**The mass and width differences of the
neutral and charged $\rho(770)$, $\rho(1450)$ and $\rho(1700)$ mesons from
 $e^+e^- \rightarrow \pi^+\pi^-$ and $\tau^- \rightarrow \nu_\tau\pi^-\pi^0$ processes**

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Unlike the Review of Particle Physics, the clear nonzero values of the mass and width differences of the neutral and charged $\rho(770)$ mesons are determined by an application of the Unitary & Analytic model of the pion electromagnetic and the pion weak form factor in a description of the data on $e^+e^- \rightarrow \pi^+\pi^-$ and $\tau^- \rightarrow \nu_\tau\pi^-\pi^0$ processes, respectively. Also the mass and the width differences of the neutral and charged $\rho(1450)$ and $\rho(1700)$ mesons are determined for the first time, however, they are masked by large evaluated errors. Therefore, in order to come to a definite conclusion, more precise data on $e^+e^- \rightarrow \pi^+\pi^-$ and $\tau^- \rightarrow \nu_\tau\pi^-\pi^0$ in the region of $\rho(1450)$ and $\rho(1700)$ mesons are desirable.

Keywords: Rho meson; tau decay; electron positron annihilation.

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1. Introduction

The isovector vector meson resonances $\rho(770)$, $\rho(1450)$ and $\rho(1700)$ to be specified by their proper mass m and decay width Γ , possessing the isospin one, exist in three charged states, ρ^0 , ρ^+ and ρ^- . While masses (and also decay widths) of ρ^+ and ρ^- are due to CPT invariance equal, the expected small difference between ρ^0 and $\rho^\pm$ parameters is considered to be a consequence of the mass differences of the corresponding constituent quarks and the effects of the electromagnetic (EM) interaction.

A theoretical estimation of differences between the ρ^0 and $\rho^\pm$ masses and decay widths is beyond the scope of this paper, and further we are interested only in their phenomenological determination from the existing data on the pion EM form factor (FF) to be obtained in a measurement of the electron–positron annihilation into $\pi^+\pi^-$ and from the existing data on the pion weak FF to be obtained in a measurement of the decay rate of the τ^- -lepton into two pions $\pi^-\pi^0$.

In order to find all these differences for $\rho(770)$, $\rho(1450)$ and $\rho(1700)$ vector meson parameters, first one has to find the proper masses and decay widths of these charged and neutral particles. But generally there is no uniform approach for the realization of such a program and one has to use for it some model.

A first step towards reconsidering the proper extraction of the vector-meson resonance parameters from experiments can be found in the recent paper of Jegerlehner and Szafron,¹ where, however, the authors have a goal of further improving the original idea promoted in the paper of Alemany, Davier and Höcker² by the study of the relevant $\rho^0 - \gamma$ mixing effect for a correction of the τ -decay data to be applicable for a more involved evaluation of the muon $g - 2$ anomaly instead of the less precise $e^+e^- \rightarrow \pi^+\pi^-$ data. All other important issues in obtaining of the total equivalence of the τ -decay data and the $e^+e^- \rightarrow \pi^+\pi^-$ data, like isospin violation effects and radiative corrections, are presented in an excellent review of Davier, Höcker and Zhang³ and references therein.

However, we are now not interested in this phenomenon, but in using a high-statistics measurement of the branching fraction of the weak decay $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ with the data sample to be about 50 times larger than those of previous experiments, which delivers the very precise pion weak FF data⁴ with visible contributions of all three charged vector mesons $\rho^-(770)$, $\rho^-(1450)$, $\rho^-(1700)$ and in the improved, and corrected pion EM FF data from $e^+e^- \rightarrow \pi^+\pi^-$ and $e^+e^- \rightarrow \pi^+\pi^-\gamma$ (Refs. 5–8) in which also the contributions of the first three neutral vector mesons $\rho^0(770)$, $\rho^0(1450)$, $\rho^0(1700)$ are clearly seen.

Starting from the conserved-vector-current (CVC) hypothesis $\partial V^\mu = 0$, which now naturally follows from the standard electroweak theory, the relation $\langle \pi^0 | V^\mu | \pi^- \rangle = \sqrt{2} \langle \pi^+ | J_E^\mu | \pi^- \rangle$ between a matrix element $\langle \pi^0 | V^\mu | \pi^- \rangle$ of the vector part of the weak charged hadronic current and a corresponding matrix element $\langle \pi^+ | J_E^\mu | \pi^- \rangle$ of the EM current has been proven in Ref. 17. Then, parametrizing both matrix elements through corresponding FFs, one finally obtains a very useful

Table 1. Mass difference between the neutral ρ^0 and charged $\rho^\pm$ states of the $\rho(770)$.

$\Delta m = m_{\rho^0} - m_{\rho^\pm}$ [MeV]	Processes	References
2.4 ± 2.1	$\pi^\pm p \rightarrow \rho N$	9
-5.0 ± 5.0	$\bar{p} p \rightarrow \rho \pi \rightarrow \pi^+ \pi^- \pi^0$	10
-4.0 ± 4.0	$\pi^- p \rightarrow \rho N$	11
$1.6 \pm 0.6 \pm 1.7$	$\bar{p} p \rightarrow \pi^+ \pi^- \pi^0$	12
$1.3 \pm 1.1 \pm 2.0$	$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$	13
$0.4 \pm 0.7 \pm 0.6$	$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$	14, 15
-0.7 ± 0.8	PDG weighted average	16

relation between the pure isovector part of the pion EM FF $F_\pi^{E,I=1}(t)$ and the pion weak FF $F_\pi^W(t)$ in the following form:

$$F_\pi^W(t) = \sqrt{2} F_\pi^{E,I=1}(t), \quad (1)$$

which authorizes us to employ the same theoretical formula for both FFs in the determination of the charged and neutral vector meson parameters.

Now, there is a question, what explicit form of the pion EM and pion weak FF, depending on the isovector vector meson parameters, has to be used in order to describe both sets of experimental data and to determine the masses and widths of charged and neutral $\rho(770)$, $\rho(1450)$ and $\rho(1700)$ vector meson resonances with the aim of a determination of the corresponding differences.

The current situation about these differences stated in the Review of Particle Physics¹⁶ concerns only the ground state $\rho(770)$ vector meson (see Table 1). The average value of the mass difference for the neutral ρ^0 and charged $\rho^\pm$ mesons is $\Delta m = m_{\rho^0} - m_{\rho^\pm} = (-0.7 \pm 0.8)$ MeV, and the average value of their total decay width difference is $\Delta\Gamma = \Gamma_{\rho^0} - \Gamma_{\rho^\pm} = (0.3 \pm 1.3)$ MeV.

These parameter differences have been obtained predominantly from the analysis of the strong interaction $\pi^\pm p \rightarrow \pi\pi N$ (Refs. 9 and 11), $\bar{p} p \rightarrow \rho\pi \rightarrow \pi^+ \pi^- \pi^0$ (Refs. 10 and 12) processes and also from the electron–positron annihilation into three pions $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$ (Refs. 13–15) by an application of various standard Breit–Wigner (BW) functions, bringing in this way into obtained results a considerable model dependence and large errors.

There is no doubt that the clearest determination of the ρ^0 and $\rho^\pm$ masses and widths comes from the electron–positron annihilation and the τ -lepton decay, respectively, both into two pions, since, as we have already mentioned, a high-statistics measurement of the branching fraction $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ delivers the invariant mass spectrum of the produced $\pi^- \pi^0$ (Ref. 4) with visible contributions of all three charged vector mesons $\rho(770)$, $\rho(1450)$, $\rho(1700)$ and the same is seen also in the precise data from $e^+ e^- \rightarrow \pi^+ \pi^-$ and $e^+ e^- \rightarrow \pi^+ \pi^- \gamma$ (Refs. 5–8) concerning the above-mentioned neutral vector mesons.

But commonly a description of pion FFs data measured in the neutral channel $e^+e^- \rightarrow \pi^+\pi^-$ and in the charged channel $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ is carried out by the Novosibirsk extended Gounaris–Sakurai (GS) formula

$$F_\pi(t) = \frac{1}{1 + \beta + \gamma} \left[\text{BW}_{\rho(770)}^{\text{GS}}(t) \cdot \left(1 + \delta \frac{t}{m_\omega^2} \text{BW}_\omega(t) \right) + \beta \text{BW}_{\rho(1450)}^{\text{GS}}(t) + \gamma \text{BW}_{\rho(1700)}^{\text{GS}}(t) \right], \quad (2)$$

which is a sum of the amplitudes of the mixing isovector and also isoscalar states, each described by a GS BW-type of the amplitude.

We do not use such an approach to describe the data, since the Novosibirsk extension of the GS model is a pure phenomenological ansatz without any deeper physical background, as the original GS model¹⁸ has been constructed from the P -wave isovector $\pi\pi$ scattering phase shift given by a generalized effective-range formula of the Chew–Mandelstam type, which is evidently valid only in the elastic region. As a result, the GS model perfectly reproduces the $\rho(770)$ -meson contribution and it cannot be used to include the $\rho(1450)$ and $\rho(1700)$ contributions, which can lead only to a deformation of the ρ -meson parameters. The extrapolated curve of the original GS model falls much lower than the high-energy data in the timelike region and it is somewhat higher than the data in the spacelike region.

Further we present the Unitary & Analytic (U&A) pion FF model,¹⁹ in which all previous shortcomings are eliminated.

Here $\rho(770)$, $\rho(1450)$ and $\rho(1700)$ resonances are defined by the fixed mass and width of the complex conjugate poles on the second Riemann sheet of the corresponding EM and weak FFs.

An advantage of this approach is that it also utilizes the existing data on the pion EM FF in the spacelike region.^{20–22} As a result, the errors of the determined masses and widths are thereby remarkably reduced.

In Sec. 2, the U&A model of the pion EM FF is constructed which reflects all the known properties of $F_\pi^{\text{EM}}(t)$ and represents a well-matched unification of the vector meson poles contributions from the unphysical sheets of the Riemann surface and the continua to be represented by the cuts on the positive real axis in the t -complex plane. Afterward it is employed for a description of the existing data on the pion EM FF in the spacelike and timelike regions simultaneously and the resonance parameters of all neutral $\rho^0(770)$, $\rho^0(1450)$ and $\rho^0(1700)$ mesons are determined.

Section 3 is devoted to an application of the U&A model of the pion weak FF to a description of existing data⁴ obtained from the $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ decay into two pions with the aim of determining resonance parameters for the determination for charged $\rho^-(770)$, $\rho^-(1450)$ and $\rho^-(1700)$ mesons.

The results for the mass and width differences of the neutral and charged $\rho(770)$, $\rho(1450)$ and $\rho(1700)$ mesons are presented in the fourth column of Table 2.

Conclusions are given in the last section.

Table 2. The values of ρ -meson parameters obtained in a fitting of the corresponding data by the U&A model. The difference $\Delta = m_{\rho^0} - m_{\rho^\pm}$ is presented in the fourth column.

Parameter	ρ^0 [MeV]	U&A $\rho^\pm$ [MeV]	Δ [MeV]	Belle-10 $\rho^\pm$ [MeV]	Belle-11 $\rho^\pm$ [MeV]
$m_{\rho(770)}$	758.23 ± 0.46	761.60 ± 0.95	-3.37 ± 1.06	774.6 ± 0.5	774.9 ± 0.6
$m_{\rho(1450)}$	1342.31 ± 46.62	1373.83 ± 11.37	-31.53 ± 47.99	1446.0 ± 28.9	1428.0 ± 30.0
$m_{\rho(1700)}$	1718.50 ± 65.44	1766.80 ± 52.36	-48.30 ± 83.81	1728.0 ± 90.6	1694.0 ± 98.0
$\Gamma_{\rho(770)}$	144.56 ± 0.80	139.90 ± 0.46	4.66 ± 0.85	148.1 ± 1.7	148.6 ± 1.8
$\Gamma_{\rho(1450)}$	492.17 ± 138.38	340.87 ± 23.84	151.30 ± 140.42	434.0 ± 62.1	413.0 ± 58.2
$\Gamma_{\rho(1700)}$	489.58 ± 16.95	414.71 ± 119.48	74.87 ± 120.67	164.0 ± 91.4	135.0 ± 61.6
t_{in}	1364.60 ± 19.80	1243.20 ± 15.70	—	—	—
W_Z	0.1857 ± 0.0004	0.4078 ± 0.0013	—	—	—
W_P	0.2335 ± 0.0005	0.6197 ± 0.0007	—	—	—
$f_{\rho\pi\pi}/f_\rho$	1.0009 ± 0.0001	0.9998 ± 0.0002	—	—	—
R	0.0125 ± 0.0103	—	—	—	—

2. U&A Model of the Pion EM Structure

The U&A model reflects all the known theoretical properties of $F_\pi^{E,I=1}(t)$, which can be piled together into the following points:

- $F_\pi^{E,I=1}(t)$ is normalized to the electric charge to be put 1 at $t = 0$.
- $F_\pi^{E,I=1}(t)$ is an analytic function in the whole complex t -plane besides the cut from $t = 4m_\pi^2$ up to ∞ .
- Discontinuity across the latter cut is given by the unitarity condition of $F_\pi^{E,I=1}(t)$

$$\frac{1}{2i} \left\{ F_\pi^{E,I=1}(t+i\varepsilon) - F_\pi^{*E,I=1}(t+i\varepsilon) \right\} = [A_1^1(t+i\varepsilon)]^* F_\pi^{E,I=1}(t+i\varepsilon) + \sigma(t+i\varepsilon),$$

where $A_1^1(t)$ is the P -wave isovector $\pi\pi$ scattering amplitude. Higher contributions from the intermediate states $|n\rangle$ are represented by $\sigma(t)$.

- A branch point corresponds to every intermediate state $|n\rangle$ in the unitarity condition.
- The reality condition $F_\pi^{E,I=1}(t+i\varepsilon) = F_\pi^{*E,I=1}(t-i\varepsilon)$ follows from the hermiticity of the EM current.
- By means of the analytic continuation of the elastic unitarity condition through the upper and lower boundaries of the elastic cut on the unphysical sheet, one can prove the branch point $t = 4m_\pi^2$ to be a square-root type. Moreover, one obtains the expression of the $F_\pi^{E,I=1}(t)$ on the second Riemann sheet, from which one can reveal that the pion EM FF is on the II. Riemann sheet besides the right-hand unitary cut possesses also the left-hand cut $-\infty < t < 0$, which cannot be neglected in any analysis of experimental data.

- All poles corresponding to the vector meson resonances are forbidden by the unitarity condition, for to be on the first sheet and can be placed only on the II. or other unphysical sheets of the Riemann surface.
- As a consequence of the reality condition, for every resonance pole the complex-conjugate pole also exists.
- Directly from the elastic unitarity condition the equality $\delta_\pi = \delta_1^1$ follows, where δ_π is the phase of $F_\pi^{E,I=1}(t)$ and δ_1^1 is P -wave isovector $\pi\pi$ scattering phase shift with the threshold behavior $\delta_1^1 = a_1^1 q^3$, where q is the absolute value of the pion c.m. three-momentum.
- The imaginary part of $F_\pi^{E,I=1}(t)$ can be expressed through δ_1^1 and by employing the threshold behavior of the latter one obtains $\text{Im } F_\pi^{E,I=1}(t) = |F_\pi^{E,I=1}(t)| a_1^1 q^3$, from which the three conditions follow:

$$\begin{aligned} \text{Im } F_\pi^{E,I=1}(t) \Big|_{q=0} &= 0, \\ \frac{d}{dq} \text{Im } F_\pi^{E,I=1}(t) \Big|_{q=0} &= 0, \\ \frac{d^2}{dq^2} \text{Im } F_\pi^{E,I=1}(t) \Big|_{q=0} &= 0. \end{aligned}$$

- The asymptotic behavior of $F_\pi^{E,I=1}(t)$ predicted by the theory of strong interactions, QCD, is

$$F_\pi(t) \Big|_{t \rightarrow -\infty} \sim -\frac{16\pi f_\pi^2 \alpha_s(t)}{t}, \quad (3)$$

$f_\pi = (92.4 \pm 0.2)$ MeV is the weak pion decay constant, $\alpha_s(t)$ is the QCD running coupling constant.

According to our knowledge, there are no any other properties of $F_\pi^{E,I=1}(t)$ for the time being.

This approach leads to U&A model of the pion EM FF²³ represented by the following expression:

$$\begin{aligned} F_\pi^{\text{EM},I=1}[W(t)] &= \left(\frac{1 - W^2}{1 - W_N^2} \right)^2 \frac{(W - W_Z)(W_N - W_P)}{(W_N - W_Z)(W - W_P)} \\ &\times \left\{ \frac{(W_N - W_\rho)(W_N - W_\rho^*)}{(W - W_\rho)(W - W_\rho^*)} \frac{(W_N - 1/W_\rho)(W_N - 1/W_\rho^*)}{(W - 1/W_\rho)(W - 1/W_\rho^*)} \left(\frac{f_{\rho\pi\pi}}{f_\rho} \right) \right. \\ &\left. + \sum_{v=\rho',\rho''} \left[\frac{(W_N - W_v)(W_N - W_v^*)}{(W - W_v)(W - W_v^*)} \frac{(W_N + W_v)(W_N + W_v^*)}{(W + W_v)(W + W_v^*)} \right] \left(\frac{f_{v\pi\pi}}{f_v} \right) \right\} \quad (4) \end{aligned}$$

with the conformal mapping $W(t) = i \frac{\sqrt{q_{\text{in}}+q} - \sqrt{q_{\text{in}}-q}}{\sqrt{q_{\text{in}}+q} + \sqrt{q_{\text{in}}-q}}$, $q = \sqrt{\frac{t-t_0}{4}}$, $q_{\text{in}} = \sqrt{\frac{t_{\text{in}}-t_0}{4}}$ of the four-sheeted Riemann surface into one W -plane, W_Z and W_P are the normalized

zero and the normalized pole, by means of which the left-hand cut of $F_\pi^{E,I=1}(t)$ on the II. Riemann sheet is taken into account. The ratio $f_{\rho\pi\pi}/f_\rho$ comprises the vector-meson-pion $f_{\rho\pi\pi}$ and the universal vector-meson f_ρ coupling constant, whereby the ratios of the coupling constants for the higher resonances ρ' and ρ'' can be expressed as

$$\begin{aligned} \left(\frac{f_{\rho'\pi\pi}}{f_{\rho'}}\right) &= \frac{1}{\frac{N_{\rho'}}{|W_{\rho'}|^4} - \frac{N_{\rho''}}{|W_{\rho''}|^4}} \left\{ 1 - \left(\frac{N_{\rho'}}{|W_{\rho'}|^4} - \left[1 + 2 \frac{W_Z W_P}{W_Z - W_P} \right. \right. \right. \\ &\quad \left. \left. \left. \times \operatorname{Re} W_\rho (1 - |W_\rho|^{-2}) \right] N_\rho \right) \left(\frac{f_{\rho\pi\pi}}{f_\rho} \right) \right\}, \\ \left(\frac{f_{\rho''\pi\pi}}{f_{\rho''}}\right) &= \frac{1}{\frac{N_{\rho'}}{|W_{\rho'}|^4} - \frac{N_{\rho''}}{|W_{\rho''}|^4}} \left\{ -1 + \left(\frac{N_{\rho''}}{|W_{\rho''}|^4} - \left[1 + 2 \frac{W_Z W_P}{W_Z - W_P} \right. \right. \right. \\ &\quad \left. \left. \left. \times \operatorname{Re} W_\rho (1 - |W_\rho|^{-2}) \right] N_\rho \right) \left(\frac{f_{\rho\pi\pi}}{f_\rho} \right) \right\}, \\ N_\rho &= (W_N - W_\rho)(W_N - W_\rho^*)(W_N - 1/W_\rho)(W_N - 1/W_\rho^*), \\ N_v &= (W_N - W_v)(W_N - W_v^*)(W_N + W_v)(W_N + W_v^*); \quad v = \rho', \rho''. \end{aligned}$$

The latter expressions are obtained²³ from the solution of the equations

$$\begin{aligned} &[(W_z - W_p) + 2W_z W_p \operatorname{Re} W_\rho (1 + |W_\rho|^{-2})] N_\rho \left(\frac{f_{\rho\pi\pi}}{f_\rho} \right) \\ &+ \sum_{v=\rho', \rho''} \frac{W_z - W_p}{|W_v|^4} N_v \left(\frac{f_{v\pi\pi}}{f_v} \right) = 0, \end{aligned} \quad (5)$$

obtained from the first derivative of the $\operatorname{Im} F_\pi^{E,I=1}[W(t)]$ of (4) at $q = 0$ and the normalization condition

$$\sum_{v=\rho, \rho', \rho''} \left(\frac{f_{v\pi\pi}}{f_v} \right) = 1. \quad (6)$$

The model is defined on the four-sheeted Riemann surface with complex-conjugate poles (corresponding to unstable ρ -meson resonances) on unphysical sheets, and reflecting all known properties of the EM pion FF.

The model depends on 10 physically interpretable free parameters, t_{in} — inelastic threshold, m_ρ , Γ_ρ , $m_{\rho'}$, $\Gamma_{\rho'}$, $m_{\rho''}$, $\Gamma_{\rho''}$ — masses and widths of ρ -mesons, $f_{\rho\pi\pi}/f_\rho$ — ratio of coupling constants, W_Z — and W_P — the zero and pole values of the W -mapping.

The fitting procedure is carried out to the pion EM FF experimental data,⁵⁻⁸ by means of the expression

$$\left| F_{\pi}^{\text{EM}, I=1}(s) + R e^{i\Phi} \frac{m_{\omega}^2}{m_{\omega}^2 - s - i m_{\omega} \Gamma_{\omega}} \right|^2, \quad (7)$$

$$\Phi = \arctan \frac{m_{\rho} \Gamma_{\rho}}{m_{\rho}^2 - m_{\omega}^2},$$

where we have included also the term corresponding to the ρ - ω interference mixing. Here the ρ - ω interference phase Φ and the interference amplitude R are free parameters.

There exists an almost continuous interval of 381 experimental points in the region $-9.77 \text{ GeV}^2 \leq t \leq 13.48 \text{ GeV}^2$, which are described by the EM pion FF model (see Eq. (4)) in the spacelike and the timelike regions simultaneously.

In extraction of the used pion EM FF data from $e^+e^- \rightarrow \pi^+\pi^-$ and $e^+e^- \rightarrow \pi^+\pi^-\gamma$ all important EM radiative corrections have been taken into account. In the previous Novosibirsk data the contribution from the leptonic loop in the t -channel was omitted, however, this shortcoming was later removed and the correct data on pion EM FF have been published in Refs. 7 and 8.

Parameter results are presented in the second column of Table 2.

3. U&A Model of the Pion Weak FF

The relation (1) enables us to use the same U&A model of the pion EM FF for the description of the weak pion FF data, however, without an isotopic violation term of the ω contribution, as there is no charged equivalent contribution in the $\tau^- \rightarrow \nu_{\tau} \pi^- \pi^0$ process.

We used only the high-statistics measurement⁴ of the weak pion FF from $\tau^- \rightarrow \nu_{\tau} \pi^- \pi^0$ decay with the Belle detector at the KEKB asymmetric-energy e^+e^- collider as it has been charged by the lowest total uncertainties in comparison with the previous measurements done by ALEPH²⁴ and CLEO.²⁵

While obtaining the weak pion FF data⁴ from the measured normalized invariant mass-squared distribution, the short-distance electroweak radiative corrections $S_{\text{EW}}^{26,27}$ have been taken into account by the value²⁸ 1.0235 ± 0.0003 consistent with the isospin-breaking corrections.²⁹⁻³¹

The results for the $\rho^{\pm}$ parameters are presented in the third column of Table 2.

4. Conclusions

The values of the masses and widths of the $\rho(770)$, $\rho(1450)$ and $\rho(1700)$ vector mesons determined by applying our Unitary & Analytic model of EM and weak pion FF to the set of 381 ($e^+e^- \rightarrow \pi^+\pi^-$) and 62 ($\tau^- \rightarrow \pi^-\pi^+\nu_{\tau}$) experimental data points are presented in Table 2.

Fig. 1. The status on the experimental values of the mass difference of the neutral ρ^0 and charged $\rho^\pm$ mesons for the state $\rho(770)$ compared with our result.

From these results one comes to the conclusion that the differences Δ of masses and widths have a rising tendency for heavier resonances.

The comparison of the obtained mass difference of the neutral and charged $\rho(770)$ with other determinations is presented in Fig. 1.

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