NONSTANDARD FINITE DIFFERENCE MODELS OF DIFFERENTIAL EQUATIONS

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NONSTANDARD FINITE DIFFERENCE MODELS OF DIFFERENTIAL EQUATIONS

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This book is dedicated
to my wife
Maria,
my son
James Williamson,
my daughter
Leah maria.

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Preface

This book was written in response to a large number of requests for copies of the author's papers on nonstandard finite difference schemes for the numerical integration of differential equations. The book provides a general summary of the methods used for the construction of such schemes. The major goal is to show that discrete (finite-difference) models exist for which the elementary types of numerical instabilities do not occur. The guiding philosophy behind this work is to get the qualitative details correct while not being overly concerned, at this level of the analysis, with the quantitative numerical results. (In any case, for most applications, the values of the various step-sizes are generally determined by the physical scales of the particular phenomena being studied.) The theoretical basis of our nonstandard discrete modeling methods is centered at the concepts of "exact" and "best" finite difference schemes. A set of rules is presented for constructing nonstandard finite difference schemes. The application of these rules often leads to an "essentially" unique finite difference model for a particular differential equation. It is expected that additional rules and restrictions will be discovered as research proceeds in this area.

An important feature of this book is the illustration of the various discrete modeling principles by their application to a large number of both ordinary and partial differential equations. The background requirements needed to fully understand the text are satisfied by the knowledge acquired in an introductory course on the numerical integration of differential equations.

I thank my many colleagues for their interest in my work. Again, I am particularly grateful to Ms. Annette Rohrs for typing the complete manuscript. Both she and Maria Mickens provided valuable editorial assistance. Finally, I thank the National Aeronautics and Space Administration for providing funds that allowed me to do research on nonstandard finite difference schemes.

Ronald E. Mickens
Atlanta, Georgia
August 1993

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