A NONLINEAR DYNAMICS PERSPECTIVE OF WOLFRAM'S NEW KIND OF SCIENCE

Volume V

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Volume V

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To

Diana

On Our Fiftieth Wedding Anniversary

December 24, 2011

ODE TO 137

We have picked the number 137 as the cover inset to dramatize its universal significance. In the context of Cellular Automata, rule 137 is one of the four globally-equivalent Universal Turing Machines {137, 110, 124, 193} in one-dimensional cellular automata. We have opted for rule 137 over its much more well-known sibling, namely, rule 110, in view of its serendipitous relationships to several unrelated disciplines, including physics, biology, chaos and numerology.

In the context of physics, 137 is equal to the integer part of the inverse of the fine structure constant α : $1/\alpha = \hbar c/e^2 = 137.035999084\dots$

The fine structure constant α is the key to the physicist's quest for a Grand Unified Theory (GUT) of physics. Remarkably, it links electromagnetics, quantum mechanics, and relativity via the electron charge e (signature of electromagnetics), Planck's constant $\hbar$, the smallest measurement possible (signature of quantum mechanics), and the speed of light c , largest possible speed in vacuum (signature of relativity theory).

The number 137 determines the strength of the interaction between two electrons because the electron charge e occurs twice in the equation. In particular, 137 is equal to the ratio between the strong interaction (nuclear) force over the electromagnetic force. The fine structure constant α is also equal to the probability of an electron emitting, or absorbing, a single photon. Unlike other constants of nature (such as electron charge, speed of light, Planck's constant, electron mass, proton mass, gravitational constant, cosmological constant, etc.), the fine structure constant α is dimensionless in the sense that it is invariant regardless of the choice of units. Hence, the number 137 could be a message whereby a superior alien civilization from some distant galaxy can make contact with humans, or vice-versa! The fine structure constant is also "scale free" in the sense that if we double the value of $\hbar$, c , and e , we would still get the same value 137 (a signature of "fractals").

The number 137 has intrigued numerous prominent theoretical physicists:

- Max Born had written a paper in 1935 entitled "The mysterious number 137"
- Werner Heisenberg had famously said: "All the quandaries of quantum mechanics would shrivel up when 137 was finally explained."
- Richard Feynman had written "On the hand of God wrote that number 137, and we don't know how he pushed his pencil... It's one of the greatest damn mysteries of physics — a magic number that comes to us with no understanding by man... 137 has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it."

- Enrico Fermi deliberately wrote on the blackboard the incorrect equation $\alpha = h^2/ec$ as a parody, and has since been immortalized in a commemorative US postage stamp, inadvertently showing the error.
- Wolfgang Pauli was a great fan of 137 until the very end of his life — the hospital room in Zurich where Pauli died after an unsuccessful surgery bears the room number 137.
- Leon Lederman's home address is 137 Eola Road!

Chemists would be thrilled to learn that element 137 (affectionately dubbed the *Feynmanium* in honor of Feynman) is the last possible element of the Periodic Table because electrons captured by an element with more than 137 protons would move faster than light. It is also interesting to note that the chlorophyll molecules, which are the principal sites of light absorption in the light-dependent reactions of photosynthesis, is made of exactly 137 atoms.

The fine structure constant α has deep biological significance: If the numerical value of 137 were to differ from its value by a mere 4%, almost all oxygen and carbon would disappear from the universe and life as we knew it on earth would not exist.

The number 137 is also related to chaos via the serendipitous relationship (pointed out by A. Tamashevich)

$$137 \approx 2\pi\delta^2 = 136.982510\dots$$

where

$$\delta \approx 4.669201691\dots$$

is the Feigenbaum number.

Unlike many serendipitous examples related to the number 137, the one pertaining to cellular automata is a whole number. This is because 137 is the exact decimal equivalent of the three “firing” patterns

$$(000) = 0, \quad (011) = 3, \quad \text{and} \quad (111) = 7$$

They correspond to the three red vertices of the Boolean cube for rule 137, with weights equal to 1, 8 and 128, respectively. Adding these weights, we obtain the corresponding local rule number

$$137 = 2^0 + 2^3 + 2^7$$

As a mnemonic aid, one could either remember the three binary codes $\{0, 3, 7\}$ of the firing patterns, or the decimal rule number it encodes, namely 137.

Another mnemonic is to associate 137 with the age of the universe, namely, 13.7 billion years.

Numerologists would relish to learn that 137 is made of 3 prime numbers, and that it is a *strong prime* in the sense that it is more than the arithmetic mean of its two neighboring primes. The number 137 is also the 33rd prime number, as well as the fifth prime number p such that $19^{p-1} \equiv 1 \pmod{p^2}$. The number 137 is also the largest prime factor of the palindrome 123456787654321, as well as smallest possible value of the largest prime factor of $n^4 + 1$ for $n \geq 4$. It is also one of only two prime numbers p (the other being 73) with the property that any number of the form $abcdabcd$ is divisible by p . The number 137 is only one of six prime numbers (called Stern numbers) which cannot be written as $n = p + 2a^2$ for some prime number p and some number a .

Finally, we note that the reciprocal of 137 is equal to $0.007299270072992700\dots$ where the 8 integers “00729927” is repeated *ad infinitum*. Note also that the first 10 digits 0072992700 is a palindrome.

Historians would remember that $E = mc^2$ was first demonstrated experimentally in 1937. They might even recall that on October 13, 1307, King Philip had ordered the arrest of French Templars and pressured Pope Clement to issue the *Papal bull Pastoralis Praeeminentiae* on November 22, 1307 to arrest all Templars in Europe and seize their assets.

Finally, adepts of the philosophical system known as Gematria would recognize that 137 is equal to the sum of the numerical code of the 4 letters of the Hebrew word *Kabbalah*.

Yet another reason for choosing 137 over 110 as the prototype of a Universal Machine for one-dimensional cellular automata is its ease of training neurons with the desired firing patterns. Imagine a very, very long line of monkeys, each holding a blue flag (code for 0) with one hand, and a red flag (code for 1) with the other hand. Each monkey is trained to raise the red flag if, and only if, the flag color of

each monkey, and that of his left and right neighbors, coincides with one of the above three firing patterns. Otherwise, the monkey would raise the blue flag, during each clock cycle, signaled by a “clocking” monkey ringing a bell. It is quite incredible that the above line of monkeys would in fact constitute a Universal Turing Machine, capable of solving all problems that are solvable by any supercomputer currently existing, or in the future.

All told, we believe that it is much easier, and more motivating, to remember a number that has deep significance in numerous disciplines, than some obscure number that has no other relevance. Let us therefore close this foreword with the following terse ode to 137:

Bethe was mischievous with 137
Bohr was intrigued by 137
Born was mystified by 137
Fermi was frisky with 137
Feynman was mesmerized by 137
Heisenberg was fascinated by 137
Lederman was enchanted by 137
Pauli was consumed by 137
Turing was matched by 137

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