Canonical Quantization and Renormalization

YANG – MILLS THEORIES IN ALGEBRAIC NON-COVARIANT GAUGES

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PREFACE CUM INTRODUCTION

In recent years, gauge theories have been successfully applied to the description of all the fundamental interactions. The basic common feature of these theories is the requirement of the invariance of the Action under local phase transformations on the matter fields. This requirement, which is operatively realized by defining a covariant derivative, entails in a natural way the introduction of gauge bosons, which mediate the fundamental interaction under consideration. The transformations on the gauge bosons, which are necessary to ensure the invariance of the Action, are called gauge transformations and form a group. According to the group they belong to, we have different models for the fundamental interactions: electromagnetism ($U(1)$), quantum chromodynamics (color $SU(3)$), unified electro-weak interactions ($SU(2) \otimes U(1)$) and so forth. When the gauge group is a non-Abelian one, we have a Yang-Mills theory.

A peculiar feature of gauge theories is the redundancy of the degrees of freedom appearing in the Lagrangian, which turns out to be of a singular type: owing to the presence of constraints between fields and conjugate momenta, it is impossible to express all the velocities in terms of the canonical variables. As a consequence, a complete equivalence class of solutions exists. The choice of a representative element of the class (gauge choice) is a delicate matter and is usually done by imposing a condition on the gauge fields. A gauge choice will be more or less convenient depending on the problem under consideration.

Among the possible gauge choices, a great interest has been devoted to the so called algebraic non-covariant gauges. The main reasons are that, in these gauges

- i) the Faddeev-Popov sector is decoupled;
- ii) already at the classical level, it is possible to restrict the theory, at least in some particular cases, to the phase subspace containing only physical degrees of freedom, leading to an intuitive partonic interpretation of the theory at the quantum level;
- iii) no Gribov's ambiguities are present.

Algebraic non-covariant gauges are essentially of two types: axial and planar gauges. In both cases the manifest Lorentz covariance of the theory is broken by the introduction of a privileged direction in the Minkowski space-time, namely of a constant gauge vector $n_\mu = (n_0, \vec{n})$. In the axial choice one imposes the conditions $n^\mu A_\mu^a = 0$, A_μ^a being the gauge potentials, whereas in the planar gauge such quantities are set equal to suitably defined scalar fields Φ^a ; for this reason, the planar option is also called in the literature “inhomogeneous axial gauge”. All these choices do not fix the gauge completely; some arbitrariness on the potentials still remains, which is usually referred to as “residual gauge freedom”. In the present book we shall mainly confine ourselves to a perturbative framework; moreover, we will never consider theories with spontaneous symmetry breaking.

There are several ways of classifying algebraic non-covariant gauges; the most obvious one we have already mentioned, namely axial and planar choices. Another useful way is related to the sign of the square of the gauge vector: axial and planar temporal gauges ($n^2 > 0$), axial and planar spatial gauges ($n^2 < 0$) and the light-cone gauge ($n^2 = 0$). In the last case, the axial and planar choices collapse. We shall see that quite different canonical procedures are in order when quantizing the theory in those cases.

Upstream, there is a wider and subtler framework, based on the degree of violation of the manifest Lorentz covariance of the theory, in which the previous classifications can be settled. This introduction is essentially devoted to describe this framework.

A common feature of algebraic non-covariant gauges, is the presence of the so called spurious poles in the vector propagator $(nk)^{-\alpha}$, $\alpha = 1, 2$. The first problem one faces is to give a prescription for handling those singularities. Several options are possible: Cauchy principal value (CPV), advanced or retarded prescriptions and any linear combination thereof. All those prescription, for a given sign of n^2 , are related by residual gauge transformations on the gauge potentials and can be derived by means of a canonical quantization procedure in which one tries to single out, already at the classical level, a “minimal” phase space, namely one tries to eliminate as many as possible unphysical degrees of freedom. For instance, in the space-like axial gauge, people thought that it would have been possible to get rid of all redundant degrees of freedom and obtain a manifestly unitary theory; in this scheme, using the Dirac bracket formalism, the manifest Lorentz covariance of the theory can be recovered already at the classical level. However, all the mentioned options are affected by serious pathologies, that can be summarized in the following list

- 1) difficulties are present in the definition of the double spurious pole in the vector propagator (e.g. the square of the CPV distribution is meaningless from a mathematical point of view);
- 2) a related difficulty concerns the sum over the polarization vectors in the axial case, which turns out to be ill-defined;
- 3) always in the axial case, a perturbative evaluation of the Wilson loop does not exhibit its expected exponential behavior, when computed along a rectangular contour whose sides orthogonal to the gauge vector n_μ are sent to infinity;
- 4) in the light-cone gauge, inconsistencies appear in one-loop integral calculations: some of them are ill-defined even when dimensionally regularized;
- 5) when $n_0 \neq 0$, extra contributions arise from the Wick's rotation so that the power counting criterion for the convergence of Feynman integrals (and the related Weinberg theorem) does not naively apply;
- 6) it is not always possible to follow the Dirac procedure and recover, at the classical level, the "minimal" phase subspace;
- 7) there are difficulties in defining the vector propagator in the temporal gauge.

Some of these pathologies can be cured following some general rules for perturbative calculations in such a way that the procedure turns out to be consistent from a *mathematical* viewpoint. Two main points have to be kept in mind

- one has always to introduce the function ξ which adiabatically switches the interaction; the limit $\xi \rightarrow 1$ has to be performed only at the very end of the calculations;
- all the N -point Green's functions $G(k_1, \dots, k_N)$ have to be considered, recursively in the loop expansion, as distributions acting on a Besov space $\mathcal{T}$ of test functions suitably vanishing at $nk_i = 0$, $i = 1, \dots, N$, and then continued to the space L_2 after a suitable regularization, when needed.

This procedure allows us to systematically perform perturbative calculations in space-like gauges, and in particular to obtain the correct exponentiation of the Wilson loop in the planar case, up to the fourth order in the coupling constant. There is however no guarantee of full consistency from a *physical* point of view, as some singularities which are washed out by the use of the $\mathcal{T}$ space, might be physically relevant. This is likely to happen, for instance, in the axial case (see point 3) above).

As far as the light-cone gauge is concerned, it is not even clear whether this procedure can apply. To overcome this *impasse*, a new prescription for handling the

spurious singularities was proposed, in which the spurious poles turn out to have a position, in the complex k_0 -plane, coherent with the one of the Feynman poles, so that the Wick's rotation does not give rise to extra contributions. The price to be paid for getting this prescription is the introduction of a "conjugate" gauge vector $n_\mu^* = (n_0, -\vec{n})$, entailing a further violation of the manifest Lorentz covariance.

This prescription successively received a justification in the framework of canonical quantization; eventually, also the renormalization program was successfully performed at any order in the loop expansion, in spite of the conceptual and technical difficulties involved. As a matter of fact, we shall see, for instance, that the introduction of the vector n_μ^* entails the presence of divergent terms in dimensionally regularized subtracted one particle irreducible (1PI) vertices with a non-polynomial character with respect to the external momenta.

It is important to remark that a prescription leading to a second violation of the manifest Lorentz covariance originates from a canonical quantization scheme which necessarily contains unphysical ghost-like degrees of freedom. One has to abandon the manifest unitarity of the formulation, introducing a vector space of states with a metric which is not positive definite; the physical Hilbert subspace will be eventually selected by imposing suitable conditions.

Since this two-vector prescription has much milder analyticity properties, people were led to consider it also outside the light-cone, so that the axial and planar gauges could be viewed as continuous deformations of the light-cone gauge. These formulations are still at a tentative stage, in our opinion; as a matter of fact, there are doubtful points concerning canonical quantization we shall discuss in the relevant sections, whereas even to undertake a renormalization program in those schemes seems to be a formidable task, owing to the burden of polynomial and non-polynomial divergent tensorial structures appearing in the dimensionally regularized subtracted 1PI vertices.

The work is organized in three main parts, according to the degree of violation of the manifest Lorentz covariance which occurs in the different cases.

In Chapt. 1 covariant formulations (0-degree of violation) have been briefly recalled, essentially for the sake of completeness and in order to be able to perform comparisons in the successive chapters.

Chapt. 2 is devoted to the classical treatment of the algebraic non-covariant gauges (1-degree of violation) according to the Dirac brackets formalism. Boundary conditions and related residual gauge freedom are discussed.

In Chapt. 3 the theory is canonically quantized. Peculiar difficulties arising in the axial cases are carefully examined. The last part of the chapter is devoted to the proof of the decoupling of the Faddeev-Popov sector and of the absence of Gribov's ambiguities.

The renormalization in the space-like gauges is discussed in Chapt. 4, together with a careful mathematical procedure to handle higher loop calculations.

In Chapt. 5 the perturbative test of gauge invariance based on the asymptotic exponential behavior of the Wilson loop is performed for the space-like planar and axial choices. In particular, its failure in the axial case is carefully discussed.

Chapt. 6 is completely devoted to the light-cone gauge. After having given arguments showing the inconsistency of the one-vector prescription for the spurious singularities, the two-vector formulation (2-degree of violation) is fully analyzed in all its aspects: canonical quantization in the free and in the interacting cases, one-loop structures and the corresponding one-loop renormalization and, finally, the completion of the renormalization program at any order in the loop expansion.

Possible extensions of this two-vector formulation outside the light-cone are discussed in Chapt. 7, where the main open issues are also pointed out.

Technical developments are deferred to the Appendices.

As a concluding remark, we would like to emphasize that our work has not been conceived as a review of the existing literature on the subject, but, rather, it represents a personal (and hopefully coherent) point of view we have developed in the course of our activity in the field.

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NOTATIONS

We employ units such that $\hbar = c = 1$. Moreover, when not explicitly said, the following conventions on the symbols will be used

- points in the Minkowski space-time will be denoted by $x, y, z, \dots$, and the corresponding spatial components by $\vec{x}, \vec{y}, \vec{z}, \dots$;
- points in the momentum-space will be denoted by $p, q, k, \dots$, and the corresponding spatial components by $\vec{p}, \vec{q}, \vec{k}, \dots$;
- Greek indices $\mu, \nu, \rho, \dots$ will take the values 0, 1, 2, 3 and label the components of a given four-vector (or tensor); in the Euclidean space they will take the values 1, 2, 3, 4; Latin indices $i, j, k, \dots$ will take the values 1, 2, 3 and label the corresponding spatial components; Greek indices $\alpha, \beta, \gamma, \dots$ usually will label the 1, 2 components of a given four-vector (or tensor); such components will be also denoted by the subscript $\perp$;
- the metric tensor will be $g_{\mu\nu} = g^{\mu\nu} = \text{diag}\{+, -, -, -\}$;
- if not specified, we shall consider $SU(N)$ as gauge group; the indices $a, b, c, \dots$ will take the values $1, \dots, N^2 - 1$ (dimension of the group);
- the Einstein convention of sum over repeated indices will be always used;
- the superscripts $\dagger$ and $*$ will denote hermitian and complex conjugations, respectively;
- commutators and anticommutators will be denoted by square and curly brackets, respectively; sometimes, in order to compactify the notations, they will be also denoted collectively by square brackets with the subscript $\mp$;
- Poisson and Dirac brackets will be denoted by curly brackets; a subscript $\mathcal{D}$ will distinguish the Dirac brackets.