ERRATA

GEOMETRIC GALOIS THEORY, NONLINEAR NUMBER FIELDS AND A GALOIS GROUP INTERPRETATION OF THE IDELE CLASS GROUP

[*Internat. J. Math.* **16**(6) (2005) 567–593]

T. M. GENDRON and A. VERJOVSKY

*Instituto de Matemáticas
 Universidad Nacional Autónoma de México
 Av. Universidad s/n, Lomas de Chamilpa
 Cuernavaca CP 62210, Morelos
 México*

Received 22 April 2010

In this errata we correct several mistakes Secs. 1 and 3–6 that appear in the published version of our paper. The corrections have been implemented in the revised version [1]. In addition, in Sec. 2 we clarify an important point which was not adequately addressed in the published version; in Sec. 7 we point out an enhancement of the hyperbolization scheme included in [1]. The reader may also wish to consult [2].

Keywords: Nonlinear number field; character field; Minkowski torus; idele class group.

Mathematics Subject Classification 2000: 11R56, 11R37, 11R32

1. Definition of Nonlinear Number Field

In the original publication, on page 582, line 15, the definition of $\mathbb{C}^*[K]$ should read:

$$\mathbb{C}^*[K] = \mathbb{C}[K] - I_K \quad (1.1)$$

and not “ $\mathbb{C}[K]/I_K$.” The set (1.1) is invariant with respect to both Cauchy and Dirichlet multiplication and the space $\mathbb{CP}[K]$ is then defined to be $\mathbb{C}^*[K]/\mathbb{C}^*$. This correction must be carried out in the more general definition, e.g. on page 586, line 30 should read $H_\bullet^*[K] = H_\bullet[K] - I_K$, where I_K is the kernel of the trace map on its domain of definition (see [1, Sec. 9]).

2. Dirichlet Product Structure

Our treatment of the Dirichlet product structure of $\text{Char}(\hat{S}_K)$ (part 1 of Theorem 5, page 581) was incomplete. Let $\psi_\infty(\mathbf{z}) = \exp(2\pi i \text{Tr}(\mathbf{z}))$ be the *standard character* on

$K_\infty \subset \hat{S}_K$, which extends continuously to a character ψ on $\hat{S}_K$ that the monomorphism

$$\alpha \mapsto \psi_\alpha, \quad \psi_\alpha(\hat{\mathbf{z}}) = \psi(\alpha\hat{\mathbf{z}}) \quad (2.1)$$

defines an isomorphism of additive groups $(K, +) \cong \text{Char}(\hat{S}_K)$ that is classical [3]. The Dirichlet product $\otimes$ on $\text{Char}(\hat{S}_K)$ is then defined by pushing forward $\times_K$. The crucial observation is that the definition of $\otimes$ is natural with respect to the inclusions $\text{Char}(\hat{S}_K) \hookrightarrow \text{Char}(\hat{S}_L)$ induced by the trace maps $\text{Tr}_{L/K} : \hat{S}_L \rightarrow \hat{S}_K$. See [1, Sec. 7] for more details.

3. Character Group of the Minkowski Torus

In part 2 of Theorem 5, the additive group $(O_K, +)$ was mistakenly identified with the character group $\text{Char}(\mathbb{T}_K)$ of the Minkowski torus $\mathbb{T}_K$ via the map (2.1). Instead, (2.1) identifies the inverse different $\mathfrak{d}_K^{-1} \supset O_K$ with $\text{Char}(\mathbb{T}_K)$. The character group $\text{Char}(\mathbb{T}_K)$ is thus an O_K -module extension of O_K (see [1, Sec. 7]).

4. Domain of Definition of Cauchy; Dirichlet Products

On line 15, page 583, it was incorrectly asserted that the Cauchy and Dirichlet products are fully defined on the Hilbert space $L^2(\hat{S}_K, \mathbb{C})$. These operations only extend partially so that for each element $f \in L^2(\hat{S}_K, \mathbb{C})$ one must specify the Cauchy and Dirichlet domains $\text{Dom}_\oplus(f)$, $\text{Dom}_\otimes(f)$ of elements g for which $f \oplus g$, respectively $f \otimes g$, makes sense. Note that since the double group algebra $\mathbb{C}[K]$ is contained in $\text{Dom}_\oplus(f)$, $\text{Dom}_\otimes(f)$, we have that the latter spaces are dense in $L^2(\hat{S}_K, \mathbb{C})$ (see [1, Sec. 8]). The definition of nonlinear number field, which appears in Definition 2 on page 587, must be adjusted accordingly by replacing everywhere the phrase “double semigroup” by “partial double semigroup” to take into account this correction (see [1, Sec. 9]).

5. Proof of Lemma 1

The proof of Lemma 1 on page 589 was incorrect. Here is a correct proof (which appears as [1, Sec. 10, proof of Lemma 2]). Suppose first that $K = \mathbb{Q}$ and without loss of generality $F \in \mathbb{H}[\mathbb{Q}]$. Let $\tau_0 \in \mathbb{H}_\mathbb{Q} \subset \hat{\mathbb{S}}_\mathbb{Q}$ be such that $F(\tau_0) \neq 0$. Then there exists a complex number α with $\exp(2\pi i \alpha \tau_0) = F(\tau_0)$. By hypothesis, $F(r\tau) = F(\tau)^r$, so $F(\tau)$ and $\exp(2\pi i \alpha \tau)$ agree on the ray $\mathbb{R}_+ \cdot \tau_0$. Then by holomorphicity, they coincide. Since F is a function of $\hat{\mathbb{S}}_\mathbb{Q}$, it follows that $\alpha \in \mathbb{Q}$.

Now let the extension $K/\mathbb{Q}$ be totally real of finite degree. Let $\Delta \subset \mathbb{H}_K$ be the diagonal hyperbolic sub-plane (the dense leaf of the image of $\hat{\mathbb{S}}_\mathbb{Q}$ under the diagonal embedding $\hat{\mathbb{S}}_\mathbb{Q} \hookrightarrow \hat{\mathbb{S}}_K$). Then by the previous paragraph, the restriction of F to Δ is an exponential $\exp(2\pi i q \tau)$ for $q \in \mathbb{Q}$. Let $\Delta_1 \subset \mathbb{H}_K$ be the diagonal $\Delta_1 = \{(\tau, \dots, \tau, \tilde{\tau}) \mid \tau, \tilde{\tau} \in \mathbb{H}\} \supset \Delta$. For each τ fixed, we can (using the same argument

employed in the previous paragraph) write the function $\tilde{\tau} \mapsto F(\tau, \dots, \tau, \tilde{\tau})$ as

$$F(\tau, \dots, \tau, \tilde{\tau}) = \exp(2\pi i q \tau) \cdot \exp(2\pi i \beta(\tau) \tilde{\tau})$$

where $\beta(\tau) \in \mathbb{C}$. Moreover we have

$$F(r \cdot (\tau, \dots, \tau, \tilde{\tau})) = \exp(2\pi i q r \tau) \cdot \exp(2\pi i \beta(\tau) r \tilde{\tau})$$

by hypothesis. As we vary τ , $\beta(\tau)$ varies holomorphically and we obtain that on a real codimension one subspace of Δ_1 ,

$$F(\tau, \dots, \tau, \tilde{\tau}) = F_1(\tau, \dots, \tau, \tilde{\tau}) := \exp(2\pi i q \tau) \cdot \exp(2\pi i \beta(\tau) \tilde{\tau}),$$

hence they are equal on Δ_1 . But this means that F_1 must also satisfy the functional equation $F_1(r \cdot (\tau, \dots, \tau, \tilde{\tau})) = F_1(\tau, \dots, \tau, \tilde{\tau})^r$, which implies that $\beta(\tau) = \beta$ is a constant. Inductively, we obtain that F restricted to $\mathbb{H}_K$ is an exponential function, and since F is a function of $\tilde{\mathfrak{S}}_K$, this restriction must be of the form $\xi^\alpha = \exp(2\pi i \text{Tr}(\alpha \cdot \tau))$ for $\alpha \in K_+$. The case of a totally complex field extension, that of mixed type, or that of infinite degree, is handled in exactly the same manner.

6. Idele Class Group Action

The discussion of the idele class group found on pages 591 and 592 is only valid for $K = \mathbb{Q}$, so that the idele class group C_K should be replaced by $C_{\mathbb{Q}}$ (see [1, Sec. 10]).

7. Hyperbolization

Apart from implementing the above corrections, the revised version [1] contains an enhancement of the hyperbolization $\mathbb{H}_K$ (page 579 and 580 of the original article), in which for each complex place μ , one takes the product of $K_\mu \cong \mathbb{C}$ with the *complex* quarter plane $\{z = x + iy \mid x, y > 0\}$, rather than the factor $(0, \infty) \times (0, \infty)$ (see [1, Sec. 6]). Furthermore, the θ -holomorphic grading of functions on $\mathbb{H}_K$ (described in page 585) has been expanded by a *complex sign set* along the complex places (this is described in [1, Sec. 8.2]).

Acknowledgments

We thank McQuillan for having brought to our attention Secs. 1, 3 and 4 above.

References

- [1] T. M. Gendron and A. Verjovsky, Geometric Galois theory, nonlinear number fields and a Galois group interpretation of the idele class group, revised version [arXiv:math/0506519].
- [2] T. M. Gendron and A. Verjovsky, Notes on nonlinear number fields [arXiv:1007.3070v1] [math.NT].
- [3] J. T. Tate, Fourier analysis in number fields, and Hecke's zeta-functions, in *1967 Algebraic Number Theory (Proc. Instructional Conf., Brighton, 1965)* (Thompson, Washington D.C.), pp. 305–347.