Erratum

Congruence schemes

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Problem. As was pointed out to me by Peiyi Cui, the proof of Theorem 3.12 of that paper is flawed. A sesquiad homomorphism of the given type cannot exist, since the element (-1) is contained in the sesquiad A and as we have $(-1)^2 = 1$ as well as $1 + (-1) = 0$, the element (-1) can neither be mapped to 0 nor to 1, hence a morphism $\phi : A \rightarrow \mathbb{F}_1$ does not exist.

Solution. We give a different construction of a Tits model for GL_n by starting off from a different description of the coordinate ring R of GL_n as follows:

$$R = \mathbb{Z}[X_{ij}, Y_{ij}]/I,$$

where $1 \leq i, j \leq n$, so we attach $2n^2$ variables to $\mathbb{Z}$ and I is the ideal generated by all elements of the form

$$\sum_{k=1}^n X_{ik}Y_{kj} - \delta_{ij},$$

where δ_{ij} is the Kronecker delta. Let $A \subset R$ be the multiplicative submonoid generated by all X_{ij} , all Y_{ij} and all elements of the form

$$X_{i1} + X_{i2} + \cdots + X_{in}$$

for $1 \leq i \leq n$. Let $\phi : A \rightarrow \mathbb{F}_1$ be a sesquiad morphism. Write $\phi(X_{ij}) = x_{ij}$ and $\phi(Y_{ij}) = y_{ij}$. Since the sum $X_{i1} + \cdots + X_{in}$ is in A , but $\mathbb{F}_1$ has trivial addition, at most one of the elements $x_{i1}, \dots, x_{in}$ can be nonzero. We prolong ϕ by $\mathbb{F}_1 \rightarrow \mathbb{Z} \rightarrow \mathbb{Q}$ to a sesquiad homomorphism $\eta : A \rightarrow \mathbb{Q}$ which therefore maps X to an invertible

matrix which has all entries equal to 0 or 1 with at most one entry nonzero in each row, therefore it must be a permutation matrix. On the other hand, each permutation matrix defines a sesquiad morphism ϕ and so

$$\mathrm{Hom}(A, \mathbb{F}_1) \cong \mathrm{Per}(n) = W_G. \quad \square$$