CHAOS

IN SYSTEMS WITH NOISE

Second Edition

Tomasz Kapitaniak

Technical University of Lodz
Lodz, Poland

World Scientific

Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 9128

USA office: 687 Hartwell Street, Teaneck, NJ 07666

UK office: 73 Lynton Mead, Totteridge, London N20 8DH

Library of Congress Cataloging-in-Publication data is available.

CHAOS IN SYSTEMS WITH NOISE – SECOND EDITION

Copyright © 1990 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

ISBN 981-02-0409-4

981-02-0410-8 (pbk)

Printed in Singapore by JBW Printers & Binders Pte. Ltd.

To

MARTIN

my little son

an expert in making

noise and chaos

This page is intentionally left blank

PREFACE

In this book the influence of the random noise on the chaotic behaviour of dissipative dynamical systems is investigated. We generally do not deal with discrete - time dynamical systems e.g., iterative maps, but we consider continuous - time dynamics. The problems presented here are illustrated by some mechanical examples.

The main part of this book is based on author's post-doctoral thesis 'Chaotic stochastic processes in nonlinear dynamics' which has been presented at the Technical University of Lodz in 1988, but it also presents the results which are the combined achievements of many investigators.

The author is very grateful to Prof. W.-H. Steeb of Rand Afrikaans University in Johannesburg and Prof. A. Tylikowski of Warsaw Technical University for their suggestions during the discussion of many problems treated in this book.

PREFACE TO THE SECOND EDITION

This is a revised and updated version of the first edition, to which new sections on the summary of probability theory, homoclinic chaos, Melnikov method, routes to chaos, stabilization of period-doubling and Hopf bifurcation by noise and new examples have been added. Also the chapters on Poincare maps, Lyapunov exponents and probability density functions have been rewritten.

I would like to acknowledge the discussion remarks of M.S. El Naschie, J.C. Antoranz, J. Brindley, H. Nusse and again W.-H. Steeb which helped me in the preparation of the second edition.

I also wish to thank my wife Margaret for copyediting the book. This work forced her to be as deterministic and predictable as possible having around two very chaotic men.

Finally, I wish to thank the King Abdul Aziz City for Science and Technology, Riyadh, Kingdom of Saudi Arabia for support of my research program.

CONTENTS

Preface	vii
Preface to the second edition	viii
Chapter 1 Introduction	1
Chapter 2 Probability Theory - Brief Summary	5
2.1 Random Variable	5
2.2 Several Random Variables	8
2.3 Stochastic Processes	11
Chapter 3 Chaotic and Stochastic Processes	19
Chapter 4 Noisy Dynamical Systems	25
Chapter 5 Fokker-Planck-Kolmogorov Equation	33
Chapter 6 Multi-Maxima Probability Density Functions	42
6.1 The Stochastic Process with Bifurcation	42
6.2 The Relation Between Stochastic Process with Bifurcation and Deterministic Bifurcation	44
6.2.1 Non-linear Pendulum Perturbed by White Noise	44
6.2.2 Duffing's Equation Perturbed by Large Narrow-Band Noise	48
Chapter 7 Random Lyapunov Exponents	53
7.1 Deterministic Lyapunov Exponents	53
7.2 Lyapunov Exponents for Noisy Systems	61

Chapter 8	Poincare Maps for Noisy Systems	65
	8.1 Definition of Poincare Map	65
	8.2 Mean Poincare Maps	71
Chapter 9	Regular and Chaotic Stochastic Processes	76
	9.1 Definitions	76
	9.2 Properties of the Probability Density Function of the Noisy Chaotic Response	79
	9.3 The Loss of Chaos	87
Chapter 10	Stochastic Sensitivity Functions and Chaos	95
Chapter 11	Homoclinic Chaos Perturbed by Noise	99
	11.1 Stable and Unstable Manifolds	99
	11.2 Averaging Theorem	101
	11.3 Melnikov Method	103
	11.4 Melnikov Method for Noisy Systems	111
Chapter 12	Examples	117
	12.1 A Simple Mechanical System with Stress Relaxation	117
	12.2 Non-Linear Oscillator with Quasi-Periodic Excitation	126
	12.3 Chaotic Oscillations of the Buckled Beam	129
	12.4 Noisy Chaotic Movement of a Simple Mechanical Machine	135
	12.5 The Dynamical Elastica	141
	12.6 Instability Waves in an Elastic Structure	150
Chapter 13	Noisy Routes to Chaos	154
	13.1 Typical Routes to Chaos	154

13.2 The Influence of Noise on Period-Doubling	161
13.3 The Influence of Noise on Hopf Bifurcation	166
13.4 The Influence of Noise on the Logistic Map	177
13.5 Noisy Route to Chaos in Non-Linear Oscillator with Quasi-Periodic Excitation	179
13.5.1 The Deterministic Case	179
13.5.1 The Stochastic Case	195
13.6 Feedback Control System with Time Delay	201
13.7 Stabilization of Period-Doubling Bifurcation and the Loss of Chaos	205
References	219