NONLINEAR LIAPUNOV DYNAMICS

Also by J M Skowronski

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NONLINEAR LIAPUNOV DYNAMICS

■ J M Skowronski ■

*University of Southern California.
Los Angeles, California
and
University of Queensland
St. Lucia, Australia*

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PREFACE

Realistic models of the physical world are nonlinear, involving large amplitudes of motion and thus usually several equilibria of the system concerned. Hence in the majority of applications nonlinearities may not be truncated without seriously effecting the adequacy of the model. In spite of this, nonlinear dynamics has remained without much interface with applications for a long time. The reason for this seems to be at least two-fold. First, there was lack of practical methods implementing the nonlinear results in everyday applied dynamics. Second, the massive investment of time and resources in the linearized techniques generated a potent disincentive for the change, even at the expense of accurate modelling. The first aspect lost its ground with the arrival of fast computers, but the second still persists. Indeed, changes in the attitudes of people are always slower than changes in technology. There is, however, good news. Not only is nonlinear dynamics stimulated by computers, but the converse occurs as well. The nonlinear techniques speed up computation by identifying the types of dynamic trajectories concerned and, perhaps even more importantly, by helping to check the results. Furthermore, the Liapunov formalism, fundamental for nonlinear dynamics, becomes an essential part of the theory of parallel computing and neural networks. The latter are presently successfully replacing artificial intelligence and expert systems in most progressive applications, and thus generating considerable demand for Liapunov type algorithms and for nonlinear dynamics in general.

The book gives the Liapunov background for the analysis and synthesis (design) of dynamic behaviour of general networks which represent

a large class of nonlinear systems, predominantly physical, and in particular mechanical. It is meant to be a self-learning and thought provoking reference text. It has grown from junior graduate level lectures in Engineering Science and Applied Mathematics over a number of years at several universities (Notre Dame, Ottawa, Queensland, Alberta, Southern California) and circulated for a while in the format of lecture notes. The first, introductory chapter refers to the basic concepts of static characteristics and dynamic processes. The second and third describe various formats of dynamic models and give a reference frame for their behaviour. The fourth chapter introduces basic energy relations, fundamental to the dynamic use of the Liapunov method. The method itself is described in chapter five, with implementations in chapters six and seven. The methods of Liapunov Design (synthesis) and Control in chapter eight close the text.

The prerequisite background is not above elementary analytic dynamics (mechanics) and differential equations. Readers not interested in the systems interpretation of dynamics may leave out the first chapter without consequence to later reading. On the other hand chapters four and five are fundamental to the whole text. The material is slightly longer than for a semester course, giving the instructor the option of cutting off some sections which are less useful to his particular group of students.

The author is indebted to his wife Elzbieta and children, Joanna and Michal for their patience and help while writing this text.

Los Angeles, January 1990

J.M. Skowronski

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