KNOTS AND PHYSICS

Fourth Edition

This page intentionally left blank

Series on Knots and Everything — Vol. 53

KNOTS

AND

PHYSICS

Fourth Edition

Louis H. Kauffman

University of Illinois, Chicago, USA

World Scientific

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

Series on Knots and Everything — Vol. 53

KNOTS AND PHYSICS (4th Edition)

Copyright © 2013 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

ISBN 978-981-4383-00-4

ISBN 978-981-4383-01-1 (pbk)

Printed in Singapore by B & Jo Enterprise Pte Ltd

To Diane

This page intentionally left blank

Preface to the First Edition

This book has its origins in two short courses given by the author in Bologna and Torino, Italy during the Fall of 1985. At that time, connections between statistical physics and the Jones polynomial were just beginning to appear, and it seemed to be a good idea to write a book of lecture notes entitled **Knots and Physics**.

The subject of knot polynomials was opening up, with the Jones polynomial as the first link polynomial able to distinguish knots from their mirror images. *We were looking at the tip of an iceberg!* The field has grown by leaps and bounds with remarkable contributions from mathematicians and physicists – a wonderful interdisciplinary interplay.

In writing this book I wanted to preserve the flavor of those old Bologna/Torino notes, and I wanted to provide a pathway into the more recent events. After a good deal of exploration, I decided, in 1989, to design a book divided into two parts. The first part would be combinatorial, elementary, devoted to the bracket polynomial as state model, partition function, vacuum-vacuum amplitude, Yang-Baxter model. The bracket also provides an entry point into the subject of quantum groups, and it is the beginning of a significant generalization of the Penrose spin-networks (see Part II, section 13⁰.) Part II is an exposition of a set of related topics, and provides room for recent developments. In its first incarnation, Part II held material on the Potts model and on spin-networks.

Part I grew to include expositions of Yang-Baxter models for the Homfly and Kauffman polynomials – as discovered by Jones and Turaev, and a treatment of the Alexander polynomial based on work of Francois Jaeger, Hubert Saleur and the author. By using Yang-Baxter models, we obtain an induction-free introduction to the existence of the Jones polynomial and its generalizations. Later, Part I grew some more and picked up a chapter on the 3-manifold invariants of Reshetikhin and Turaev as reformulated by Raymond Lickorish. The Lickorish model is completely elementary, using nothing but the bracket, trickery with link diagrams, and the tangle diagrammatic interpretation of the Temperley-Lieb algebra. These 3-manifold invariants were foretold by Edward Witten in his landmark paper on quantum field theory and the Jones polynomial. Part I ends with an introduction to Witten's functional integral formalism, and shows how the knot polynomials

arise in that context. An appendix computes the Yang-Baxter solutions for spin-preserving R -matrices in dimension two. From this place the Jones and Alexander polynomials appear as twins!

Part II begins with Bayman's theory of hitches – how to prove that your horse can not get away if you tie him with a well constructed clove hitch. Then follows a discussion of the experiments available with a rubber band. In sections 3⁰ and 4⁰ we discuss attempts to burrow beneath the usual Reidemeister moves for link diagrams. There are undiscovered realms beneath our feet. Then comes a discussion of the Penrose chromatic recursion (for colorations of planar three-valent graphs) and its generalizations. This provides an elementary entrance into spin networks, coloring and recoupling theory. Sections 7⁰ and 8⁰ on coloring and the Potts model are taken directly from the course in Torino. They show how the bracket model encompasses the dichromatic polynomial, the Jones polynomial and the Potts model. Section 9⁰ is a notebook on spin, quantum mechanics, and special relativity. Sections 10⁰ and 11⁰ play with the quaternions, Cayley numbers and the Dirac string trick. Section 11⁰ gives instructions for building your very own topological/mechanical quaternion demonstrator with nothing but scissors, paper and tape. Sections 12⁰ and 13⁰ discuss spin networks and q -deformed spin networks. The end of section 13⁰ outlines work of the author and Sostenes Lins, constructing 3-manifold invariants of Turaev-Viro type via q -spin networks. Part II ends with three essays: Strings, DNA, Lorenz Attractor. These parting essays contain numerous suggestions for further play.

Much is left unsaid. I would particularly like to have had the space to discuss Louis Crane's approach to defining Witten's invariants using conformal field theory, and Lee Smolin's approach to quantum gravity – where the states of the theory are functionals on knots and links. This will have to wait for the next time!

It gives me great pleasure to thank the following people for good conversation and intellectual sustenance over the years of this project.

Corrado Agnes	Steve Bryson
Jack Armel	Scott Carter
Randy Baadhio	Naomi Caspe
Gary Berkowitz	John Conway
Joan Birman	Paolo Cotta-Ramusino
Joe Birman	Nick Cozzarelli
Herbert Brun	Louis Crane
William Browder	Anne Dale

Tom Etter	Maurizio Martellini
Massimo Ferri	John Mathias
David Finkelstein	Ken Millett
Michael Fisher	John Milnor
James Flagg	Jose Montesinos
George Francis	Hugh Morton
Peter Freund	Kunio Murasugi
Larry Glasser	Jeanette Nelson
Seth Goldberg	Frank Nijhoff
Fred Goodman	Pierre Noyes
Arthur Greenspoon	Kathy O'Hara
Bernard Grossman	Eddie Oshins
Victor Guggenheim	Gordon Pask
Ivan Handler	Annetta Pedretti
John Hart	Mary-Minn Peet
Mark Hennings	Roger Penrose
Kim Hix	Mario Rasetti
Tom Imbo	Nicolai Reshetikhin
Francois Jaeger	Dennis Roseman
Herbert Jehle	Hubert Saleur
Vaughan Jones	Dan Sandin
Alice Kauffman	Jon Simon
Akio Kawauchi	Isadore Singer
Milton Kerker	Milton Singer
Doug Kipping	Diane Slaviero
Robion Kirby	Lee Smolin
Peter Landweber	David Solzman
Ruth Lawrence	George Spencer-Brown
Vladimir Lepetic	Sylvia Spengler
Anatoly Libgober	Joe Staley
Raymond Lickorish	Stan Tenen
Sostenes Lins	Tom Tieffenbacher
Roy Lisker	Volodja Turaev
Mary Lupa	Sasha Turbiner

David Urman	Randall Weiss
Jean-Michel Vappereau	Steve Winker
Rasa Varanka	Edward Witten
Francisco Varela	John Wood
Oleg Viro	Nancy Wood
Heinz von Foerster	David Yetter
Miki Wadati	

(As with all lists, this one is necessarily incomplete. Networks of communication and support span the globe.) I also thank the Institute des Hautes Études Scientifiques of Bures-Sur-Yvette, France for their hospitality during the academic year 1988-1989, when this book was begun.

Finally, I wish to thank Ms. Shirley Roper, Word Processing Supervisor in the Department of Mathematics, Statistics, and Computer Science at the University of Illinois at Chicago, for a great typesetting job and for endurance above and beyond the call of duty.

This project was partially supported by NSF Grant Number DMS-8822602 and the Program for Mathematics and Molecular Biology, University of California at Berkeley, Berkeley, California.

Chicago, Illinois, January 1991

Preface to the Second Edition

This second edition of *Knots and Physics* contains corrections to misprints that appeared in the first edition, plus an appendix that contains a discussion of graph invariants and Vassiliev invariants followed by a reprinting of four papers by the author. This discussion and the included papers constitute an update and extension of the material in the original edition of the book.

Chicago, Illinois
September 1993

This page intentionally left blank

Preface to the Third Edition

This third edition of *Knots and Physics* contains corrections to misprints in the second edition plus a new article in the appendix entitled “Knot Theory and Functional Integration”. This article provides an introduction to the relationships between Vassiliev invariants, Kontsevich integrals and Witten’s approach to link invariants via quantum field theory.

Chicago, Illinois and
Calais, France
November 15, 2000

This page intentionally left blank

Preface to the Fourth Edition

In this fourth edition of *Knots and Physics*, we have corrected typographical errors, rearranged the position of the index and we have added a new article on Virtual Knot Theory and Khovanov Homology. Both virtual knot theory and Khovanov homology are new developments in the theory of knots. Virtual knot theory was discovered by the author in 1996. It is a generalization of knot theory to a theory of knots in thickened surfaces taken up to changing these surfaces by adding or subtracting 1-handles in the complement of the knot or link. This three-dimensional theory has a diagrammatic theory that is exactly analogous to the corresponding theory for classical knots and links. In this article we give an introduction to virtual knot theory and we also give an introduction to Khovanov homology, Khovanov's great and startling generalization of the bracket state sum to a homology theory whose graded Euler characteristic is equal to the original bracket polynomial. With these developments we reach the physical limitations of this book about knots and physics. It will take the writing of another volume to give a full account of recent developments and their implications for physics, topology and mathematics.

Chicago, Illinois
May 18, 2012

This page intentionally left blank

Table of Contents

Preface to the First Edition	vii
Preface to the Second Edition	xi
Preface to the Third Edition.....	xiii
Preface to the Fourth Edition	xv

Part I. A Short Course of Knots and Physics

1. Physical Knots	4
2. Diagrams and Moves	8
3. States and the Bracket Polynomial	25
4. Alternating Links and Checkerboard Surfaces	39
5. The Jones Polynomial and its Generalizations	49
6. An Oriented State Model for $V_K(t)$	74
7. Braids and the Jones Polynomial	85
8. Abstract Tensors and the Yang-Baxter Equation	104
9. Formal Feynman Diagrams, Bracket as a Vacuum-Vacuum Expectation and the Quantum Group $SL(2)q$	117
10. The Form of the Universal R -matrix	148
11. Yang-Baxter Models for Specializations of the Homfly Polynomial ..	161
12. The Alexander Polynomial	174
13. Knot-Crystals – Classical Knot Theory in a Modern Guise	186
14. The Kauffman Polynomial	215
15. Oriented Models and Piecewise Linear Models	235
16. Three Manifold Invariants from the Jones Polynomial	250
17. Integral Heuristics and Witten's Invariants	285
18. Appendix – Solutions to the Yang-Baxter Equation	316

Part II. Knots and Physics — Miscellany

1. Theory of Hitches	323
2. The Rubber Band and Twisted Tube	329
3. On a Crossing	332
4. Slide Equivalence	336
5. Unoriented Diagrams and Linking Numbers	339
6. The Penrose Chromatic Recursion	346
7. The Chromatic Polynomial	353
8. The Potts Model and the Dichromatic Polynomial	364

9. Preliminaries for Quantum Mechanics, Spin Networks and Angular Momentum	381
10. Quaternions, Cayley Numbers and the Belt Trick	403
11. The Quaternion Demonstrator	427
12. The Penrose Theory of Spin Networks	443
13. Q -Spin Networks and the Magic Weave	459
14. Knots and Strings – Knotted Strings	475
15. DNA and Quantum Field Theory	488
16. Knots in Dynamical Systems – The Lorenz Attractor	501
Coda	511
References	513

Appendix

Introduction	533
Gauss Codes, Quantum Groups and Ribbon Hopf Algebras	543
Spin Networks, Topology and Discrete Physics	589
Link Polynomials and a Graphical Calculus	630
(<i>with P. Vogel</i>)	
Knots, Tangles, and Electrical Networks	676
(<i>with J. R. Goldman</i>)	
Knot Theory and Functional Integration	716
Introduction to Virtual Knot Theory and Khovanov Homology	763
Index	835