**LATTICE GAUGE THEORIES
AND
MONTE CARLO SIMULATIONS**

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Lattice Gauge Theories and Monte Carlo Simulations

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PREFACE

Quantized gauge theories play a central role in the description of particle interactions at a fundamental level. As for all quantum fields, quantized gauge systems must be regularized for a proper mathematical definition. Conventional schemes of regularization are based on a perturbative expansion: they can be used to derive a variety of theoretical predictions, but are not suitable for a study of those phenomena which are governed by a strong coupling constant or are of nonperturbative nature. The formulation of gauge theories on a space-time lattice was proposed by Wilson in 1974 to overcome the limitations of perturbative regularization. The lattice regularized quantum gauge theory is a very elegant mathematical construct, which does not rely on any weak coupling for its definition. In particular, strong coupling techniques can be applied and in the strong coupling regime sought for phenomena such as quark confinement are seen to occur. The regularization given by the lattice must, however, eventually be removed by letting the lattice spacing go to zero. In this process the coupling constant gets renormalized and results originally obtained for strong coupling may have to be extrapolated to weak coupling. While on the basis of strong coupling expansions alone this extrapolation may be problematic, information on the outcome of the renormalization procedure can be obtained exploiting another important property of lattice gauge models. These systems, which have many formal analogies with statistical systems used in the description of thermodynamical behavior, like their statistical counterparts can be investigated by numerical methods known as Monte Carlo simulations. Use of the lattice regularization in conjunction with Monte Carlo simulations has produced invaluable results during the last few years, especially for the gauge theory of strong interactions known as Quantum Chromodynamics.

The purpose of this book is to illustrate the lattice formulation of a gauge theory, the techniques which can be used to derive its *predictions, with particular emphasis on Monte Carlo simulations*, and the major results obtained. A collection of papers, selected so as to provide a global view of what has been accomplished in

the field, is presented to the reader. The selection is however not exhaustive of all important contributions: there are many more than could have fit into a volume of reasonable size. A few introductory chapters precede the reprints. Chapter 2, on the formulation of a quantum gauge theory, and Chapter 3, on the Monte Carlo method, have been written to provide the reader who is not familiar with the subject with basic notions about the topics considered in this book. Chapter 4 ought to serve as a guide through the collected papers. Also, additional references can be found in Chapter 4.

I wish to express my gratitude to Dr. Phua and WSPC for inviting me to edit this book, to my colleague Mike Creutz for assistance in the selection of the reprints, to Mrs. Isabell Harrity for her prompt and accurate typing of the manuscript and to the authors and publishers of the journals for their permission to reproduce the papers.

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