Fractional Statistics and Quantum Theory

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Fractional Statistics and Quantum Theory

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Dedicated to My Parents
Tai and Late Bhayasahel

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Preface

Sometime in early 1994, I received a letter from World Scientific, Singapore, inviting me to write a book on anyons. That made me think about the whole thing. Initially I was not sure if I should spend about two years of my research career in such a venture. At that time I went through the literature on this field and I had a careful look at the only monograph on this subject by Lerda as well as the book on anyon superconductivity edited by Wilczek. While I liked parts of these books (and their influence is apparent in parts of the book), I also felt that our perceptions are somewhat different. Having worked on both the non-relativistic as well as the relativistic anyon models, I was finally convinced that perhaps the time had come to accept this opportunity and put forward my point of view and hence this book.

As I started the planning of the book, I realized that a lot of work has been done in the last two decades in this field and it was not possible to cover everything in this monograph. I then decided that instead of pretending to be objective, it was better to include those topics which I consider important. However, I felt that I should at least give a brief description of the omitted topics and also give a couple of references for each of these topics so that the interested reader can follow the developments in these fields. I am somewhat lucky in that many of these topics have been adequately covered in the literature.

Even though so much work has been done in this field in the last two decades, I feel that this area is still in its early developmental stage and it is not completely clear as to what direction it will take in the future. This is because even the basic problem of the statistical mechanics of a non-interacting anyon gas is still an unsolved problem. I strongly believe that unless one can solve this basic problem, no qualitative progress is

possible in this field, since in the absence of this bench-mark study, any calculation including interactions will always be unreliable.

I am grateful to R.K. Patra who typed the first draft of the book, and to Umesh Salian, Sandip Joshi, and Koushik Ray for help with drawing figures and putting everything together properly. I also thank my daughter Anupama for a careful proof reading. Some of the chapters were written while I was visiting Los Alamos National Laboratory, Univ. of Oslo and Univ. Of Trondehim. It is my pleasure to thank Fred Cooper, Jon Magne Leinaas and Jan Myrheim for the kind invitations and for a warm hospitality.

It is a great pleasure to thank all my colleagues and collaborators who, over the years, have enhanced my knowledge of theoretical physics in general and anyons in particular. I would specially like to thank Surjyo Behera, Rajat Bhaduri, Alan Comtet, Fred Cooper, Trilochan Pradhan and Uday Sukhatme.

Further, I am extremely grateful to all the members of Institute of Physics, Bhubaneswar, and specially the three Directors, Trilochan Pradhan, V.S. Ramamurthy and Surjyo Behera who in the last twenty two years have given all the help and encouragement and most importantly provided a very stimulating atmosphere to carry out research.

I am grateful and indebted to my parents, Tai and late Bhayasaheb who in spite of difficult economic condition always encouraged us to look for intellectual pursuits and emphasized the need for hard work with single minded devotion. It is only because of their sacrifices that I am today what I am. Finally, I am grateful to my wife Pushpa who always set very high standards and encouraged and inspired me to try to achieve these standards.

Avinash Khare
Bhubaneswar, July, 1997

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