RELATIVITY

An Introduction to the Special Theory

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An Introduction to the Special Theory

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Dedicated to

my wife

RABIYA QADIR

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PREFACE

This book is based on a series of lectures I gave for M.Sc. students repeatedly over the years at the Department of Mathematics, Quaid-i-Azam University, Islamabad. These lectures evolved according to the needs of the students whom I taught. The background of the students was very different from that of Western students at a similar level. The problem was, keeping the lectures largely self-contained, to develop the background required for Special Relativity more or less from scratch. Of course, this development must be made in a finite time – one semester, to be precise. On the presumption that these problems are faced throughout the Third World, I decided to publish this book with a press that could make it available for the Third World.

It is hoped that this book will be useful, not only in the Third World, but everywhere. It is aimed at an audience making its first real contact with the Special Theory of Relativity. A background of matrices and vectors, of differential and integral calculus and the rudiments of group theory is assumed. Virtually no background in Physics is assumed except for some Classical Mechanics and a nodding acquaintance with the formalisms of Lagrange and Hamilton.

A word of explanation is in order as to why the General Theory of Relativity is not included *at all* in this volume. In Pakistan, this subject is dealt with separately in a one-semester course. The background required for it is more extensive. All things considered,

it did not seem useful for most readers to be forced to 'buy' the General Theory when they are only interested in the Special Theory. This is particularly true for students of Physics who intend to work on the experimental side, or even in branches of Theoretical Physics other than Particle Physics, Cosmology, Astrophysics or General Relativity itself. While General Relativity is steadily gaining importance in Physics, it is by no means as basic for it as the Special Theory is.

I have tried to maintain, throughout, a historical perspective of the development of the Theory of Relativity. This is done for two reasons. First, I believe that it helps to build an interest in the subject and give credit where it is due. Second, I believe that it helps to give a better 'feel' for the concepts on which the theory is based. Many nuances are 'lost in non-translation' in carrying forward earlier terms out of their original context.

Finally, I would like to record my indebtedness to my late father, Mr. Manzur Qadir, who introduced me to the pleasures of Relativity; to my Ph.D. supervisor, Professor Roger Penrose, from whom I learnt precision in thinking (particularly in the field of Relativity); to Professor John Archibald Wheeler, who brought home to me the importance of clear and attractive presentation of ideas; and to my numerous students on whom I experimented in an effort to find the best method of teaching Relativity to students with the background available in Pakistan. Thanks are also due to many colleagues and students who refused to 'see the Emperor's new clothes' till they were put on. Of course, my gratitude goes to my family who were neglected because of this work and in particular to my wife, Rabiya Qadir, for giving me continual support and encouragement in writing this book. Finally, I wish to thank Mr. Shabahat Ullah Khan for his excellent typing of the manuscript.

ASGHAR QADIR

TABLE OF CONTENTS

CHAPTER 1 — INTRODUCTION

1. Historical Background of Motion	1
2. The Measurement of Time	6
3. Classical Motion	9
4. Pre-Relativistic Mechanics	13
5. A Digression on Scientific Method	18
Exercise 1	19

CHAPTER 2 — DERIVATION OF SPECIAL RELATIVITY

1. Einstein's Formulation of Special Relativity	21
2. Reformulation of Relativity	25
3. Length Contraction, Time Dilation and Simultaneity	26
4. The Velocity Addition Formulae	30
5. Three Dimensional Lorentz Transformations	31
Exercise 2	34

CHAPTER 3 — DIGRESSION INTO TENSOR THEORY

1. Invariant Quantities	39
2. Tensors	43

3. Coordinate Transformations	46
Exercise 3	51
 CHAPTER 4 — THE FOUR-VECTOR FORMULATION OF SPECIAL RELATIVITY	
1. The Four-Vector Formalism	53
2. The Lorentz Transformations in 4-Vectors	55
3. The Lorentz and Poincaré Groups	59
4. The Null Cone Structure	62
5. The Search for Absolutes – Proper Time	65
Exercise 4	67
 CHAPTER 5 — APPLICATIONS OF SPECIAL RELATIVITY	
1. Relativistic Kinematics	69
2. The Doppler Shift in Relativity	73
3. The Compton Effect	77
4. Particle Scattering	80
5. Binding Energy, Particle Production and Particle Decay	83
Exercise 5	89
 CHAPTER 6 — ELECTROMAGNETISM IN SPECIAL RELATIVITY	
1. Review of Electromagnetism	93
2. The Electric and Magnetic Field Intensities	94
3. The Electric Current	96
4. Maxwell's Equations and Electromagnetic Waves	98
5. The Four-Vector Formulation	99
6. The Maxwell Equations Again	103
Exercise 6	105

CHAPTER 7 — SPECIAL RELATIVITY WITH SMALL ACCELERATIONS

1. Some 'Paradoxes' in Special Relativity	107
2. The Range of Validity of Special Relativity	111
3. The Gravitational Red-Shift	112
4. Gravitational Deflection of Light	113
5. Four-Vector Acceleration and Force	115
6. Restatement of the Principle of Special Relativity	119
7. Change of Metric Due to Acceleration	119
Exercise 7	122

INDEX	125
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