AN ERRATUM TO “ON THE G_2 LINK INVARIANT”

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The author is indebted to Francois Jaeger for pointing out the following error in “On the G_2 -Link invariant” which appeared in JKTR, Vol. 2 No. 4 (1993) 431–451: In remark 3.1 the total coefficient of the element $H_{i+1}K_iH_{i+1}$ is zero. Moreover, it turns out that $H_{i+1}K_iH_{i+1}$ cannot be expressed as a linear combination of elements of the form $a\chi b$, where a, b involve elements with indices up to i and $\chi \in \{1, G_{i+1}, E_{i+1}, H_{i+1}, K_{i+1}\}$ in the algebra $F_n(q)$. Hence, the defining relations of $F_n(q)$ are not enough to independently define and calculate the G_2 link invariant in the class of braids.

The way to get around this is the following:

We consider a suitable quotient $AT_n \cong F_n(q)$, by adding new relations which follow from Kuperberg’s “pentagon rule” and from graph isotopy. Hence, the trace τ_n , obtained in theorem 2.1, factors through this quotient. On the other hand, one can see that on this quotient all the steps of the construction of the trace tr carry through.

More presicely, let $\bar{F}_n(q)$ be the quotient of $AT_n \cong F_n(q)$ by:

$$\begin{aligned} H_{i+1}H_iK_{i-1}H_iH_{i+1} &= cK_iE_{i+1}K_{i-1}K_i + K_{i-1}H_{i+1}K_iH_{i+1} + H_{i+1}K_iH_{i+1}K_{i-1} + \\ &+ cK_{i-1}K_iE_{i+1}K_i + cK_iE_{i+1}K_iK_{i-1} - cK_{i-1}Y_{i+1} - \\ &- cY_{i+1}K_{i-1} - cK_{i-1}H_{i+1} - cK_iE_{i+1}Z_i - cW_iE_{i+1}K_i \end{aligned}$$

$$\begin{aligned} H_{i+1}K_iH_{i+1}K_{i-1}H_i &= cK_{i-1}K_iE_{i+1}H_i + H_{i+1}H_iK_{i-1}H_i + cK_iE_{i+1}Z_iH_i + \\ &+ cK_{i-1}H_{i+1}H_i + cK_iE_{i+1}K_{i-1}H_i - cH_{i+1}E_iK_{i-1} - \\ &- cH_{i+1}W_i - Y_{i+1}K_{i-1}H_i - cK_{i-1}Y_{i+1} - cK_{i-1}Z_{i+1} \end{aligned}$$

$$K_{i+1}H_iK_{i-1}H_iK_{i+1} = cK_iH_{i+1}K_{i-1}K_i$$

$$K_iH_{i+1}H_{i-1}K_i = K_{i-1}H_iE_{i+1}H_iK_{i-1}$$

$$K_i H_{i+1} H_{i-1} K_{i-2} H_{i-1} K_i = c K_{i-1} H_i E_{i+1} H_i K_{i-2} K_{i-1}$$

where,

$$Y_i = \text{diagram} = b^{-1} H_{i-1} H_i H_{i-1} - b^{-1} a H_{i-1} E_i H_{i-1} - b^{-1} a c^{-1} H_{i-1} K_i H_{i-1} - H_{i-1}$$

$$Z_i = \text{diagram} = b^{-1} K_{i-1} K_i H_{i-1} - b^{-1} a K_i H_{i-1} - b^{-1} a c^{-1} H_{i-1} K_i H_{i-1} - E_{i-1} K_i$$

$$W_i = \text{diagram} = b^{-1} H_{i-1} K_i K_{i-1} - b^{-1} a H_{i-1} K_i - b^{-1} a c^{-1} H_{i-1} K_i H_{i-1} - K_i E_{i-1}$$

It turns out that the defining relations on $\bar{F}_n(q)$ are enough to show:

(Correction of Proposition 3.3): Any element of $\bar{F}_n(q)$ can be written as a linear combination of elements of the form $a\chi b$, where $a, b \in \bar{F}_{n-1}(q)$ and $\chi \in \{1, H_{n-1}, E_{n-1}, K_{n-1}, H_{n-1} K_{n-2} H_{n-1}\}$.

Proof. Observe that in addition to (1)–(29) we get the following relations in $\bar{F}_n(q)$, and proceed by induction on n exactly as described in Proposition 3.3.

$$H_{i+1} H_i K_{i-1} H_i K_{i+1} = H_{i+1} K_i H_{i+1} K_{i-1} K_i$$

$$K_{i+1} H_i K_{i-1} H_i H_{i+1} = K_i H_{i+1} K_{i-1} K_i H_{i+1}$$

$$H_{i+1} K_i H_{i+1} K_{i-1} K_i H_{i+1} = c H_{i+1} H_i K_{i-1} H_i H_{i+1}$$

$$H_{i+1} H_i K_{i-1} H_i E_{i+1} = c^{-1} H_{i+1} K_i H_{i+1} K_{i-1} H_i E_{i+1}$$

$$H_{i+1} K_i H_{i+1} H_{i-1} K_i = c K_{i-1} K_i E_{i+1} H_i K_{i-1}$$

$$H_{i+1} K_i H_{i+1} H_{i-1} K_{i-2} H_{i-1} K_i = c^2 K_{i-1} K_i E_{i+1} H_i K_{i-2} K_{i-1}$$

$$H_{i+1} K_i H_{i+1} E_{i-1} K_i = H_{i-1} K_i E_{i+1} K_i E_{i-1} K_i$$

$$K_{i+1} H_i K_{i-1} H_i E_{i+1} = c K_i K_{i-1} K_i E_{i+1}$$

$$H_{i+1} K_i H_{i+1} K_i = c H_{i+1} H_i K_{i+1}$$

$$H_{i+1} K_i H_{i+1} E_i = (ax + ac + b) H_{i+1} E_i$$

$$H_{i+1} K_i H_{i+1} K_{i-1} E_i = ac K_i E_{i+1} K_{i-1} E_i + ac H_{i+1} K_{i-1} E_i + bc Y_{i+1} E_{i-1} E_i + bc W_i E_{i+1} E_i$$

$$H_{i+1} K_i H_{i+1} K_{i-1} = ca K_i E_{i+1} H_i + ac H_{i+1} H_i + bc Z_{i+1} + cb Y_{i+1}$$

Since, $E_n a H_{n-1} K_{n-2} H_{n-1} b E_n = c^2 a K_{n-2} b E_n$, for $a, b \in \bar{F}_{n-1}(q)$, we get that lemma 3.5 is true for $\bar{F}_{n-1}(q)$. Finally, theorem 3.6 is true for $\bar{F}_{n-1}(q)$. The only change that one needs to make in the statement is to add in e) that $\text{tr}(a H_{n-1} K_{n-2} H_{n-1} b) = \frac{c^2}{z} \text{tr}(a K_{n-2} b)$.