DESIGN AND ANALYSIS OF **EXPERIMENTS**

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DESIGN AND ANALYSIS OF EXPERIMENTS

Arjun K Gupta

Bowling Green State University, USA

D G Kabe

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

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ISBN 978-981-4522-53-3

Printed in Singapore

Dedication

For Samik
The Indo-Irish Significant Boy
AKG

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Preface

Design of experiments holds a central place in statistics. The aim of this book is to present in a readily accessible form certain theoretical results of this vast field. This is intended as a text book for a one-semester or two-quarter course for undergraduate seniors or first-year graduate students or as a resource of supplementary reading and references. Basic knowledge of algebra, calculus and statistical theory is required to master the techniques presented in this book. We believe that students and most teachers of design of experiments will find the book both interesting and useful.

The book contains 9 chapters. Chapter 1 provides the basic statistical tools that are used in the remaining chapters. It can be read first or can be used as reference for readers who can begin with Chapter 2. The main body of the material is presented in Chapters 2–7. Chapter 8 is different in the sense that it is pure mathematics and provides the elements of modern algebra which are used in Chapter 9. The last chapter discusses the construction of designs. The topics in a book as this always come from many sources. We are grateful to many teachers, students and authors who directly or indirectly have contributed to this book. Our special thanks go to our graduate students, Hu and Ngoc for their help in proofreading the manuscript and for suggesting many improvements.

The authors would like to acknowledge the support of the Bowling Green State University and the St. Mary's University. The sad demise of Professor D. G. Kabe during the preparation of this volume has delayed the completion of the book. We are

thankful to Ms. Lai Fun Kwong and the staff of World Scientific for their help and cooperation in bringing this book to publication. Finally, we would like to thank Lei Kou and Nathan Baxter whose skills at word processing made the preparation of this book far less painful than it might have been. However, any errors or omissions remain entirely the responsibility of the authors.

Arjun K. Gupta
D. G. Kabe

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