

# Diagnosics of Laboratory and Astrophysical Plasmas Using Spectral Lineshapes of One-, Two-, and Three-Electron Systems

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# Diagnostics of Laboratory and Astrophysical Plasmas Using Spectral Lineshapes of One-, Two-, and Three-Electron Systems

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*Published by*

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

*USA office:* 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

*UK office:* 57 Shelton Street, Covent Garden, London WC2H 9HE

### **Library of Congress Cataloging-in-Publication Data**

Names: Oks, E. A. (Evgenii Aleksandrovich), author.

Title: Diagnostics of laboratory and astrophysical plasmas using spectral lineshapes of one-, two-, and three-electron systems / Eugene Oks, Auburn University, USA.

Description: Singapore ; Hackensack, NJ : World Scientific, [2017] |

Includes bibliographical references and index.

Identifiers: LCCN 2016054152 | ISBN 9789814699075 (hardcover ; alk. paper) |

ISBN 9814699071 (hardcover ; alk. paper)

Subjects: LCSH: Plasma spectroscopy. | Plasma diagnostics. | Plasma astrophysics. | Spectral line formation.

Classification: LCC QC718.5.S6 O365 2017 | DDC 530.4/4--dc23

LC record available at <https://lccn.loc.gov/2016054152>

### **British Library Cataloguing-in-Publication Data**

A catalogue record for this book is available from the British Library.

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Typeset by Stallion Press

Email: [enquiries@stallionpress.com](mailto:enquiries@stallionpress.com)

Printed in Singapore

In memory of recently passed away

*G.V. Sholin and V.P. Gavrilenko*

with appreciation of their contribution to the area  
of the Stark broadening of spectral lines in plasmas

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## Introduction

“Thaumas took as his wife Electra, daughter of Okeanos,  
whose stream is deep, and she bore swift Iris. . .”

(Hesiod, *Theogony*, lines 265-6 — about *Iris, the goddess of rainbow*  
and thus, the first spectroscopist)

Studies of plasmas have very broad practical applications across various areas of physics and technology. Examples are (but not limited to) magnetically-controlled fusion, laser-controlled fusion, X-ray lasers, powerful Z-pinchs (used for producing X-ray and neutron radiation, ultra-high pulsed magnetic fields, and for X-ray lasing), low-temperature technological discharges for plasma chemistry (including plasma-surface processing for manufacturing microchips and for nano-technologies), interaction of powerful laser radiation or coherent microwave radiation with plasmas and gases, and astrophysics (including solar physics). An indispensable part of plasma research and its applications is *plasma diagnostics*, i.e., a compendium of methods for the experimental determination of various plasma parameters and of parameters of various electric and magnetic fields in plasmas.

In other areas of physics, the corresponding subarea is called *measurements methods*. The reason why in plasma physics it is called *diagnostics* is the following. The same observed signal from a

plasma (such as a spectrum of the electromagnetic radiation) could correspond to many different sets of parameters of the plasma and/or of fields in it — like in the medicine, where a set of patient symptoms could correspond to different illnesses or disorders. Because of this analogy, i.e., because of the absence of one-to-one correspondence between the signals/symptoms and the underlying cause in both plasma physics and medicine, the medical term *diagnostics* is used in plasma physics.

Among various methods of plasma diagnostics, the methods employing lineshapes of spectral lines emitted from plasmas play a very important role for at least four reasons. First, these methods are *non-intrusive* (sometimes called *non-perturbing*) because they did not affect parameters of a plasma (and/or of fields in it) to be measured — in distinction to the overwhelming majority of other diagnostic methods. Second, these methods often are the *most informative*. Third, they do not depend on model assumptions about the plasma state (such as local thermodynamic equilibrium, or partial local thermodynamic equilibrium, or the corona model, or the collisional-radiative model, etc.) — in distinction to the diagnostic methods using various ratios of intensities (such as ratios of integrated intensities of several spectral lines, or line-to-continuum ratios, ratios of intensities of recombination continua, etc.). Fourth, sometimes there is *no other method* for the experimental determinations of a particular parameter of the plasma and/or a particular field in the plasma.

In these methods, the role of “probes” is played by radiating atoms or ions. Probes have to be well-calibrated for achieving the required accuracy in the experimental determination of a particular parameter. Among atoms and ions, the best-calibrated are those having the simplest electronic structure: atoms or ions having one or two or three electrons and thus the most accurately described by quantum mechanics both as isolated systems and as the systems immersed in various plasmas.

The absence of one-to-one correspondence between the observed lineshapes and the underlying parameters of a plasma and/or of fields in it poses a significant challenge for the interpretation of the

experimental data and leads to a kind of a “gap” between theorists and experimentalists. Typically, theorists solve the so-called “*direct problem*”: given a set of parameters of a plasma and/or of fields in it, they calculate, e.g., a Stark profile of a particular spectral line. They can repeat it for many other sets of parameters and produce a huge multi-parametric set of Stark profiles of a particular spectral line. Then theorists calculate a convolution of these profiles with other broadening mechanisms and thus dramatically increase both the size of the set of calculated profiles and the number of parameters, on which the set depends. However, interpreting an experimental profile of the spectral line is an “*inverse problem*”: given the experimental profile, *to find a unique set of the parameters of a plasma and/or fields in it corresponding to the observed profile.*

Books on plasma spectroscopy, usually focus on presenting solutions of the direct problem, but pay zero or little attention to the solutions inverse problem — despite it is the latter that is most necessary in practice. An exception is Griem’s book of 1974 (*Spectral Line Broadening by Plasmas*, Academic Press, New York), where Chapter 4 presented solutions of the inverse problem for certain parameters to be measured. However, first, the scope of parameters was limited to the electron density, the temperature and some parameters of some plasma waves. Second, but more importantly, over the subsequent four decades, very significant advances were made both in the theory of spectral line broadening by plasmas and in approaches to solving the corresponding inverse problem.

Hutchinson’s book of 2005 (*Principles of Plasma Diagnostics*, Cambridge University Press, Cambridge) discussed the inverse problem in spectral lineshapes from plasmas only in a couple of sections of only one out of nine chapters (Chapter 6), and the discussion was cursory. Kunze’s book of 2009 (*Introduction to Plasma Spectroscopy*, Springer, Berlin) discussed the inverse problem in spectral lineshapes from plasmas only in a few sections of only one out of 10 chapters (Chapter 10), and the discussion was relatively brief as well. Those are fine books — just they had a focus different from presenting solutions of the inverse problem in spectral lineshapes from plasmas.

As for other books having “plasma spectroscopy” in the title, such as Oks’ book of 1995 (*Plasma Spectroscopy: The Influence of Microwave and Laser Fields*, Springer, Berlin), Griem’s book of 1997 (*Principles of Plasma Spectroscopy*, Cambridge University Press, Cambridge), and Fujimoto’s book of 2004 (*Plasma Spectroscopy*, Clarendon Press, Oxford), they presented lots of theoretical results, but paid zero or little attention to solutions of the inverse problem in spectral lineshapes from plasmas — because this was not the focus of those books.

The present monograph has the following three important features distinguishing it from the above-mentioned books. First, practically its *entire focus* is on presenting solutions for the inverse problem in spectral lineshapes from plasmas. Second, this monograph significantly *expands the scope of parameters* of plasmas and/or fields in it to be measured (such as effective charge of ions, parameters of low-frequency electrostatic turbulence in magnetized plasmas, parameters of Langmuir turbulence in magnetized plasmas, parameters of transverse laser-induced electromagnetic fields in plasmas). Third, for some parameters of plasmas and/or fields in it, this monograph presents *new, more advanced diagnostic methods* than the methods covered (though briefly) in the previous books.

In summary, the present book is an advanced tool for experimentalists using spectral lineshapes for diagnostics and for theorists helping the experimentalists in interpreting the experimental line profiles. This concerns both laboratory and astrophysical plasmas.

The book is divided into two parts. Part 1 is dedicated to plasmas that do not contain the electrostatic turbulence, i.e., turbulence represented by oscillatory electric fields. Part 2 is devoted to plasmas containing oscillatory electric fields. Both parts contain practical advice on how to interpret spectral line profiles observed in varieties of laboratory and astrophysical plasmas. They also contain numerous examples of the corresponding interpretations of various laboratory plasma experiments and astrophysical observations. The underlying theories are presented mostly in Appendices.