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QUANTUM DISSIPATIVE SYSTEMS

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To my family

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Preface

This book is an outgrowth of a series of lectures which I taught at the ICTP at Trieste and at the University of Stuttgart during the spring and summer of 1991. The purpose of my lectures was to present the approaches and techniques that accurately treat quantum processes in the presence of frictional influences.

The problem of *open* quantum systems has been around since the beginnings of quantum mechanics. Important contributions to this general area have been made by researchers working in fields as diverse as solid-state physics, chemical physics, biophysics, quantum measurement theory, quantum optics, nuclear and particle physics. Often, there has been used, and still is used, a language well known in one context or one field, yet sufficiently different from others that it is not altogether easy to make out the connection. Here, I offer a collection of ideas and examples rather than a comprehensive review of the topic and the history.

The central theme is the space-time functional integral or path integral formulation of quantum theory. This approach is particularly well suited for treating the quantum generalization of friction. Here we are faced to understand the behavior of a system with few quantal degrees of freedom coupled to a thermal reservoir. After integrating out the bath while keeping the system's coordinates fixed we get the influence functional describing the influence of the many bath degrees of freedom on the few relevant ones. This leads to an effective action weighting the paths of the open system in the functional integral description. Indeed, if one wishes to perform numerical computations on a rigorous level, there are no alternatives to this approach at present.

Path integration in condensed matter and chemical physics has become a growth industry in the last one or two decades. A newcomer to this thriving field may not yet be very familiar with the path integral method. Here, I do assume a knowledge of standard text book quantum mechanics and statistical mechanics augmented by a knowledge of Feynman's approach on a first introductory level. The books by Baym [1], Chandler [2], Feynman and Hibbs [3], and by Feynman [4] provide the elementary material in this regard. Further background and supplementary material on the path integral method are contained in the books by Schulman [5] and by Kleinert [6]. However, advanced mastery of these subjects is not necessary.

Some of the more sophisticated concepts, such as preparation functions, propagating functions, and correlation functions, are basic to the development as it is presented here. To cover this material at an introductory level, I make frequent use of simplified models. In this way, I can keep the mathematics relatively simple.

Many of the problems, methods, and ideas which are discussed here have become essential to the current understanding of quantum statistical mechanics. I

have made a considerable effort to make the material largely self-contained. Thus, although the theoretical tools are not developed systematically and in its full beauty, the material may be useful to many graduate students to become familiar with the field and learn the methods. For the most part, I refrain from just quoting results without explaining where they come from. With regard to citations, I have tried to give references to the historical development and also to provide a selection of the very recent important ones. But the list is surely not a comprehensive bibliography.

This book exists because of the physics I learned and enjoyed from the fertile collaboration with Hermann Grabert, Peter Hänggi, Gert-Ludwig Ingold, Peter Riseborough, and Maura Sassetti. I am particularly indebted to Maura who took time off her research to weed out points of confusion and who persistently encouraged me to finish this venture. I am also grateful to my students Reinhold Egger, Manfred Milch, Jürgen Stockburger, and Dietmar Weinmann for helpful comments concerning the presentation of many subjects discussed here and for preparing the figures.

In writing this book, I have benefited from the discussion with many companions; in particular Uli Eckern, Enrico Galleani d'Agliano, Anthony J. Leggett, Hajo Leschke, Franco Napoli, Albert Schmid, Gerd Schön, Larry Schulman, Peter Talkner, Valerio Tognetti, Andrei Zaikin, and Wilhelm Zwerger, who helped me in increasing my understanding of many of the subjects which are discussed here.

It is a pleasure to thank my teacher, colleague, and friend Wolfgang Weidlich for many fruitful discussions over the years.

Finally, and most importantly, I am deeply grateful to my wife Christel and my children Ulrike, Jan, and Meike for their infinite patience and omnipresent sympathy.

Stuttgart
October 1992

Ulrich Weiss

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