Multiscale Problems

Theory, Numerical Approximation
and Applications

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Theory, Numerical Approximation
and Applications

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Preface

The ISFMA Symposium “Multiscale Problems: Theory, Numerical Approximation and Applications” was held in May, 4–16, 2009 at Fudan University in Shanghai. Its aim was to introduce graduate students and post-doctors to the newest developments related to the analysis of problems in which several scales are presented.

This volume gathers the notes corresponding to the lectures given by the following professors: Alain Damlamian (Laboratoire d’Analyse et Mathématiques Appliquées, Université Paris-Est, Paris12-Val de Marne, France), Gabriel Nguetseng (Department of Mathematics, University of Yaounde 1, Cameroon), Georges Griso (Laboratoire d’Analyse Numérique J.L. Lions, Université P. et M. Curie, Paris, France), Patrizia Donato (Université de Rouen, St Etienne du Rouvray, France), Dominique Blanchard (Université de Rouen, Saint Etienne du Rouvray, France), Bernadette Miara (Département de Modélisation et Simulation Numérique, Université Paris-Est, Ecole Supérieure d’Ingénieurs en Electronique et Electrotechnique, Noisy-le-Grand, France) and Assyr Abdulle (Section of Mathematics, Swiss Federal Institute of Technology, Switzerland).

The contributions listed below cover a wide range of topics in theory, numerical approximation with finite elements and applications in the fields of elasticity and fluid mechanics.

Chapter 1, by Alain Damlamian, presents a general introduction to the theory of homogenization in the periodic case.

Chapter 2, by Alain Damlamian, is on the periodic unfolding method, a very efficient and recent method for periodic homogenization.

Chapter 3, by Gabriel Nguetseng and Lazarus Signing, presents the homogenization of the stationary Navier-Stokes equations in fixed or variable domains occupied by porous media.

Chapter 4, by Patricia Donato, concerns the homogenization of a class of imperfect transmission problems.

Chapters 5 and 6, by Georges Griso, present a new approach to the decomposition of displacements in the case of thin structures (chapter 5) and rods with applications to the asymptotic behavior of nonlinear elastic rods (chapter 6).

Chapter 7, by Dominique Blanchard, concerns the elastic behavior for the junction of a periodic family of rods with a plate as the limit of a 3D elastic body.

Chapter 8, by Bernadette Miara, presents the theory and numerical simulation for the multi-scale modelling of new composites.

Chapter 9, by Assyr Abdulle, is a presentation and analysis of heterogeneous multiscale finite element methods (HMFEM); the a priori and a posteriori analysis of such numerical methods is investigated and a general framework to perform such analysis is given.

We are very grateful to our colleagues who carefully prepared the paper version of their talks. It is also our pleasure to thank Ms. Zhou Chunlian whose kindness and professionalism guaranteed the success of this Symposium.

We thank the Mathematical Center of Ministry of Education of China, the National Natural Science Foundation of China, the School of Mathematical Sciences, Fudan University, Shanghai Key Laboratory for Contemporary Applied Mathematics, the Research Center of Scientific Computing and Engineering and the Nonlinear Mathematical Modeling and Methods Laboratory, which sponsored the Symposium.

Finally, the editors would like to express their gratitude to Fudan University and the “Institut Sino-Français de Mathématiques Appliquées” (ISFMA) for their help and support.

Alain Damlamian, Bernadette Miara, Tatsien Li

January 2011

Contents

Preface

Alain Damlamian

An Introduction to Periodic Homogenization 1

Alain Damlamian

The Periodic Unfolding Method in Homogenization 28

Gabriel Nguetseng and Lazarus Signing

Deterministic Homogenization of Stationary Navier-Stokes
Type Equations 70

Patricia Donato

Homogenization of a Class of Imperfect Transmission Problems .. 109

Georges Griso

Decompositions of Displacements of Thin Structures 148

Georges Griso

Decomposition of Rods Deformations. Asymptotic Behavior of
Nonlinear Elastic Rods 171

Dominique Blanchard

Junction of a Periodic Family of Rods with a Plate
in Elasticity 198

Bernadette Miara

Multi-scale Modelling of New Composites: Theory and
Numerical Simulation 234

Assyr Abdulle

A Priori and a Posteriori Error Analysis for Numerical
Homogenization: A Unified Framework 280