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ELEMENTS OF QUANTUM MECHANICS OF INFINITE SYSTEMS

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PREFACE

These notes arose from lectures at the Scuola Normale Superiore (Pisa) and at the International School for Advanced Studies (Trieste) in 1977-78, 1981-82, 1982-83. The lectures were addressed to students of the last year of undergraduate studies or of the first year of graduate studies. The course was planned to discuss some of those basic features of quantum mechanics of systems with infinite degrees of freedom (QM_{∞}) like collective phenomena, spontaneous symmetry breaking, etc. that in the author's opinion should be part of the common education of every theoretical physics student. No pretention of completeness is made about the subject covered in these lectures. The present notes are only meant to serve as an introduction to the problems and results of QM_{∞} . The mathematical precision has been reduced to the minimum in order to communicate the main ideas to a larger audience, including people not mathematically minded. The hope is that once the basic structures are known each student may eventually implement the arguments by mathematical rigour without much difficulty, according to his taste, following for example the references as a guide.

The main motivation for writing down the lecture notes was to help the students who apparently found some difficulty in finding an accessible and compact exposition of the material in standard textbooks.

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Elements of Quantum Mechanics of Infinite Systems

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INTRODUCTION

In its original formulation, Quantum Mechanics (QM) was essentially dealing with systems with a finite number of degrees of freedom. Invented to explain the atomic levels, QM has been able to explain very different phenomena and its level of completeness and consistency, even from a rigorous point of view, is very satisfactory. There are, however, physical systems whose theoretical description requires infinite degrees of freedom and the treatment of quantum phenomena associated to them requires an extension of the original structure of QM. Such a formulation goes under the various names of QM of systems with infinite degrees of freedom, second quantization, field theory, many-body theory etc. according to the branches of physics to which it has been applied. Contrary to the case of ordinary QM, the QM of systems with infinite degrees of freedom (QM_∞) still presents unsolved questions of principle and its foundations are still the object of investigations and research. Most of what is known about that theory is based on the perturbative expansion and the recently proven triviality of the ϕ^4 theory (in four space-time dimensions) shows how misleading the perturbation expansion may be. Nevertheless, in many of the fields in which the ideas and methods of QM_∞ have been used, the success has been so remarkable (often beyond the theoretical expectations) that it is not unreasonable to regard it as a trustable theory, whose physical relevance cannot be denied.

The historical motivations for extending ordinary QM to systems with infinite degrees of freedom mainly came from the problem of combining QM and (special) Relativity. The need of describing interactions (or forces)

for relativistic systems no longer as "actions at a distance", but as contact actions, mediated by "fields", necessarily leads to considering systems with infinite degrees of freedom. The well-known classical example is the expansion of the electromagnetic field into infinite normal modes. High energy physics (in particular elementary particle physics) is therefore the physics of systems with infinite degrees of freedom and the theoretical quantum description of them goes under the name of quantum field theory. Its successful applications go beyond the realm of high energy physics, like hyperfine structure of atomic levels (Lamb shift), nuclear physics, etc.

The theoretical description of macroscopic systems in terms of forces between their "elementary" constituents (like atoms, electrons, molecules etc.) also involves systems with infinite degrees of freedom. The essential features of macroscopic bodies is that of consisting of a very large number N of particles and that their physical description in general involves "intensive" or "thermodynamical" properties of such systems (like particle density, mean energy per particle, etc.), i.e. properties for which effects of order $1/N$ and/or $1/V$ can be neglected, provided the density $n = N/V$ is kept finite. The above physical properties are therefore essentially the same as for the limiting situation of infinite degrees of freedom ($N \rightarrow \infty$) in an infinite volume ($V \rightarrow \infty$). This limiting case (also called thermodynamical limit) is at the basis of theories like Statistical Mechanics and Thermodynamics it not only leads to great technical simplifications, like the exploitation of Euclidean invariance, the neglect of surface or finite volume effects etc., but it also allows a simple treatment of concepts like temperature,

collective excitations, quasi particles, condensation etc., all of which involve the $N \rightarrow \infty$ limit and would hardly be treatable otherwise. The quantum treatment of such branch of physics goes under the name of Many Body Theory. The fields in which many body theory has been successfully applied run from nuclear physics to superconductivity, superfluidity, plasma physics, electron structure of metals, ferromagnetism, etc.

The above remarks should make it clear that QM_{∞} should be regarded as a very useful theory especially for its interdisciplinary aspects, as relevant for physics as ordinary QM. As mentioned before, many questions of principle for QM_{∞} are still unsolved, especially from a non-perturbative point of view, but the main features and structure of the theory are becoming more and more established. In particular important phenomena like collective phenomena, spontaneous symmetry breaking, which appear to play such a crucial role in modern theoretical physics, have no counterpart in ordinary QM. So that what was historically viewed as the origin of the many difficulties of QM_{∞} , in comparison with ordinary QM, is now emerging as the basis for (partly) unexpected and welcome structures, which appear as common features of very different physical phenomena.

The aim of these lectures is to emphasize those general features of QM_{∞} which have an interdisciplinary interest and which are strictly related to the foundations of the theory itself. As much as possible the discussion will not rely on perturbative expansions and non-perturbative effects will actually be emphasized.