938\zeta(3) + 21], \quad (34)$$

$$k_2 = 2,613\zeta(3) + \frac{3}{4}, \quad (35)$$

$$k_3 = 507\zeta(3), \quad (36)$$

$$c_0 = \frac{9}{14} \left[337\zeta(3) + \frac{1}{2} \right], \quad (37)$$

$$c_2 = 2,436\zeta(3) + 6, \quad (38)$$

$$c_3 = 753\zeta(3) + \frac{15}{4}, \quad (39)$$

$$c_4 = \frac{123}{2}\zeta(3) + \frac{15}{16}. \quad (40)$$

(13) On p. 850, in line 6, printed as “28.6 – 369” should read “19.8 – 82.0”

(14) On p. 850, in line 9, printed as “ c_0 is $225 \equiv 9 \times 175/7 \equiv 9(11,737 - 11,562)/7$ ” should read “ c_0 is $3,033/14 \equiv (108,927 - 105,894)/14$ ”

(15) On p. 850, in line 10, printed as “ $1/f = 175/11,562 \approx 1/66.1$ ” should read “ $1/f = 1,011/35,298 \approx 1/34.9$ ”

(16) On p. 850, in line 15, printed as “ $f \approx 369$ ” should read “ $f \approx 82.0$ ”

(17) On p. 850, in Eq. (42), printed as

$$\left\{ \left[6(4M^2 - 3M - 3) - M(M+1)(M-3) - \frac{1}{32}[\dots] \right] \zeta(3) + \dots \right.$$

should read

$$\left\{ \left[M(M+1)(M-3) - 6(4M^2 - 3M - 3) - \frac{1}{32}[\dots] \right] \zeta(3) + \dots \right.$$

(18) On p. 850, Table 1 should read

Coefficient	Power of k	f_{exact}	$f_{\text{approximate}}$
λ'_0	4	2,259/114	19.8
a'_0	3	3,018/67	45.0
b_0	2	110,223/2,974	37.1
b'_2	2	5,751/182	31.6
k_0	1	63,675/1,823	34.9
k_2	1	24,048/871	27.6
k_3	1	4,182/169	24.7
c_0	0	35,298/1,011	34.9
c_2	0	85,131/2,436	34.9
c_3	0	10,902/251	43.4
c_4	0	10,080/123	82.0

- (19) On p. 852, in line 26, printed as “... $f_0 A_r^2 k^2$... $h_0 A_r^5 k^8$ ” should read “... $f_0 A_r^2 k^2$... $h_0 A_r^5 k^8$ ”
- (20) On p. 852, in line 27, printed as “ $7\zeta_0 \xi^8$ ” should read “ $7C_0 \xi^8$ ”
- (21) On p. 852, lines 29 and 30 should read “ignorable provided that $f_0 \ll 5.4\{[337\zeta(3) + \frac{1}{2}]/18\}^{2/3} \approx 43.1$ and $h_0 \ll (729/11)\{[337\zeta(3) + \frac{1}{2}]/18\}^{5/3} \approx 11,912$.”
- (22) On p. 853, in line 2, printed as “anti-de Sitter space” should read “de Sitter space”
- (23) On p. 853, in line 3, printed as “negative” should read “positive”
- (24) On p. 853, Eq. (59) should read

$$\Lambda = 3\xi^2 = \left[\frac{18}{337\zeta(3) + \frac{1}{2}} \right]^{1/3} A_r^{-1} \kappa^{-2}, \tag{59}$$

- (25) On p. 856, in line 5, printed as “anti-de Sitter” should read “de Sitter”
- (26) On p. 856, line 6 should read

$$\ddot{\alpha}_0(t) = \ddot{\alpha}_0(t) = 0. \tag{82}$$

- (27) On p. 856, from line 7 onwards, and on p. 857, the stability analysis of the de Sitter-space solution should read as follows:
The Lagrangian and Hamiltonian functions take the forms

$$\mathcal{L} = a^3(A_0 \dot{\alpha}^2 + C_0 \dot{\alpha}^8) \tag{83}$$

and

$$\mathcal{H} = \dot{\alpha} \partial \mathcal{L} / \partial \dot{\alpha} - \mathcal{L} = a^3 (A_0 \dot{\alpha}^2 + 7C_0 \dot{\alpha}^8) = 0, \quad (84)$$

the solution to Eq. (84) being

$$\dot{\alpha}_0 = \pm (-A_0/7C_0)^{1/6} = \pm \left\{ \frac{2}{3 \left[337\zeta(3) + \frac{1}{2} \right]} \right\}^{1/6} A_r^{-1/2} \kappa^{-1}. \quad (85)$$

In this case, the coefficients A' , B' and C defined from Eqs. (74), (75) and (76), respectively, after substitution from the solution Eq. (57) or (84), reduce to

$$A' = 2A_0 \dot{\alpha}_0 + 56C_0 \dot{\alpha}^7 = -6A_0 \dot{\alpha}_0 = 18\kappa^{-2} \dot{\alpha}_0, \quad (86)$$

$$\begin{aligned} B' &= -6(B_2 \dot{\alpha}_0^2 + C_2 \dot{\alpha}^6) = -6[B_2(-A_0/7C_0)^{1/3} + C_2(-A_0/7C_0)] \\ &= -144 \left\{ \frac{2}{3 \left[337\zeta(3) + \frac{1}{2} \right]} \right\}^{1/3} \\ &\quad \times \left\{ B + \left[\frac{203\zeta(3) + \frac{1}{2}}{337\zeta(3) + \frac{1}{2}} \right] \left[\frac{337\zeta(3) + \frac{1}{2}}{18} \right]^{1/3} A_r \right\} A_r^{-1} \kappa^{-2} \end{aligned} \quad (87)$$

and

$$C = -2[B_2 + C_2(-A_0/7C_0)^{2/3}] \dot{\alpha}_0 = B'/3\dot{\alpha}_0, \quad (88)$$

where $\dot{\alpha}_0$ is given by Eq. (85).

Dividing Eq. (73) by $\dot{\alpha}_0 \delta(t)$ after substituting from Eq. (79), we derive the characteristic equation

$$\mathcal{A}\lambda \pm \mathcal{B}\lambda^2 + \mathcal{C}\lambda^3 = 0, \quad (89)$$

where

$$\mathcal{A} = A'/\dot{\alpha}_0 = 18\kappa^{-2}, \quad (90)$$

$$\mathcal{B} = -|B'/\dot{\alpha}_0|$$

$$\begin{aligned} &= -144 \left\{ \frac{2}{3 \left[337\zeta(3) + \frac{1}{2} \right]} \right\}^{1/6} \\ &\quad \times \left\{ B + \left[\frac{203\zeta(3) + \frac{1}{2}}{337\zeta(3) + \frac{1}{2}} \right] \left[\frac{337\zeta(3) + \frac{1}{2}}{18} \right]^{1/3} A_r \right\} A_r^{-1/2} \kappa^{-1} \end{aligned} \quad (91)$$

and

$$\mathcal{C} = C/\dot{\alpha}_0 = -2[B_2 + C_2(-A_0/7C_0)^{2/3}]$$

$$= -48 \left\{ B + \left[\frac{203\zeta(3) + \frac{1}{2}}{337\zeta(3) + \frac{1}{2}} \right] \left[\frac{337\zeta(3) + \frac{1}{2}}{18} \right]^{1/3} A_r \right\}, \quad (92)$$

in which the $\pm$ sign of the coefficient $\mathcal{B}$ in Eq. (89) refers to the $\pm$ sign in Eq. (85). The solutions to Eq. (89) are correspondingly

$$\lambda = 0, \lambda \equiv \lambda_{\pm\pm} = \frac{\mp \mathcal{B} \pm \sqrt{\mathcal{B}^2 - 4\mathcal{A}\mathcal{C}}}{2\mathcal{C}} = \frac{\mp |\mathcal{B}| \mp \sqrt{\mathcal{B}^2 + 4\mathcal{A}|\mathcal{C}|}}{2|\mathcal{C}|}, \quad (93)$$

since $\mathcal{A} > 0, \mathcal{B} < 0, \mathcal{C} < 0$, which show that there are not only decaying modes $\lambda_{\pm+} < 0$, but also growing modes $\lambda_{\pm-} > 0$, irrespective of the choice of sign $\pm$ in Eq. (85) for $\dot{\alpha}_0$.

We thus obtain the important result that the de Sitter-space solution (58) is unstable to linear perturbations, essentially on the Planck time scale

$$\tau = \frac{1}{\lambda} = \frac{2|\mathcal{C}|}{|\mathcal{B}| + \sqrt{\mathcal{B}^2 + 4\mathcal{A}|\mathcal{C}|}} \approx \frac{A_r^{1/2} \kappa}{5} \approx t_P. \quad (94)$$

(28) On p. 858, in lines 16 and 22, printed as “anti-de Sitter” should read “de Sitter”

(29) On p. 858, Eq. (103) should read

$$\mathcal{R}_0^2 = \frac{8\Lambda^2}{3} = \frac{8}{3} \left[\frac{18}{337\zeta(3) + \frac{1}{2}} \right]^{2/3} A_r^{-2} \kappa^{-4}. \quad (103)$$

(30) On p. 858, Eq. (104) should read

$$t_c \approx \frac{3^{1/2}}{2} \left[\frac{337\zeta(3) + \frac{1}{2}}{18} \right]^{1/6} A_r^{1/2} \kappa \approx 10 t_P. \quad (104)$$

(31) On p. 859, in line 24, printed as “anti-de Sitter” should read “de Sitter”

(32) On p. 859, lines 32 and 33 should read “that $S/\hbar \approx 1$ at t_c .^{18–20}”

(33) On p. 860, lines 1–30 do not apply.

(34) On pp. 861–865, Section 7 does not apply.

(35) On p. 866, in lines 2 and 3, Eq. (A.6) printed as

$$\left\{ - \left(L_1 - 2L_2 + \frac{1}{6}L_3 \right) + \frac{1}{32}[\dots] \right\}$$

should read

$$\left\{ \left(L_1 - 2L_2 + \frac{1}{6}L_3 \right) + \frac{1}{32}[\dots] \right\}$$

(36) On p. 866, in line 15, Eq. (A.9) printed as

$$-L_1 = -\frac{3}{7}M(M-1)(M-7)\dot{\alpha}^8 + \dots$$

should read

$$L_1 = -\frac{3}{7}M(M-1)(M-7)\dot{\alpha}^8 + \dots$$

(37) On p. 866, in line 17, Eq. (A.11) printed as

$$-\frac{1}{6}L_3 = \frac{1}{42}M^2 \dots$$

should read

$$\frac{1}{6}L_3 = \frac{1}{42}M^2 \dots$$

(38) On p. 867, in lines 6 and 7, Eq. (A.18) printed as

$$\left\{ \left[6(4M^2 - 3M - 3) - M(M+1)(M-3) - \frac{1}{32}M^3[\dots] \right] \zeta(3) + \dots \right.$$

should read

$$\left\{ \left[M(M+1)(M-3) - 6(4M^2 - 3M - 3) - \frac{1}{32}M^3[\dots] \right] \zeta(3) + \dots \right.$$

(39) On p. 867, in line 9, printed as “The (negative) expression (37)” should read
“The expression (37)”