NONSYMMETRIC FIELDS THEORY AND ITS APPLICATIONS

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TO MY PARENTS

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FOREWORD

We present the Nonsymmetric Kaluza-Klein and Jordan-Thiry Theories as interesting propositions of physics in higher dimensions. We consider separately 5-dimensional (electromagnetic) and nonabelian cases.

Part 3 of the work is devoted to a five-dimensional unification of NGT (Nonsymmetric Theory of Gravitation), electromagnetism and scalar forces in a Jordan-Thiry manner. We found "interference effects" between gravitational and electromagnetic fields which appear to be due to the skewsymmetric part of the metric. Our unification called the Nonsymmetric Jordan-Thiry Theory becomes the classical Jordan-Thiry Theory if the skewsymmetric part of the metric is zero. It becomes a classical Kaluza-Klein Theory if the scalar field $\rho = 1$ (Kaluza's Ansatz).

We also deal with material sources in the Nonsymmetric Kaluza-Klein Theory for an electromagnetic case. We consider phenomenological sources with a non-zero fermion current, a non-zero electric current and a non-zero spin density tensor. From the Palatini variational principle we find equations for the gravitational and electromagnetic fields. We also consider the geodetic equations in the theory and the equation of motion for charged test particles.

We consider some numerical predictions of the Nonsymmetric Kaluza-Klein Theory with non-zero (and with zero) material sources. We prove that they do not contradict any experimental data in the solar system and on the surface of a neutron star. We deal also with spin sources in the Nonsymmetric Kaluza-Klein Theory.

We find an exact, static, spherically symmetric solution in the Nonsymmetric Kaluza-Klein Theory in the electromagnetic case. This solution has the remarkable property of describing "mass without mass" and "charge without charge". We examine its properties and a physical interpretation.

Part 4 of this work is devoted to a $(n+4)$ -dimensional unification of NGT and Yang-Mills' theory in a Jordan-Thiry manner. We found "interference effects" between gravitational and Yang-Mills' fields which appear to be due to the skewsymmetric part of the metric on the $(n+4)$ -dimensional manifold (nonsymmetrically metrized principal fibre bundle). Our unification, called the Nonsymmetric-Nonabelian Jordan-Thiry Theory, becomes classical if the skewsymmetric part of the metric is zero.

We find the electromagnetic and Yang-Mills' field

lagrangians up to the second order of approximation in $h_{\mu\nu} = g_{\mu\nu} - h_{\mu\nu}$. We prove that in the electromagnetic case there is no skewon-photon interaction up to the first order of approximation.

We also deal with the lagrangian for the scalar field (connected to the "gravitational constant"). We prove that in the zeroth and first orders of approximation in the electromagnetic case the lagrangian vanishes. We consider the spin content of the theory and a relationship between the cosmological constant and the coupling constant between the skewon field and the gauge field in the first order of approximation.

We show how to derive a dielectric model of a confinement from "interference effects" in these theories. We underline some similarities between the Nonsymmetric Jordan-Thiry lagrangian in the flat space limit and the soliton bag model lagrangian.

In the 5th part of the work we construct the Nonsymmetric Jordan-Thiry Theory unifying NGT, the Yang-Mills' field, the Higgs' field, and scalar forces in a geometric manner. We discuss spontaneous symmetry breaking, the Higgs' mechanism and mass generation in the theory. The scalar field (as in classical Jordan-Thiry Theory) is connected to the effective gravitational constant. This field is massive and has Yukawa-type behaviour. We discuss the relation between R_+ invariance and $U(1)_F$ from GUT within Einstein λ -transformation, and fermion number conservation. In this way we connect $\bar{W}_\mu$ -field from nonsymmetric theory of gravitation with a gauge field $\tilde{A}_\mu^F$ from GUT. We derive the equation of motion for a test particle from conservation laws in the hydrodynamic limit. We consider an approximation procedure for the lagrangian and the geodetic equations. We deal also with a tower of scalar fields appearing in the theory.

Summing up we proceed in three stages:

- 1^o. Electromagnetic (abelian) case
- 2^o. General Nonabelian (massless Yang-Mills' field) case
- 3^o. General nonabelian case with Higgs' mechanism and spontaneous symmetry breaking (massless and massive Yang-Mills' case)

to construct a Nonsymmetric Kaluza-Klein (Jordan-Thiry) Theory and to find its consequences.

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