High-Frequency Trading and Probability Theory

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Zhaodong Wang
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Foreword to *East China Normal University Scientific Reports*

Founded in 1951, as a result of merging of several old universities in Shanghai (including St. John's University founded in 1879), East China Normal University (ECNU) becomes one of the most prestigious universities in China. Our faculty members have acquired high academic reputations in a number of scientific research areas.

ECNU Scientific Reports, a book series edited by the editorial office of *Journal of ECNU* and published by World Scientific Publishing Co., present valuable results and significant progresses in scientific researches. We hope that this book series will serve as an international platform for scientific research, particularly inter-disciplinary research.

I am delighted to see the publication of the first volume, *High-Frequency Trading and Probability Theory*; and would like to sincerely congratulate the authors. I expect that more and more valuable volumes of the series will appear in the near future, so that the international scientific world will benefit from our research.

Qun Chen
President of East China Normal University
Shanghai, China

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Preface

There have been quite a few publications on **High-Frequency Trading (HFT)**. However, few of them discussed the detailed strategies and algorithms of HFT. From our point of view, HFT is an application of high technology and mathematics to the financial market. Based on our practice in the Chinese futures market, we will show mainly three things in this book:

- (1) Some facts and technical details of the IT system design for HFT, some of which have not been seen in literature until this point.
- (2) HFT has the ergodic theory of stationary process as its mathematical background.
- (3) A trader can use technical analysis to repeatedly trade one financial derivative and make statistical arbitrage according to the strong ergodic theorem.

Due to its remarkable profitability, the algorithms of HFT were considered highly confidential and have not been well-studied in the academic world. Just as in Chinese proverb “throwing out a brick in order to attract the others throwing gems”, we hope that this book will attract more researchers to study the scientific background of HFT. We will concentrate our discussions of the examples of the Chinese futures market. However, the general principles apply to other markets as well.

We assume that our readers are aware of some basic calculus. The material of this book was discussed at the School of Finance and Statistics, East China Normal University three times in three semesters. We would like

to thank Professor Shujin Wu, PhD students Si Bao, Shi Chen, Chang Liu, Shuai Wang and Yu Zhou for various insightful discussions and technical assistances. We particularly thank Ms. Xiaolin Guo who videotaped the first author's lectures which form the first part of this book, and thank Ms. Eileen Raney who assisted us to edit the original English manuscript. This work is partially supported by the 111 project (B14019) in China.

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Zhaodong Wang obtained his PhD degree in Computer Science from Shanghai Jiao Tong University. He is an expert in high performance system design. He was the CEO of Shanghai Futures Information Technology Corp. Ltd. At that time, he designed the trading system for Shanghai Futures Exchange, China Finance Futures Exchange and many other systems in financial area. Later, he set up a hedge fund focus on high-frequency trading.

Weian Zheng is a State Endowed Professor at ECNU and Professor Emeriti at University of California, Irvine, USA. He had Docteur d'état ès Sciences Mathématiques degree from University of Strasbourg, France. He has a few known results in probability theory, including (with P. A. Meyer) the tightness criteria for laws of semi-martingales and (with T. J. Lyons) the forward-backward decomposition of symmetric Markov processes. With Y. Shen, he used probability theory to reduce the

famous Monge–Kantorovich mass transfer problem to a boundary value problem of partial differential equation and solved this problem in the plane. He was born in Shanghai and grew up during the epoch of Chinese Cultural Revolution. He was unable to continue his study after the first year of middle school. He studied mainly by himself to become a mathematician.