Inverse Proportional Relationship Between Switching-Time Length and Fractal-Like Structure for Continuous Tracking Movement

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We investigate the relationship between the switching-time length T and the fractal-like feature that characterizes the behavior of dissipative dynamical systems excited by external temporal inputs for tracking movement. Seven healthy right-handed male participants were asked to continuously track light-emitting diodes that were located on the right and left sides in front of them. These movements were performed under two conditions: when the same input pattern was repeated (the periodic-input condition) and when two different input patterns were switched stochastically (the switching-input condition). The repeated time lengths of input patterns during these conditions were 2.00, 1.00, 0.75, 0.50, 0.35, and 0.25 s. The movements of a lever held between a participant's thumb and index finger were measured by a motion-capture system and were analyzed with respect to position and velocity. The condition in which the same input was repeated revealed that two different stable trajectories existed in a cylindrical state space, while the condition in which the inputs were switched induced transitions between these two trajectories. These two different trajectories were considered as excited attractors. The transitions between the two excited attractors produced eight trajectories; they were then characterized by a fractal-like feature as a third-order sequence effect. Moreover, correlation dimensions, which are typically used to evaluate fractal-like features, calculated from the set on the Poincaré section increased as the switching-time length T decreased. These results suggest that an inverse

proportional relationship exists between the switching-time length T and the fractal-like feature of human movement.

Keywords: Excited attractor; external temporal input; dynamical systems.

1. Introduction

The famous early works in which dynamical systems theory was applied to human movement were the experimental verification by Kelso [1984] and the mathematical model of Haken *et al.* [1985]. They examined the dynamic state of the relative phase between two limbs. Their main point of view is the phase transition from out-of-phase to in-phase states by intrinsic dynamics [Schöner & Kelso, 1988], which is an internal human parameter. These phases are understood as attractors that are important for human motor control. Several studies by Kelso and his colleagues are considered to be based on an autonomous system or an autonomous dynamical system [Yamamoto & Gohara, 2000] because of the closed system that is independent of other systems. These studies addressed tasks in which participants moved limbs periodically to metronome sounds that were changed gently, i.e. a control parameter. In contrast, a dynamical system, which is open to other systems, is considered a nonautonomous system or a nonautonomous dynamical system [Gohara & Okuyama, 1999a]. In this system, movement patterns are excited by external inputs that are external parameters. An attractor, which is excited with an external input, underlies a movement pattern; i.e. this is considered an excited attractor. Recent studies based on this dynamical systems theory, for example not only the physical phenomenon [Arecchi *et al.*, 1986; Constantin *et al.*, 1991; Maas *et al.*, 1997; Matias *et al.*, 1997; Mestl *et al.*, 1997; Schmiegel & Eckhardt, 1997; Tanii *et al.*, 1999; Tanii *et al.*, 1991] but also human movement [Hirakawa *et al.*, 2016; Suzuki & Yamamoto, 2015; Yamamoto & Gohara, 2000], have revealed that the spatiotemporal structure in various natural phenomena is fractal-like and/or self-similar. These studies on human movement entailed tasks in which participants switched several different movement patterns continuously with the external inputs being abruptly switched.

This implies that, “for autonomous dynamical systems, the basic law of evolution is static in the sense that the environment does not change with

time. In many applications, however, such a static approach is too restrictive and a temporally fluctuating environment must be taken into account” [Kloeden & Pötzsche, 2013]. Consequently, “in reasonable models adapted to and well-studied for problems in temporally fluctuating environments, the evolutionary equations have to depend explicitly on time through time-dependent parameters or external inputs” [Kloeden & Pötzsche, 2013]. Therefore, nonautonomous dynamical systems have been applied to biological systems, even life sciences widely, in recent years owing to the importance of utilization of dissipative dynamical systems theory. This perspective in their studies, based on nonautonomous dynamical systems, differs from that of the series of studies by Kelso mentioned above. Moreover, Nishikawa and Gohara [2008, 2008] established that we are able to quantify a state in a continuous dynamical system with a fractal dimension, although the fractals are generated by an iterated function system, e.g. the Sierpinski gasket, if we design an experiment by a theoretical framework for a continuous dynamical system stochastically excited by an external temporal input. In this study, we apply the hypothesis with a dissipative dynamical system by Gohara and Okuyama [1999a, 1999b], Nishikawa and Gohara [2008, 2008] to our experiment. This theoretical framework is detailed in the next section.

Figure 1 shows schematics highlighting the difference between autonomous and nonautonomous systems [Hirakawa *et al.*, 2016]. For instance, Figs. 1(a) and 1(b) show autonomous systems. Figure 1(a) implies a state in which the same tracking movement pattern toward the left side (L) is repeatedly excited with temporal input in this

Fig. 1. Schematics of the interplay between systems [Hirakawa *et al.*, 2016]. See the body of the text for further details.

study. Similarly, Fig. 1(b) shows a state toward the right side (R). These figures depict systems proposed by Kelso and his colleagues and take into account the assumption in their studies to estimate the motor control of human movement as mentioned above. That is, an attractor underlies the system shown in Figs. 1(a) and 1(b), and generates the same movement pattern repeatedly and continuously. Therefore, Kelso *et al.* treated the small change in input as a perturbation, and examined the stability of the system “in terms of relaxation time and critical slowing down” [Yamamoto & Gohara, 2000]. On the other side, Fig. 1(c) shows the dynamics when a human switches two attractors abruptly. This mixed state shows that we are required to switch rapidly and stochastically two attractors excited with two external inputs. The dynamics of the switching state in this system implies that not only are L and R excited with the external input but also the dynamics is affected by the temporal order and the temporal interval of the two external inputs. In this system, the number of the attractors corresponds to the number of the external inputs. This phenomenon is called the switching dynamics, wherein the human has to switch several attractors [Yamamoto & Gohara, 2000]. We need to adopt the latter approach and the corresponding theory to examine this dynamics in order to study human motor control, and the difference between this study and the studies by Kelso and his colleagues is that the experiment in our study is based on the nonautonomous dynamical systems theory.

The first study in which the dynamical systems theory [Gohara & Okuyama, 1999a] involving switching stochastically with several external inputs was applied to human movement was carried out by Yamamoto and Gohara [2000]. They focused on the periodic movement patterns of forehand and backhand strokes of tennis players as attractors were excited with external inputs. They confirmed that a fractal-like structure exists in the transition state between the two movement patterns when players switch patterns stochastically and rapidly. They also provided evidence that the fractal-like geometric structure is a Cantor set. Suzuki and Yamamoto [2015] examined the relationship between transition states, which occurred by excited attractors, and skill levels for table tennis by the switching-time length T as a parameter of the external input. The theoretical background

is that the transition states are influenced by T , and the authors quantified these states by using a fractal dimension, i.e. the value of the correlation dimension. As a result, the values of the correlation dimension, which are an indicator of the complexity of behavior in movement patterns, for unskilled players when T is longer are the same as values for skilled players when T is shorter. This study suggested that T and the state of complexity, i.e. the correlation dimension, are closely related to each other. Furthermore, a regular relationship exists between T and the fractal dimension [Hirakawa *et al.*, 2016].

Previous studies [Nishikawa & Gohara, 2008; Sato & Gohara, 2001; Wada & Gohara, 2001a, 2001b] in applied physics have examined this kind of regular relationship in various physical phenomena. Until now, few previous studies have been conducted in which human movement has been analyzed [Hirakawa *et al.*, 2016]; hence, accumulating examinations of various human movements within the context of nonautonomous dynamical systems theory is essential. For example, Hirakawa *et al.* [2016] analyzed continuous trajectories of a fingertip in pointing movements toward left and right directions. However, such a task might be considered a discrete task, because it involves a reaching movement toward two preset locations. Therefore, in this study we investigate continuously moving left or right target locations as the external inputs in the task. This entails developing a task in which participants must switch a fingertip movement toward the left and right directions while they track the target continuously. The purpose of this study is to estimate the transition state between attractors that are excited by T , which is the external parameter, and to examine the regular relationship between T and the transition state when participants track a moving target continuously as the external input. Furthermore, based on these results, we will be able to demonstrate that the dissipative dynamical system, which is applied for human movement, is stochastically excited by plural temporal inputs characterized by the trajectory set $\Gamma(C)$ starting from the initial set C . We also show the relationship between Eq. (2) as the discrete dynamical system and Eq. (3) as the continuous dynamical system from the perspective of the nonautonomous dynamical system by analyzing the set on the Poincaré section Σ in the hyper-cylindrical state space $\mathcal{M}$.

2. Dynamical System Excited by Switching Inputs

From the discussion in the preceding section, it seems not appropriate to study human motor control with only the perspective of autonomous dynamical system that is incapable of addressing the switching between several movement patterns following an abrupt change in the temporal input [Yamamoto & Gohara, 2000]. Hence, we perform the experiment in this study with the dissipative nonautonomous dynamical system theory [Gohara & Okuyama, 1999b; Nishikawa & Gohara, 2008]. We briefly survey this theory in this section.

An autonomous system is expressed as $\dot{x} = f(x)$. A nonautonomous system depending on a continuous external input $I(t)$ is defined by the following ordinary differential equation:

$$\dot{x} = f[x, I(t)], \quad x, I \in R^N, \quad (1)$$

where x and f are the state and the vector fields, respectively. In the nonautonomous system, we consider the manner in which the vector field changes with the external input $I(t)$ over time. The input is the set $\{I\}_{l=1}^L$ under a finite time length, and is stochastically and continuously fed into the system. The attractor is excited by $I(t)$ in this system. We assume several attractors in a nonautonomous system, and we examine the transition states between attractors. The trajectories in the state space are unable to converge sufficiently onto those attractors if the external input $I(t)$ is switched rapidly and stochastically. If a sequence of the input is repeated randomly, the set C on the Poincaré section Σ satisfies the following equation:

$$C = \bigcup_{l=1}^L g_l(C) \quad (2)$$

and the set C consists of $g_l(C)$. Moreover, this set C influences the trajectory in the hyper-cylindrical state space $\mathcal{M}$. The set of trajectories $\Gamma(C)$ starting from the initial set C in state space $\mathcal{M}$ is the bundle of trajectories $\gamma_l(C)$ that was influenced by each external input I_l :

$$\Gamma(C) = \bigcup_{l=1}^L \gamma_l(C). \quad (3)$$

We are able to quantify the set C in this state space as the value of the correlation dimension calculated by the typical method to estimate a feature of a

fractal state [Strogatz, 2001]. Furthermore, we are able to estimate the relationship between the set C or the set of trajectories $\Gamma(C)$ and the switching-time length T [Gohara & Okuyama, 1999a]. T is also the duration of the external input. That is, although the set on the Poincaré section Σ is of discrete points, the set on the Poincaré section Σ is originally one trajectory, and the connection between the discrete dynamical system [Eq. (2)] and the continuous dynamical system [Eq. (3)] is on the Poincaré section Σ . Therefore, it is valid to apply the nonautonomous dynamical system to an experiment designed to investigate human movement patterns switched continuously and abruptly. We omit the detailed theoretical explanation of this dynamical system in which the input pattern is stochastically changed such as for a nonautonomous system, and refer the reader to [Gohara & Okuyama, 1999a].

3. Methods

3.1. Participants

The seven participants were healthy right-handed men (according to the laterality questionnaire on the Edinburgh Handedness Inventory [Oldfield, 1971]) and had no vision problems. Their ages ranged between 21 and 39 years (mean $M = 32.6$ and standard deviation $SD = 4.2$). They had no history of neurological or musculoskeletal disorders. All participants provided written informed consent after the experimenter explained the experimental procedure. The procedure used in this study was approved by the Human Ethics Committee of Nanzan University.

3.2. Experimental setting

Figure 2 shows the experimental setup. Eight light-emitting diodes (LEDs), each with a diameter $\phi = 5$ mm, were located in a straight line at the left and right positions and at the back of the slider, respectively. The distance between two LEDs was 5 mm. The distance between the sets of eight LEDs at the left and right sides was 170 mm. A lever, used by a participant to track the LEDs lit, was set on the slider and participants could move the lever freely toward left and right directions. The slider was set on a table. Each participant sat 300 mm in front of the slider and could adjust the height of the chair. The movement of a reflective marker

Fig. 2. Experimental setup.

(15 mm in diameter) pasted onto the lever was sampled at 240 Hz by an optical motion-tracking system (Qualisys Oqus 300, Gothenburg, Sweden). The lighting of the LEDs was controlled with LabVIEW programming (Version 8.6, National Instruments, Austin TX; NI cDAQ-9174, NI9403), and was synchronized with Oqus.

3.3. Task and procedure

The experiment was conducted in a quiet room. Participants first held the lever with thumb and index finger. They were then instructed to track the LEDs lit. The order of LED lighting was as follows. When the LEDs located to the left of the participant (L) were lit, the lighting of LEDs moved to the right one by one after the LED on the leftmost was first lit. The lighting of LEDs moved to the left one by one after the rightmost LED was lit. When the LEDs located to the right of the participant (R) were lit, the order of LEDs lit was the reverse of that of L. The durations of LEDs lit on L or R, i.e. the switching-time lengths T s were 2.00, 1.00, 0.75, 0.50, 0.35, and 0.25 s. The experiment was divided into two or three days, and measurements were performed under up to three switching-time lengths per day.

Participants executed the periodic-input condition in which only L (LLL...) or only R (RRR...) was repeated as well as the switching-input

condition in which L and R were switched stochastically and randomly for each T . For each T , L or R was repeated for 62 trials during the periodic-input condition and for 496 trials during the switching-input condition. The number of trials under the switching-input condition was divided into eight blocks to equalize the number per block under the periodic-input condition, i.e. 62 trials per block. Participants performed ten blocks, i.e. two blocks under the periodic-input condition and eight blocks under the switching-input condition, for each T . For the switching-input condition, the input sequence consisted of only eight patterns, i.e. LLL, RLL, LRL, RRL, LLR, RLR, LRR, and RRR, although the sequence was made up stochastically and randomly. The first two trials for each block after the second block were the same as the last two trials for the previous block. The first two trials after the second block were removed for the data analysis. The data analyzed consisted of 60 trials for the periodic-input condition and 480 trials ($= 8 \text{ blocks} \times (62 - 2) \text{ trials} = 8 \text{ serial patterns} \times 60 \text{ data}$) for the switching-input condition. We set up the number of participants and data set by applying the previous studies [Hirakawa *et al.*, 2016; Suzuki & Yamamoto, 2015] with dynamical systems theory for human movement. The number and duration of a break between blocks was determined freely by each participant according to his need.

3.4. Data analysis

The displacement of the position of the lever was calculated using Qualisys Track Manager (Gothenburg, Sweden). The displacement data were smoothed with a second-order Butterworth digital low-pass filter using a 5-Hz cutoff frequency to obtain the positional data x . The velocity $\dot{x}$ was calculated by differentiating x . We embedded x and $\dot{x}$ into the hyper-cylindrical state space $\mathcal{M}$ to examine the trajectories in a continuous dynamical system. The Poincaré section $\Sigma(\theta = 0)$ in state space $\mathcal{M}$ is

the moment when the leftmost LED was lit (when L started) and the moment when the rightmost LED was lit (when R started), as denoted by the blue and red triangles in Figs. 3(a), 3(b), and 5(a). We unfolded state space $\mathcal{M}$ to θ -($x, \dot{x}$) space to examine the relationship between the trajectory sets $\Gamma(C)$ and $g(C)$. The correlation dimension D_2 for set $g(C)$ on the Poincaré section $\Sigma(\theta = 2\pi)$ under each T was calculated with the G-P algorithm [Grassberger & Procaccia, 1983] by using the correlation integral, which is a typical method to estimate the

Fig. 3. Dynamics for the periodic-input condition (using data from participant #7 for a switching-time length T of 0.25 s).

fractal dimension. The validity and reliability of a fitting curve were evaluated by using Akaike's information criterion (AIC) to estimate the relationship between T and D_2 .

4. Results

4.1. Periodic-input condition

Figure 3(a) shows part of the time series data of the output pattern (5 s) for participant #7 to track L continuously under the periodic-input condition when the switching-time length T (duration of L lit) is 0.25 s as a typical example. The reason why we selected data for participant #7 is that this result in Fig. 9, which is described later, was the most approximate to the fitting curve; i.e. this value is the minimum by the least-squares method. Similarly, Fig. 3(b) shows part of the result for participant #7 to track R. The vertical scale on the left of these figures is $\dot{x}$, and the scales on the right side are $\dot{x}$. The blue and red triangles indicate when L and R started to light up, respectively. The blue and red solid lines are the positional displacement x of the lever when a target LED was lit on L or R, respectively. The black solid lines are the velocity $\dot{x}$.

Figure 3(c) shows the result when output patterns were embedded into the hyper-cylindrical state space $\mathcal{M}$ when participant #7 tracked L. The Poincaré section $\Sigma(\theta = 0, 2\pi)$ is the moment that L started. The trajectory around the cylinder represents one cycle and was composed of x and $\dot{x}$ during the time the LEDs on L were lighting up. Figure 3(d) shows the result when participant #7 tracked R. The trajectories around the cylinder represent 60 cycles. Additionally, the trajectories before and after the Poincaré section $\Sigma(\theta = 0, 2\pi)$ in the state space were omitted to more easily confirm the set C on the section.

Figure 3(e) shows the result when we unfolded Fig. 3(c) in θ -($x, \dot{x}$) space to more easily examine the set of trajectories in state space $\mathcal{M}$. Figure 3(f) shows the result corresponding to Fig. 3(d). These trajectories were the output patterns when the participant tracked L or R, and the trajectories in the state space were clearly different according to the type of external input, i.e. L or R.

Figure 4 shows the results when we, respectively, embedded the trajectories set in state space $\mathcal{M}$ to confirm the output patterns during five T s (except for $T = 0.25$ s) under the periodic-input condition. Figures 4(a)–4(e) show the results for

Fig. 4. Cylindrical state space $\mathcal{M}$ during the periodic-input condition (using data from participant #7). The switching-time length T = (a) 2.00, (b) 1.00, (c) 0.75, (d) 0.50, and (e) 0.35 s.

$T = 2.00, 1.00, 0.75, 0.50$, and 0.35 s, respectively. The results for $T = 0.25$ s are divided into Figs. 3(c) and 3(d). The trajectory around the cylinder represents one cycle, and the trajectories start from the moment when L or R started, i.e. the Poincaré section $\Sigma(\theta = 0)$, and terminate at the moment when L or R ended, i.e. $\theta = 2\pi$, similar to state space $\mathcal{M}$ in Figs. 3(c) and 3(d). The blue and red trajectories are the output patterns $(x, \dot{x})$ when a participant tracked L and R, respectively. In addition, the trajectories before and after the Poincaré section $\Sigma(\theta = 0, 2\pi)$ were omitted to confirm the set C on the section more easily. These results show that the trajectories of output patterns, which were excited by the external inputs L and R, were different in the continuous dynamical system.

Furthermore, a one-way multivariate analysis of variance (MANOVA) (Wilks' λ) was performed to test for the equality of the means of the multiple variables in the cluster of the set on the Poincaré section for each T and for all participants to examine the output patterns statistically in the discrete dynamical system, because the set of trajectories in the continuous dynamical system were different qualitatively. Table 1 lists the results. These results revealed significant differences for all participants and all T s ($p < 0.001$), revealing that the output patterns excited by the external inputs L and R were clearly different in state space $\mathcal{M}$.

These qualitative and quantitative results under the periodic-input condition suggest that the set of trajectories — the continuous dynamical system — in the hyper-cylindrical state space $\mathcal{M}$ and the set C — the discrete dynamical system — on the Poincaré section $\Sigma(\theta = 2\pi)$ were basic movement patterns excited by the external input. These movement patterns are limit-cycle attractors, and the

attractors are excited by an external input, i.e. the movement patterns have obviously different excited attractors, A_L and A_R . However, we are unable to confirm whether the output patterns during a continuous tracking movement in the periodic-input condition are attracted to either attractor when we rapidly switch the tracking movements by the external inputs L and R. We examine this phenomenon in the switching-input condition.

4.2. Switching-input condition

Figure 5(a) shows part of the time series data of the output pattern (5 s) for participant #7 to track the LEDs continuously under the switching-input condition when the switching-time length T (duration of L or R lit) is 0.25 s as a typical example. The reason why we selected data for participant #7 is similar to that stated in the previous subsection. The vertical scale on the left side in Fig. 5(a) is x , and the scales on the right side are $\dot{x}$. The blue and red triangles indicate when L and R started to light up, respectively. The blue and red solid lines are the positional displacements of the lever x when an LED on L or R was lit, respectively. The black solid line is $\dot{x}$.

Figure 5(b) shows the result when the output patterns were embedded into the hyper-cylindrical state space $\mathcal{M}$ when participant #7 tracked an LED on L or R. The Poincaré section $\Sigma(\theta = 0, 2\pi)$ is the moment when L or R started. The trajectory around the cylinder was composed of x and $\dot{x}$. The set C on the Poincaré section Σ comprise all data for the participant. Only 15 trajectories $\gamma(C)$ for each serial (for example, LLL) were embedded to more easily confirm the trajectories in this state space. That is, all 120 trajectories for

Table 1. Results of a MANOVA (Wilks' λ) of differences between the mean left (L) and right side (R) input patterns on the Poincaré section $\Sigma(\theta = 2\pi)$ under the periodic-input condition.

Participant		T (s)					
		2.00	1.00	0.75	0.50	0.35	0.25
#1	L/R	0.0004	0.0016	0.0006	0.0015	0.0021	0.0009
#2	L/R	0.0003	0.0005	0.0012	0.0011	0.0010	0.0006
#3	L/R	0.0001	0.0002	0.0005	0.0002	0.0005	0.0003
#4	L/R	0.0003	0.0006	0.0005	0.0005	0.0008	0.0009
#5	L/R	0.0002	0.0004	0.0004	0.0021	0.0049	0.0035
#6	L/R	0.0006	0.0006	0.0010	0.0013	0.0017	0.0013
#7	L/R	0.0003	0.0005	0.0003	0.0007	0.0011	0.0011

$p < 0.001$

Fig. 5. Dynamics for the switching-input condition (using data from participant #7 for a switching-time length T of 0.25 s).

eight serials (LLL, RLL, LRL, RRL, LLR, RLR, LRR, and RRR) were embedded in this state space. Additionally, the trajectories before and after the Poincaré section $\Sigma(\theta = 0, 2\pi)$ in this state space were omitted to more easily confirm the set C on the section. Figure 5(c) shows the output patterns under the switching-input condition when the current external input was L. Figure 5(d) shows the result when the current external input was R. In these figures, the blue trajectories are the result when the preceding input was L, and red trajectories indicate the result when the preceding input was R. That is, LL, RL, LR, and RR, which were composed of the preceding and current inputs, are represented as blue, red, blue, and red colors, respectively.

Figure 5(e) shows the result when we unfolded Fig. 5(c) in θ -($x, \dot{x}$) space to more easily examine the set of trajectories in state space $\mathcal{M}$. Figure 5(f) shows the result corresponding to Fig. 5(d). Further, the trajectories before and after the Poincaré section $\Sigma(\theta = 0, 2\pi)$ in the state space were omitted to more easily confirm the set C on the section. This result indicates that, under the switching-input condition, the sets of trajectories $\gamma(C_L)$ and $\gamma(C_R)$ were output patterns when the

participant was tracking L and R, respectively, and that the trajectories were starting from the initial sets C_L and C_R at $\theta = 0$ and were terminated to sets $g(C_L)$ and $g(C_R)$ at $\theta = 2\pi$. For example, we are able to confirm that two clusters of the set of trajectories started from two different clusters of the initial set C on the section $\Sigma(\theta = 0)$ in both Figs. 5(e) and 5(f). These clusters show that the current output pattern was influenced by not only the current input pattern but also the preceding input pattern. That is, trajectories $\gamma(C_L)$ start from two clusters when the preceding input is L or R and the current input is L. Therefore, these clusters are composed of serials of external inputs LL and RL. Similarly, trajectories $\gamma(C_R)$ start from two clusters when the preceding input is L or R and the current input is R. Therefore, these clusters are composed of serials of external inputs LR and RR. Based on these results, the trajectories of the continuous tracking movement in state space $\mathcal{M}$ and space θ -($x, \dot{x}$) were composed of four clusters with the external input.

Figure 6 shows the result when we, respectively, embedded the set of trajectories in state space $\mathcal{M}$ to confirm the output patterns during five T s (except for $T = 0.25$ s) under the switching-input condition. Figures 6(a)–6(e) show the results for $T = 2.00$,

Fig. 6. Cylindrical state space $\mathcal{M}$ during the switching-input condition (using data from participant #7). The switching-time length $T =$ (a) 2.00, (b) 1.00, (c) 0.75, (d) 0.50, and (e) 0.35 s.

1.00, 0.75, 0.50, and 0.35 s, respectively. The results for $T = 0.25$ s are shown in Fig. 5(b). The trajectory around the cylinder represents one cycle, and the trajectories start from the moment when L or R started, i.e. the Poincaré section $\Sigma(\theta = 0)$, and terminate at the moment when L or R ended, i.e. $\theta = 2\pi$, similar to Figs. 5(b)–5(d). The blue and red trajectories are the output patterns $(x, \dot{x})$, and the colors are similar to that in Figs. 5(b)–5(d), respectively. Additionally, the trajectories before and after the Poincaré section $\Sigma(\theta = 0, 2\pi)$ were omitted to more easily confirm the set C on the section. Compared with the set of trajectories under the periodic-input condition in Figs. 3 and 4, the set of trajectories under the switching-input condition in Figs. 5 and 6 was excited by the external inputs L and R, i.e. $\gamma(C_L)$ and $\gamma(C_R)$, which were clearly different in the continuous dynamical system.

Furthermore, a MANOVA (Wilks' λ) was performed to test for the equality of the means of the multiple variables in the set on the Poincaré section (LL/RL and LR/RR, which are the serials composed of the preceding and the current external inputs, respectively) for each T and all participants to examine the output patterns statistically in a discrete dynamical system, because the sets of trajectories in the continuous dynamical system were different qualitatively. Table 2 lists the results. Figure 7 shows a scatter diagram on the section Σ ;

participants #1 to #7 are arranged by row and the six T s are arranged by column. The blue and red data points in Fig. 7 indicate that the preceding inputs are L and R, respectively. The set on the left side of the section is $g(C_L)$, and the other is $g(C_R)$. The results revealed significant differences in 78 of the 84 cases ($\approx 92.6\%$) from the 42 sections. These results suggest that tracking movements became four clusters by second-order sequence effects composed of the preceding and current inputs, i.e. LL, RL, LR, or RR.

Figure 8 shows the result when the set of trajectories in state space $\mathcal{M}$ in Figs. 5(b) and 6 was unfolded into space $\theta(x, \dot{x})$ to examine the clusters in more detail that were outputted by serials of external inputs. Figures 8(a)–8(f) show the results for $T = 2.00, 1.00, 0.75, 0.50, 0.35$, and 0.25 s, respectively. The set on the Poincaré section comprises all data (480 data points for each section), and the trajectories $\Gamma(C)$ are the averaged trajectories for every eight serials (LLL, RLL, LRL, RRL, LLR, RLR, LRR, and RRR). For example, when the input before the preceding input is L, the preceding input is L, and the current input is L, then the serial is LLL. Additionally, the trajectories after each Poincaré section $\Sigma(\theta = 0, \pi/2, \pi, 3\pi/2)$ were omitted to more easily confirm the set C on the section. The blue trajectories are for when the preceding input is L and the current inputs are

Table 2. Results of a MANOVA (Wilks' λ) of differences between the mean left side–left side (LL) and right side–left side (RL) and between the mean right side–right side (RR) and left side–right side (LR) input patterns on the Poincaré section $\Sigma(\theta = 2\pi)$ under the switching-input condition.

Participant		T (s)					
		2.00	1.00	0.75	0.50	0.35	0.25
#1	LL/RL	0.6248***	0.9591**	0.9157***	0.6714***	0.9275***	0.9488**
	LR/RR	0.6057***	0.9614**	0.6100***	0.9161***	0.9881	0.8388***
#2	LL/RL	0.9770	0.9669*	0.9266***	0.9435**	0.7281***	0.8106***
	LR/RR	0.9684*	0.9860	0.9901	0.8607***	0.7950***	0.8182***
#3	LL/RL	0.9996	0.7162***	0.7421***	0.4999***	0.6611***	0.6246***
	LR/RR	0.9032***	0.9693*	0.9363**	0.9088***	0.8418***	0.8593***
#4	LL/RL	0.9854	0.9662*	0.8779***	0.8989***	0.9462**	0.6288***
	LR/RR	0.7979***	0.8177***	0.8869***	0.8250***	0.9537**	0.7886***
#5	LL/RL	0.7868***	0.5148***	0.7914***	0.5120***	0.5234***	0.6381***
	LR/RR	0.6701***	0.6936***	0.9478**	0.6862***	0.5997***	0.8056***
#6	LL/RL	0.8568***	0.7562***	0.9460**	0.7467***	0.6439***	0.6335***
	LR/RR	0.9701*	0.9578**	0.9748*	0.9077***	0.7170***	0.8502***
#7	LL/RL	0.9303***	0.9593**	0.9063***	0.8350***	0.7397***	0.8039***
	LR/RR	0.9325***	0.8097***	0.7960***	0.8468***	0.9438**	0.9038***

***: $p < 0.001$, **: $p < 0.01$, * : $p < 0.05$.

Fig. 7. Results of $g(C)$ on the Poincaré section $\Sigma(\theta = 2\pi)$. Participants #1 to #7 are arranged by row and the six T s are arranged by column.

Fig. 8. The unfolded state space during the switching-input condition (using data from participant #7). The switching-time length T = (a) 2.00, (b) 1.00, (c) 0.75, (d) 0.50, (e) 0.35, and (f) 0.25 s.

L (LL) and R (LR). Red trajectories are for when the preceding input is R and the current inputs are L (RL) and R (RR). From these results, we are able to confirm clearly the eight trajectories as T shortened [for example, Figs. 8(e) and 8(f)], while the trajectories during longer T s [for example, Figs. 8(a) and 8(b)] seem to converge at two points. This result suggests that we are able to confirm that the current output pattern is influenced by

the preceding external input under the switching-input condition and that we are also able to confirm a second-order sequence effect by trajectory $\Gamma(C)$ in the continuous dynamical system. Furthermore, these results indicate that not only are the four clusters composed of trajectories from each serial but also that each of the four clusters (LL, RL, LR, and RR) is composed of two subclusters, respectively. That is, the set of trajectories $\Gamma(C)$

is composed of eight clusters while the initial set C has only four clusters on the Poincaré section. Therefore, the eight trajectories in state space $\mathcal{M}$ show that the human output patterns exhibit a fractal-like feature or a self-similarity structure by third-order sequence effects when humans switched rapidly between motions with two external inputs.

4.3. Switching-time length T and correlation dimension D_2

Figures 9(a)–9(f) show typical examples (from participant #7) for set $g(C)$ on the Poincaré section $\Sigma(\theta = 2\pi)$ under the switching-input condition for all six T s. Figures 9(a)–9(f) correspond to the results for $T = 0.25, 0.35, 0.50, 0.75, 1.00$, and 2.00 s, respectively. From these results, we are able to confirm that set $g(C)$ spreads over the section Σ during a shorter T [for example, Fig. 9(a)] and converges to a local area during a longer T [for example, Fig. 9(f)]. The blue and red data points in the upper part of Fig. 9(g) are the results of the

correlation dimension D_2 for each participant under each T . For all participants and T s, the value of the dimension exceeds ~ 1 under the shorter switching-time length T [for example, Figs. 8(f) and 9(a)] and decreases to near zero under the longer T [for example, Figs. 8(a) and 9(f)]. This suggests that set C on the Poincaré section Σ is spreading gradually from points to lines and from lines to a plane as T is shortened. That is, the value of the correlation dimension becomes higher because of spreading around two excited attractors when switching these two attractors rapidly as $T \rightarrow 0$, while the value of the correlation dimension was lower because of the convergence to these two attractors as $T \rightarrow \infty$.

The black open circles in the upper part of Fig. 9(g) provide the means of the correlation dimension for all participants and each T . Repeated measures of ANOVA revealed a significant effect for T [$F(5, 30) = 34.59, p < 0.01$]. Moreover, multiple comparisons revealed that the value of the correlation dimension for $T = 2.00$ s was lower than the value at other T s, the value for $T = 1.00$ s was lower

Fig. 9. Means of the correlation dimensions and the raw data under all switching-time lengths and the fitting curve, results of ANOVA, and AIC. The top panels are examples of Poincaré sections for switching-time length T s = (a) 0.25, (b) 0.35, (c) 0.50, (d) 0.75, (e) 1.00, and (f) 2.00 s. (g) The result of the T - D_2 graph and the results of ANOVA and AIC. The black solid line is a fitting curve of α/T .

than the values for 0.50 s, 0.35 s, and 0.25 s, and the value for $T = 0.75$ s was lower than the values for 0.35 s and 0.25 s [where the asterisks in the middle part of Fig. 9(g) show that the relationships between T s were significant]. This result suggests that the values of the correlation dimension increase gradually as T reduces from 2.00 s to 0.35 s, while the values for $T = 0.35$ s and 0.25 s are not different; this numerically expresses the state that humans become less able to track external inputs gradually as T becomes even shorter.

Theoretically, the relationship between the switching-time length T and the correlation dimension D_2 suggested by Gohara and Okuyama [1999b], Nishikawa and Gohara [2008] is

$$D = \frac{-\ln N}{\lambda T}, \quad (4)$$

where N and T are the number of inputs ($N = 2$) and the switching-time length respectively in this study. We are not able to estimate the value of λ because λ is the internal parameter in the system. However, assuming that the participants in this study performed similar tracking movements and λ is constant, the correlation dimension D_2 is inversely proportional to the switching time length T ($D \propto 1/T$). Accordingly, we fitted a curve with an inverse proportional function ($y = a/T + b$) to the values of the correlation dimension for all participants and all T s, and we calculated AIC to this function. First, we calculated the coefficient a , constant term b of the fitting curve, and AIC using the value of the correlation dimension for all participants under six T s (42 data = 7 participants $\times$ 6 T s). Second, we calculated a , b , and AIC again after excluding the values of the correlation dimension under $T = 0.25$ s. Third, we calculated a , b , and AIC again after excluding the values under two T s (0.25 s and 0.35 s). The line graph in the lower part of Fig. 9(g) provides the results of AIC that we obtained with these procedures. The value of AIC was the smallest when we calculated using the value of the correlation dimension for five T s, i.e. from 0.35 s to 2.00 s [the blue filled circle on the line graph in the lower part of Fig. 9(g)]. The fitting curve was $D_2 = 0.178/T + 0.738$ when the value of the residual sum of squares was the smallest. From the above theoretical analysis, we find that in the case of tracking movements performed by a human that require abrupt switching of several movement patterns, the correlation dimension D_2 is inversely

proportional to only time T , as reported in the theoretical analysis by Gohara and Okuyama [1999b], Nishikawa and Gohara [2008], if we assume that λ is constant although we are not able to estimate this value of λ in this study. These results suggest that the value of the correlation dimension increases gradually as T decreases and that a regular relationship such as an inverse proportion $D_2 \propto 1/T$ exists between the switching-time length T and the correlation dimension D_2 .

5. Discussion

Recent studies have confirmed that fractal-like and self-similar structures exist not only in various physical phenomena [Arecchi *et al.*, 1986; Constantin *et al.*, 1991; Mestl *et al.*, 1997; Maas *et al.*, 1997; Matias *et al.*, 1997; Schmiegel & Eckhardt, 1997; Tanii *et al.*, 1999; Tanii *et al.*, 1991] but also in human motor control in cases when we switch several movement patterns rapidly with a particular switching-time length T [Yamamoto & Gohara, 2000] or several T s [Suzuki & Yamamoto, 2015]. Moreover, we confirmed the inverse proportional relationship between T and the fractal-like feature D_2 in human pointing movement [Hirakawa *et al.*, 2016]. Our purpose was to develop these studies on continuous tracking movements and to examine the regular relationship between T and D_2 , which is estimated by the transition state and which occurs when a human switches several attractors stochastically, based on a theoretical model [Gohara & Okuyama, 1999b; Nishikawa & Gohara, 2008].

It is assumed that an attractor underlies a movement pattern in a nonautonomous dynamical system with an external input [Gohara & Okuyama, 1999a]. In this study, we confirmed that a movement in which a human tracks continuously an LED on L or R is the limit-cycle attractor, i.e. the excited attractors (A_L and A_R), because we qualitatively verified the trajectories in the hypercylindrical state space $\mathcal{M}$ as a continuous dynamical system (Figs. 3 and 4) and statistically and quantitatively the set C on the Poincaré section Σ as a discrete dynamical system (Table 1) under the periodic-input condition.

The results for trajectory $\Gamma(C)$ in the hypercylindrical state space $\mathcal{M}$ [Figs. 5(b)–5(d) and 6], for the unfolded state space $\theta-(x, \dot{x})$ [Figs. 5(e), 5(f) and 8], and for a MANOVA for set $g(C)$ on the Poincaré section $\Sigma(\theta = 2\pi)$ (Table 2 and Fig. 7) suggest that the output of human movement is

influenced by second-order sequence effects that are composed of the preceding input and the current input and that these are four clusters, i.e. LL, RL, LR, and RR. Moreover, the results of the averaged trajectories in the unfolded state space θ -($x, \dot{x}$) (Fig. 8) suggest that each of the four clusters consists of two subclusters, respectively, and that these are the eight clusters. These results imply that the output patterns of human movement are influenced by third-order sequence effects with the external input. Furthermore, the set $g(C)$ on the Poincaré section $\Sigma(\theta = 2\pi)$ under the switching-input condition was a noninteger value [in the upper part of Fig. 9(g)]. It is well known that a geometric structure is fractal-like if the value of the correlation dimension is a noninteger [Tufillaro *et al.*, 1992]. This signifies that a fractal-like or self-similar structure underlies the output of human continuous movement when humans switch two movement patterns stochastically and rapidly. In other words, the distribution of the outcome by human movement may not be stochastic (such as a Gaussian distribution) or random (such as a uniform distribution) while there are third-order sequence effects.

These results seem to imply the following. Studies on the traditional dynamical systems theory, e.g. that conducted by Kelso and his colleague [Kelso, 1995], enable us to investigate human motor control. In their theoretical framework, it was often difficult to apply their theory to a study on various complex human movements that involve abrupt switching of some attractors, i.e. movement patterns. In real-world phenomena, however, the external input provided from the external environment to humans should be abruptly changed and switched. Hence, in our study, human movement was examined using the theoretical framework of the dissipative and nonautonomous dynamical system proposed by Gohara and Okuyama [1999a, 1999b], Nishikawa and Gohara [2008, 2008]. Moreover, in a series of previous researches by Kelso and his colleague [Kelso, 1995], which were on the state of switching between two attractors, with the continuous dynamical system, the authors had examined the attractor strength, relaxation time, and so on. Therefore, we expanded their autonomous and continuous dynamical systems theory to suit this study, and we applied the theoretical framework of a nonautonomous and continuous dynamical system excited stochastically with an external

temporal input to human movement to quantify the state of switching rapidly and abruptly between two attractors. Validating the regular relationship between the switching-time length and the correlation dimension with the fractals generated by switching the vector fields in the hyper-cylindrical state space $\mathcal{M}$ is significant.

In addition, as the results of the inverse proportional relationship $D \propto 1/T$ in this study are almost in agreement with the theoretical analysis reported in [Gohara & Okuyama, 1999b; Nishikawa & Gohara, 2008], it is considered that the correlation dimension D_2 is inversely proportional to the switching-time length T . However, the following findings emerged when we analyzed the relationship. The noninteger values estimated by using D_2 were different among T s. The fitting curve estimated by using the ANOVA results of the multiple comparisons [see the asterisks in the middle part of Fig. 9(g)] and AIC [see the lower part of Fig. 9(g)] suggest that a regular relationship of inverse proportion exists between T and D_2 . The correlation dimension, which depends on T , can be understood intuitively if we consider both sides of the fitting curve, as Nishikawa and Gohara [2008] proposed. That is, two attractors converge to two regions [for example, set C in Fig. 9(f)] as T becomes longer and longer infinitely ($T \rightarrow \infty$) and D_2 approaches zero ($D_2 \rightarrow 0$). In contrast, trajectories $\Gamma(C)$ spread around two attractors on the Poincaré section Σ with a rapid transition between attractors while D_2 increases up to 2 if T approaches zero ($T \rightarrow 0$). However, the tracking movement with rapid switching did not increase and exceed $D_2 \approx 1.4$ as T decreased [upper part of Fig. 9(g)]. This result for human movement implies that the values of the correlation dimension do not increase without limit as T decreases and do not exceed a certain noninteger value ($D_2 \approx 1.4$) and possibly remain constant ($D_2 \approx 1.2$ for the mean value). In addition, the differences among $T = 0.50$ s, 0.35 s, and 0.25 s for the multiple comparisons were not significant, suggesting that human movement may not become random no matter how short the switching-time length is. Perhaps the reason might be that human movement is affected by biomechanical and/or neuro-anatomical constraints. Therefore, a certain amount of complexity (for example, the constant term of the fitting curve, i.e. 0.738 in this study) is inherent in the output patterns of human movement; the output from human movement is not completely

regular such as it would be for an industrial robot even if T is long. In addition, the system used by humans for tracking movements in this study might have a state of anomaly with the critical time length [Nishikawa & Gohara, 2008] between the switching-time lengths 0.35 s and 0.25 s. That is, the output or the response of the system, i.e. the trajectory set $\Gamma(C)$, forms a hierarchical tree structure, and this might suggest that the fractal transition may have relevancy in information processing, such as in motion control [Gohara & Okuyama, 1999b]. If we interpret this phenomenon from this point of view, humans seem to switch the feedback system to the feedforward system at this critical time length.

6. Summary

This study analyzed the movements of participants who continuously tracked two target LEDs located on the left and the right sides in front of them, i.e. L and R, as accurately as possible. The experimental setting involved the periodic-input condition, in which the same external input was repeated, and the switching-input condition, in which two external inputs were switched stochastically. Consequently, we confirmed that the tracking movement towards L and R, which was excited by the external input, consisted of two clearly different attractors. Although this result suggests that the relation for $D_2 \propto 1/T$ might, in general, hold for dissipative and continuous dynamical system excited by a temporal input, there is a possibility of anomaly as the switching-time length is reduced. Therefore, we should study motor responses, e.g. the value of D_2 in Fig. 9(a), from the point of view of anomaly in human movement while calculating the inverse proportional function in the future. However, we also believe and conjecture that such complex dynamics provides an origin of various complex phenomena [Nishikawa & Gohara, 2008] for human movement, and that the results reported in this paper are generally qualitatively applicable to dynamical systems excited by external temporal inputs. The conclusions of this study are as follows. (1) The structure of variability of the transition state between several movement patterns is not composed of stochastic or random errors, such that human motor control can switch several movement patterns abruptly. (2) The structure is fractal-like and/or self-similar, and (3) A regular inversely proportional relationship between the structure D_2 and the switching-time length T is possible.

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