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COMPUTER TRANSFORMATION OF DIGITAL IMAGES AND PATTERNS

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獻給我們的雙親

*Kính dâng hương hồn Thầy
Me với hết lòng quý mến của con*

A Nos Parents

To Our Parents

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Preface

This book deals with the subject of geometric transformations of digitized images and patterns by computers to develop systematically various shape models and discrete techniques, in particular for picture processing and pattern recognition. Transformations discussed in this book include linear, quadratic, cubic, bilinear, bi-quadratic, bi-cubic, Coons models and other nonlinear forms such as harmonic and perspective transformations. Since they concern only the geometrical shapes and geometrical properties of the images or patterns, they are commonly known as geometric transformations, which can be used in computer graphics, character recognition, pattern recognition, computer vision and image processing.

Computer graphics is one of the most exciting and rapidly growing fields. As usual, displayed objects are defined by sets of coordinate points. Hence, geometric transformations are procedures for calculating new coordinate positions of these points, as required by a specified change in size and orientation of the object. Most books on computer graphics cover only the basic transformations such as translation, scaling, rotation, and reflection. To achieve more complicated forms and shapes, we need other kinds of geometric transformations and efficient algorithms to carry them out without creating unacceptable errors and deviations. Also efficient discrete techniques have been exploited in this book to facilitate digital images and patterns through the geometric transformations and to avoid the superfluous holes or "measles".

In pattern and character recognition, one of the most interesting and challenging problems is the recognition of characters and patterns which are subjected to "distortions". For instance, distortions result from handwritten characters or objects scanned at an oblique

angle. Since only the standard images are ready to be recognized by computers, distorted images require "normalization" using inverse transformations. Nevertheless, by using all existing approaches, inverse transformations are difficult to realize due to the necessity of solving nonlinear equations which often produce multiple solutions. In this book we will provide new efficient methods to carry out inverse transformations which bypass the need to solve nonlinear equations.

Since three dimensional (3D) computer vision requires the analysis of 2D perspective views, perspective transformations are also introduced to analyze inverse perspective transformations which are used to normalize 2D images.

In this book, we will present a number of useful transformation models in Chapters 2, 3 and 5, provide efficient discrete techniques for image conversion in Chapter 4, and apply both models and techniques to the recognition of geometrical shapes, Roman letters and Chinese characters in Chapter 6.

It is worth pointing out that the materials in Chapters 4 - 6 have been largely selected from our recent research work, on which detailed reports can be found in some papers published in conference proceedings and journals. Furthermore, the methodology in this book for image processing and pattern recognition is mainly based on classical mathematics (e.g., calculus) and numerical methods (e.g., approximate integration). Since the approaches and results represented are new, we provide rather detailed numerical and graphical examples to verify the analysis and to show the promising applications. We therefore hope that such a style would make this book vivid and readable, and that this book would stimulate further research in the related fields.

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in collaboration with

Tien Dai Bui

Yuan Yan Tang

Ching Y. Suen

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Abbreviations

- T : Transformation or shape model.
- T^{-1} : The inverse transformation or model of T .
- $T^{-1}T$: A cycle conversion of first the transformation T and then its inverse transformation T^{-1} .
- $CSIM$: The combined algorithm for $T^{-1}T$ by first using the splitting-shooting method in the T procedure, and second using the splitting-integrating method in the T^{-1} procedure.
- $CSSM$: The combined algorithm for $T^{-1}T$ by using the splitting-shooting method for both T and T^{-1} procedures.
- $CIIM$: The combined algorithm for $T^{-1}T$ by using the splitting-integrating method in both T and T^{-1} procedures.
- $Eq.$: Equation.
- $Eqs.$: Equations.
- $Fig.$: Figure.
- $Figs.$: Figures.
- $2D$: Two dimensions or two-dimensional.
- $3D$: Three dimensions or three-dimensional.

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