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**To my mother Shui-Tan Hsia Chen and
to the memory of my father Yu-Chao Chen**

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PREFACE

Since Bode published his classical text “Network Analysis and Feedback Amplifier Design” in 1945, very few books have been written that treat the subject in any reasonable depth. The purpose of this book is to bridge this gap by providing an in-depth, up-to-date, unified, and comprehensive treatment of the fundamentals of the theory of active networks and its applications to feedback amplifier design. The guiding light throughout has been to extract the essence of the theory and to discuss the topics that are of fundamental importance and that will transcend the advent of new devices and design tools. Intended primarily as a text in network theory in electrical engineering for first-year graduate students, the book is also suitable as a reference for researchers and practicing engineers in industry. In selecting the level of presentation, considerable attention has been given to the fact that many readers may be encountering some of these topics for the first time. Thus, basic introductory material has been included. The background required is the usual undergraduate basic courses in circuits and electronics as well as the ability to handle matrices.

The book can be conveniently divided into three parts. The first part, comprising the first three chapters, deals with general network analysis. The second part, composed of the next four chapters, is concerned with feedback amplifier theory. The third part, consisting of the last two chapters, discusses the state-space and topological analyses of active networks and their relations to feedback theory.

Chapter 1 introduces many fundamental concepts used in the study of linear active networks. We start by dealing with general n -port networks and define passivity in terms of the universally encountered physical quantities *time* and *energy*. We then translate the time-domain passivity criteria into the equivalent frequency-domain passivity conditions. Chapter 2 presents a useful description of the external behavior of a multiterminal network in terms of the indefinite-admittance matrix and demonstrates how it can be employed effectively for the computation of network functions. The significance of this approach is that the indefinite-admittance matrix can usually be written down directly from the network by inspection and that the transfer functions can be expressed compactly as the ratios of the first-and/or second-order cofactors of

the elements of the indefinite-admittance matrix. In Chapter 3 we consider the specialization of the general passivity condition for n -port networks in terms of the more immediately useful two-port parameters. We introduce various types of power gains, sensitivity, and the notion absolute stability as opposed to potential instability.

Chapters 4 and 5 are devoted to a study of single-loop feedback amplifiers. We begin the discussion by considering the conventional treatment of feedback amplifiers based on the ideal feedback model and analyzing several simple feedback networks. We then present in detail Bode's feedback theory, which is based on the concepts of return difference and null return difference. Bode's theory is formulated elegantly and compactly in terms of the first- and second-order cofactors of the elements of the indefinite-admittance matrix, and it is applicable to both simple and complicated networks, where the analysis by conventional method for the latter breaks down. We show that feedback may be employed to make the gain of an amplifier less sensitive to variations in the parameters of the active components, to control its transmission and driving-point properties, to reduce the effects of noise and nonlinear distortion, and to affect the stability or instability of the network. The fact that return difference can be measured experimentally for many practical amplifiers indicates that we can include all the parasitic effects in the stability study and that stability problems can be reduced to Nyquist plots.

The application of negative feedback in an amplifier improves its overall performance. However, we are faced with the stability problem in that, for sufficient amount of feedback, at some frequency the amplifier tends to oscillate and becomes unstable. Chapter 6 discusses various stability criteria and investigates several approaches to the stabilization of feedback amplifiers. The Nyquist stability criteria, the Bode plot, the root-locus technique, and root sensitivity are presented. The relationship between gain and phase shift and Bode's design theory is elaborated. Chapter 7 studies the multiple-loop feedback amplifiers that contain a multiplicity of inputs, outputs, and feedback loops. The concepts of return difference and null return difference for a single controlled source are now generalized to the notions of return difference matrix and null return difference matrix for a multiplicity of controlled sources. Likewise, the scalar sensitivity function is generalized to the sensitivity matrix, and formulas for computing multiparameter sensitivity functions are derived.

In Chapter 8, we formulate the network equations in the time domain as a system of first-order differential equations that govern the dynamic behavior of a network. The advantages of representing the network equations in this form are numerous. First of all, such a system has been widely studied in mathematics and its solution, both analytical and numerical, is known and readily available. Secondly, the representation can easily and naturally be extended to time-varying and nonlinear networks. In fact, nearly all time-varying and nonlinear networks are characterized by this approach. Finally, the first-order differential equations are easily programmed for a digital computer or simulated on an analog computer. We then formulate the general feedback theory in terms of the coefficient matrices of the state equations of a multiple-input, multiple-output and multiple-loop feedback amplifier, and derive expressions relating the zeros and poles of the determinants of the return difference matrix and the null return difference matrix to the eigenvalues of the coefficient matrices of the state

equations under certain conditions. Finally, in Chapter 9 we study topological analysis of active networks and conditions under which there is a unique solution. These conditions are especially useful in computer-aided network analysis when a numerical solution does not converge. They help distinguish those cases where a network does not possess a unique solution from those where the fault lies with the integration technique. Thus, when a numerical solution does not converge, it is important to distinguish network instability, divergence due to improper numerical integration, and divergence due to lack of the existence of a unique solution.

The book is an outgrowth of notes developed over the past twenty-five years while teaching courses on active network theory at the graduate level at Ohio University and University of Illinois at Chicago. There is little difficulty in fitting the book into a one-semester or two-quarter course in active network theory. For example, the first four chapters plus some sections of Chapters 5, 6 and 8 would be ideal for a one-semester course, whereas the entire book can be covered adequately in a two-quarter course.

A special feature of the book is that it bridges the gap between theory and practice, with abundant examples showing how theory solves problems. These examples are actual practical problems, not idealized illustrations of the theory. A rich variety of problems has been presented at the end of each chapter, some of which are routine applications of results derived in the text. Others, however, require considerable extension of the text material. In all there are 286 problems.

Much of the material in the book was developed from my research. It is a pleasure to acknowledge publicly the research support of the National Science Foundation and the University of Illinois at Chicago through the Senior University Scholar Program. I am indebted to many graduate students who have made valuable contributions to this book. Special thanks are due to my doctoral student Hui Tang, who helped proofread Chapters 8 and 9, and to my secretary, Ms. Barbara Wehner, who assisted me in preparing the index. Finally, I express my appreciation to my wife, Shiao-Ling, for her patience and understanding during the preparation of the book.

Wai-Kai Chen

Naperville, Illinois

January 1, 1991

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