Corrigendum and addendum

DECAY BOUNDS FOR SOLUTIONS OF SECOND ORDER PARABOLIC PROBLEMS AND THEIR DERIVATIVES

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That part of Theorem 2 which makes use of hypothesis (3.2) is incorrect. In fact, if (3.2) holds, the second half of the theorem should be replaced by the following statement:

Let $\hat{\mathbf{P}} := (\hat{x}, \hat{t})$ be a point at which Φ takes its maximum value. If $\hat{x}$ is an elliptic point of $\partial\Omega$, then we must have $|\nabla u(\hat{\mathbf{P}})| = 0$. If $\hat{x}$ is a parabolic point of $\partial\Omega$, then either $|\nabla u(\hat{\mathbf{P}})| = 0$ or (1.1) is linear.

For the proof we again use (3.9)–(3.13). At an elliptic point $\hat{x}$, it follows from the second maximum principle³ applied to solutions of (2.9) that either $|\nabla u(\hat{\mathbf{P}})| = 0$ or

$$\Phi(x, t) = e^{2\alpha t} \{ |\nabla u|^2 + \alpha u^2 + 2F(u) \} = \Phi_{\max} = \text{const.} \quad \forall x \in \partial\Omega, \quad 0 < t < \hat{t}. \quad (3.14)$$

Let $\mathbf{e}_i$, $i = 1, \dots, N-1$ be $N-1$ orthonormalized eigenvectors of the curvature matrix B defined in (3.11) corresponding to the eigenvalues k_i , $i = 1, \dots, N-1$ at $\hat{x}$. If $|\nabla u(\hat{\mathbf{P}})| \neq 0$ we can write

$$\nabla u = \sum_{i=1}^{N-1} \beta_i \mathbf{e}_i \quad \text{with } (\beta_1, \dots, \beta_{N-1}) \neq (0, \dots, 0), \quad (3.15)$$

since ∇u is tangent to $\partial\Omega$ and since $\prod_{i=1}^{N-1} k_i > 0$ at $\hat{x}$. We then obtain

$$\frac{1}{2} \frac{\partial \Phi}{\partial n} = -u_{,k} B_{ki} u_{,i} = - \sum_{i=1}^{N-1} k_i \beta_i^2 < 0 \quad \text{at } \hat{x}, \quad 0 < t < \hat{t}, \quad (3.16)$$

in contradiction to (3.13), and the conclusion is established at an elliptic point $\hat{x} \in \partial\Omega$. At a parabolic point $\hat{x} \in \partial\Omega$, Eq. (3.13) does not imply that $|\nabla u(\hat{\mathbf{P}})| = 0$, so that we have two possibilities in view of the second maximum principle³: either $|\nabla u(\hat{\mathbf{P}})| = 0$, or $|\nabla u(\hat{\mathbf{P}})| \neq 0$ and $\Phi = \text{const.}$ in Ω . In the latter we have

$$u_{,i}u_{,ik} = -[\alpha u + f(u)]u_{,k}$$

from (2.3), and

$$u_{,ik}u_{,ik} = \alpha u f(u) + \alpha^2 u^2 + 2\alpha F(u) + f^2(u)$$

from (2.6). Using the Schwarz inequality

$$u_{,i}u_{,ik}u_{,j}u_{,jk} \leq |\nabla u|^2 u_{,ik}u_{,ik},$$

we obtain $uf(u) \leq 2F(u)$, which together with (1.2) implies that $uf(u) = 2F(u)$, i.e. $f(u) = \text{const. } u$, and the conclusion is established at a parabolic point $\hat{x}$ of $\partial\Omega$.

In Theorem 4 we need the further assumption that Ω is *strictly* convex. The proof indicated in the paper is correct if we replace the assumption containing (4.24) by the following one. Assume now that the maximum of $\Phi(x, t)$ does occur at an interior or boundary point $(\bar{x}, \bar{t})$ at which ∇u vanishes, i.e.

$$\Phi(x, t) \leq \alpha u^2(\bar{x}, \bar{t})e^{2\alpha\bar{t}} = \alpha u_m^2 e^{2\alpha\bar{t}}, \quad x \in \Omega, \quad 0 < t < T. \quad (4.24)$$

The remainder of the paper is unchanged.