QUANTUM MECHANICAL TUNNELLING AND ITS APPLICATIONS

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Quantum Mechanical Tunnelling And Its Applications

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PREFACE

The concept of Quantum Mechanical Tunnelling appeared to emerge as one of the immediate consequences of wave mechanics. Once, a propagating stream of material particles is represented by its wave function, according to the conventional analyses, the former is able to penetrate a reasonably thin barrier without any real tunnel existing across it even when its energy remains far smaller than the barrier height. Several physical phenomena e.g. alpha decay, high field electron emission from cold metals, auto-ionization of hydrogen atoms etc. were reported to be explained on the basis of the above-mentioned idea.

The study of Quantum Mechanical Tunnelling through solid-state junctions, however, became an area of intense research activity with the discoveries of Esaki and Josephson effects because of their innumerable applications. The inability of the conventional tunnelling theories to explain adequately the current flow through such junctions have exposed our limitations of truly understanding the physical basis of the process of quantum mechanical tunnelling itself. The primary aim of this work is to present a new theoretical account of the latter based on the principle of Quantum Measurement and Observation. The essence of this theory is that during tunnelling transition an electron acquires a state intermediate to that of a matter and a wave when it is physically impossible to assign its energy until the process of tunnelling is over. After having presented a brief review of the conventional tunnelling theories in Chapter I, the new tunnelling theory has been developed, in detail, in Chapter II where its basic parameters, such as the tunnelling probability, the tunnelling time and the tunnelling current density are derived so as to be applicable to real problems of interest. Chapters III and IV, on the other hand, review the physics of solid state devices involving rectangular and triangular barrier systems from the viewpoint of the new tunnelling theory. Chapter V, in the end, presents some of the topics whose adequate understanding is a prerequisite to proper appreciation of the contents of the text.

An earlier edition of the work entitled, "Tunnelling and Negative Resistance Phenomena in Semiconductors" appeared from Pergamon Press, Oxford, England in 1977. However, the difference of the present work from its predecessor should be mentioned at the outset. This edition contains an up-to-date account of our work on Tunnelling which was just initiated earlier and includes a derivation of Josephson Effect from the standpoint of the new theory. The discussion on negative resistance microwave devices has now been excluded. The emphasis is presently more on the development of physical principles of operations of solid-state devices rather than on the discussion of their technology of fabrication and circuit applications. An elementary understanding of wave mechanics and physics of semiconductor devices should enable the readers of the book to fully appreciate its contents. The book has primarily been intended to be a work of reference to post-graduate, doctoral and post-doctoral students along with research workers working in the area of solid-state devices. This essentially is an outcome of my teaching and research experiences at the Indian Institute of Technology, Delhi, India for nearly two decades. The efforts of my doctoral students viz. P. J. George, P. N. Roy, P. N. Singh, R. Y. Thakur, K. N. Rai, S. K. Ghosh, N. S. T. Sai, Amitabh Ghosh and G. Tiwary in contributing towards the development of some of the contents of this work are worth mentioning.

In the course of writing this book I have received assistance from several quarters. Firstly, I must acknowledge with thanks the authorities of the Rotary Foundation of the Rotary International for granting me a fellowship to visit AT&T Bell Laboratories at Murray Hill during 1986. Secondly, the authorities of the Indian Institute of Technology, Delhi deserve admiration for promptly granting me a sabbatical leave to come over to USA. Finally, I must thank the management of the Bell Laboratories for providing me a challenging environment and all the secretarial assistance necessary for the satisfactory completion of this work. Efficient typing of the manuscript by Anna H. Jacobsen is hereby being thankfully acknowledged. Thanks are also due to Federico Capasso, Gene Baraff and several other members of the Technical Staff of Bell Laboratories for a number of stimulating discussions that I had with them while writing this book. Special thanks are due to V. Narayanamurti, Director of Solid-State Electronics Research for

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D.K. Roy

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