Asymptotic Symmetry and its Implication in Elementary Particle Physics

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S. Oneda

University of Maryland

Y. Koide

University of Shizuoka

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**ASYMPTOTIC SYMMETRY AND ITS IMPLICATION
IN ELEMENTARY PARTICLE PHYSICS**

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Preface

In this book, we describe in some detail a rather unique algebraic approach to elementary particle physics which has been developed by the authors and their collaborators in the past two decades.

In elementary particle physics, there are a number of well recognized underlying symmetries which correctly predict the spectacular multiplet structures of observable particles. However, lack of a consistent method to deal with badly broken symmetries has considerably hindered us from investigating the dynamics of particle physics from symmetry point of view. Group theoretical method is usually based on the notion of exact symmetry.

With this book, however, we wish to arouse interest in this not well addressed important problem, by describing our point of view, method and its consequences in comparison with experiment.

We describe a formalism in which we deal directly only with physical particles but not virtual particles. We argue that the spectrum generating symmetries, though broken, still maintain their original linearities in a particular way, called asymptotic symmetry, which is, however, valid only for the physical particle multiplet with infinite momentum.

When combined with the presence of the equal-time commutation relations involving the generators of underlying symmetry group, which are still valid in broken symmetry, asymptotic symmetry then immediately produces constraints on physical particle spectrum, displaying a close interplay among the masses, mixing parameters and coupling constants. This is the salient feature of the approach.

In this book we pursue the scenario described above without using the Feynman diagrams which involve virtual particles. Therefore, the method is in principle non-perturbative and deals directly with observable particles. We nevertheless show that we can reproduce the successful result of naive quark model and improve the shortcoming of the non-relativistic aspects of the model. We can also trace the origin of the observed remarkable mass and mass-mixing-angle relations in QCD and electroweak theory. We also derive many of the successful results of old $SU(6)$ symmetry without assuming $SU(6)$ symmetry and simultaneously recognize the presence of the dynamical selection rules observed within the theoretical frame-

work, without imposing them from outside. We can thus essentially count quarks and anti-quarks algebraically, without really seeing them.

We are beginning to notice that one can approach particle spectroscopy also from a point of view of asymptotic $SU(2)_L \times U(1)_Y \times SU(3)_{family}$ symmetry. The method may even give us a powerful tool for the study of new physics where fundamental Lagrangian is not yet known.

This book is thus written with a spirit that in the quest of unknown, one should not be confined too rigidly to traditional thinking or method and there might be a way to approach broken symmetry in a non-perturbative way. Otherwise, the beautiful group theory remains to be a mere mathematical device to classify multiplicity in the symmetry limit. As will often be mentioned in the book, many difficult problems remain, of course, unresolved or untouched and await for fresh ideas and innovations of the method.

The book was planned to be readable for the physicists who know the basics of the concepts of quantum field theory. In Chapter 2, we give for non-experts a quick but hopefully sufficient review of the present picture of elementary particle physics and lay out the problems which have given motivations for the approach pursued in the later chapters of this book.

The project which started near the end of the sixties has evolved, along with the amazing experimental and theoretical development of elementary particle physics, gradually into a more ambitious undertaking, thanks to the efforts of many active collaborators and supports.

One of the authors (Sadao Oneda) owes his first training and introduction to the exciting world of particle physics to his outstanding teacher and dedicated physicist, Professor Shoji Ozaki.

Enthusiastic collaboration and pioneering effort of Dr. Seisaku Matsuda in the early stage was indispensable and is greatly appreciated. Professor Hiroomi Umezawa has always been a mentor and a collaborator on the pursuit of the idea related to asymptotic symmetry.

Many former students of S. Oneda have completed their Ph.D. thesis at the University of Maryland and many visitors to the University have collaborated with S. Oneda on various aspects and phases of the development of the present approach. Many of their contributions are included in the book. He also thanks his old and new colleagues at the University of Maryland for discussions, advice and, above

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Sadao Oneda

Yoshio Koide

April, 1991

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Notations and Conventions

Metric and Related Definitions

We denote the contravariant components of the four-dimensional coordinates x^μ as

$$x^\mu = (x^0, x^1, x^2, x^3) = (t, x, y, z) = (t, \mathbf{r}) , \quad (0.1)$$

and the covariant components x_μ as

$$x_\mu = g_{\mu\nu} x^\nu = (t, -\mathbf{r}) , \quad (0.2)$$

where

$$g_{00} = 1 , \quad g_{ij} = -\delta_{ij} , \quad g_{0i} = 0 \quad (i, j = 1, 2, 3) . \quad (0.3)$$

The gradient operators are expressed as

$$\partial^\mu \equiv \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial t}, -\nabla \right) , \quad (0.4)$$

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, \nabla \right) , \quad (0.5)$$

where

$$\nabla \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) . \quad (0.6)$$

The scalar product of the two Lorentz four-vectors A^μ and B^μ is expressed as

$$(AB) = A_\mu B^\mu = A_0 B_0 - \mathbf{A} \mathbf{B} . \quad (0.7)$$

The Levi-Civita symbol $\epsilon^{\mu\nu\rho\sigma}$ is defined by

$$\epsilon^{0123} = -1 \quad \text{i.e.,} \quad \epsilon_{0123} = +1 . \quad (0.8)$$

The γ Matrices

The γ matrices are defined as

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} , \quad (0.9)$$

$$\gamma^{0\dagger} = \gamma^0 , \quad \gamma^{k\dagger} = -\gamma^k \quad (k = 1, 2, 3) , \quad (0.10)$$

i.e.,

$$\gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0, \quad (0.11)$$

and

$$\gamma_\mu \equiv (\gamma_0, \gamma_k) = (\gamma^0, -\gamma^k). \quad (0.12)$$

The antisymmetric matrices $\sigma_{\mu\nu}$ are defined by

$$\sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu), \quad (0.13)$$

which satisfy $\sigma_{\mu\nu}^\dagger = \sigma_{\mu\nu}$.

The γ_5 matrix is defined by

$$\gamma_5 \equiv \gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3, \quad (0.14)$$

so that it satisfies the relations

$$\gamma_5^\dagger = \gamma_5 \quad \text{and} \quad \gamma_5^2 = +1 \quad (0.15)$$

In our definition of γ_5 , the helicity projection operators into the left-handed and right-handed states are given by $\frac{1}{2}(1 - \gamma_5)$ and $\frac{1}{2}(1 + \gamma_5)$, respectively:

$$\psi_L = \frac{1}{2}(1 - \gamma_5)\psi, \quad \psi_R = \frac{1}{2}(1 + \gamma_5)\psi, \quad (0.16)$$

where ψ is four-component spinor. Therefore, the so-called “ $V - A$ currents” are expressed by the form

$$J_\mu^{V-A}(x) = \bar{\psi}(x) \gamma_\mu (1 - \gamma_5) \psi(x), \quad (0.17)$$

where $\bar{\psi}$ is defined by

$$\bar{\psi} = \psi^\dagger \gamma_0. \quad (0.18)$$

Sometimes, we use the following expression for a Lorentz scalar product with γ_μ :

$$\not{A} \equiv A_\mu \gamma^\mu = A_0 \gamma_0 - \mathbf{A} \boldsymbol{\gamma}, \quad (0.19)$$

$$\not{\partial} \equiv \gamma^\mu \partial_\mu = \gamma_0 \partial_0 + \boldsymbol{\gamma} \boldsymbol{\nabla}. \quad (0.20)$$

Isospin Matrices

We will use τ^a matrices ($a = 1, 2, 3$) in the discussion of $SU(2)$ symmetry:

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (0.21)$$

whose form is the same as the Pauli spin matrices σ^a

The following relations hold for the τ matrices:

$$\tau^a \tau^b - \tau^b \tau^a = 2i\epsilon_{abc} \tau^c, \quad (0.22)$$

$$\tau^a \tau^b + \tau^b \tau^a = 2\delta_{ab}, \quad (0.23)$$

$$\text{Tr}[\tau^a \tau^b] = 2\delta_{ab}, \quad (0.24)$$

$$\text{Tr}[\tau^a \tau^b \tau^c] = 2i\epsilon_{abc}, \quad (0.25)$$

$$\text{Tr}[\tau^a \tau^b \tau^c \tau^d] = 2(\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} - \delta_{ac}\delta_{bd}), \quad (0.26)$$

where $\epsilon_{123} = +1$.

A scalar product $\tau \pi$ with isospin triplet fields π^a is expressed as

$$\begin{aligned} \tau \pi &\equiv \sum_{a=1}^3 \tau^a \pi^a = \sqrt{2}(\tau^+ \pi^+ + \tau^- \pi^-) + \tau^3 \pi^0 \\ &= \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}, \end{aligned} \quad (0.27)$$

where

$$\tau^\pm = \frac{1}{2}(\tau^1 \pm i\tau^2), \quad (0.28)$$

$$\pi^\pm = \frac{1}{\sqrt{2}}(\pi^1 \mp i\pi^2), \quad (0.29)$$

$$\pi^0 = \pi^3. \quad (0.30)$$

We will also denote 2×2 unit matrix as τ^0 ;

$$\tau^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (0.31)$$

Normalization

In the present book, the normalization of the state $|\mathbf{p}, \alpha\rangle$ is defined by

$$\langle \mathbf{p}, \alpha | \mathbf{p}', \beta \rangle = \delta_{\alpha\beta} \delta(\mathbf{p} - \mathbf{p}') \quad (0.32)$$

However, currently, various definitions of the normalization have been used. For convenience, we also list some examples of the general form when we define the normalization as

$$\langle \mathbf{p}, \alpha | \mathbf{p}', \beta \rangle = \delta_{\alpha\beta} N_\alpha \delta(\mathbf{p} - \mathbf{p}') . \quad (0.32')$$

The relation for creation and annihilation operators:

$$[a_\alpha(\mathbf{p}), a_\beta^\dagger(\mathbf{p}')] = \delta_{\alpha\beta} \delta(\mathbf{p} - \mathbf{p}') , \quad (0.33)$$

or

$$[a_\alpha(\mathbf{p}), a_\beta^\dagger(\mathbf{p}')] = \delta_{\alpha\beta} N_\alpha \delta(\mathbf{p} - \mathbf{p}') . \quad (0.33')$$

The completeness integrals:

$$1 = |0\rangle\langle 0| + \sum_\alpha \int d^3p |\mathbf{p}, \alpha\rangle\langle \mathbf{p}, \alpha| , \quad (0.34)$$

or

$$1 = |0\rangle\langle 0| + \sum_\alpha \int d^3p \frac{1}{N_\alpha} |\mathbf{p}, \alpha\rangle\langle \mathbf{p}, \alpha| \quad (0.34')$$

Spinless field $\phi(x)$:

$$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int d^3p \frac{1}{\sqrt{2E}} (e^{-ipx} a(\mathbf{p}) + e^{ipx} a^\dagger(\mathbf{p})) , \quad (0.35)$$

or

$$\phi(x) = \int d^3p \frac{1}{\sqrt{(2\pi)^3 2E N_\phi}} (e^{-ipx} a(\mathbf{p}) + e^{ipx} a^\dagger(\mathbf{p})) . \quad (0.35')$$

Matrix element of the bosonic vector currents:

$$\langle \pi(\mathbf{p}') | V_\mu(0) | K(\mathbf{p}) \rangle = \frac{1}{(2\pi)^3} \frac{1}{\sqrt{4E_\pi E_K}} [f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu] , \quad (0.36)$$

or

$$\langle \pi(\mathbf{p}') | V_\mu(0) | K(\mathbf{p}) \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{N_\pi}{2E_\pi}} \sqrt{\frac{N_K}{2E_K}} [f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu] , \quad (0.36')$$

where $q = p' - p$. Decay constant f_π :

$$\langle 0 | A_\mu^{\pi^+}(0) | \pi^+(\mathbf{p}) \rangle = \frac{1}{\sqrt{(2\pi)^3 2E}} i f_\pi p_\mu , \quad (0.37)$$

or

$$\langle 0 | A_\mu^{\pi^+}(0) | \pi^+(\mathbf{p}) \rangle = \sqrt{\frac{N_\pi}{(2\pi)^3 2E}} i f_\pi p_\mu . \quad (0.37')$$

For a Dirac field $\psi(x)$, we use the same normalization

$$\{a^{(r)}(\mathbf{p}), a^{(s)}(\mathbf{p}')\} = \delta_{rs} \delta(\mathbf{p} - \mathbf{p}') \quad (0.38)$$

as (0.32), where

$$\psi(x) = \frac{1}{(2\pi)^{3/2}} \int d^3p \frac{1}{\sqrt{2E}} \sum_{r=1,2} \left(a^{(r)}(\mathbf{p}) u^{(r)}(\mathbf{p}) e^{-ipx} + b^{(r)\dagger}(\mathbf{p}) v^{(r)}(\mathbf{p}) e^{ipx} \right) , \quad (0.39)$$

so that the normalizations for $u^{(r)}(\mathbf{p})$ and $v^{(r)}(\mathbf{p})$ must be

$$u^{(r)\dagger}(\mathbf{p}) u^{(s)}(\mathbf{p}) = v^{(r)\dagger}(\mathbf{p}) v^{(s)}(\mathbf{p}) = 2E \delta_{rs} , \quad (0.40)$$

or

$$\bar{u}^{(r)}(\mathbf{p}) u^{(s)}(\mathbf{p}) = -\bar{v}^{(r)}(\mathbf{p}) v^{(s)}(\mathbf{p}) = 2m \delta_{rs} , \quad (0.41)$$

The projection operators for positive and negative energy states are given by

$$\Lambda_+(\mathbf{p}) \equiv \sum_r u^{(r)}(\mathbf{p}) \bar{u}^{(r)}(\mathbf{p}) = \not{p} + m , \quad (0.42)$$

and

$$\Lambda_-(\mathbf{p}) \equiv - \sum_r v^{(r)}(\mathbf{p}) \bar{v}^{(r)}(\mathbf{p}) = -(\not{p} - m) , \quad (0.43)$$

respectively.

If we choose

$$\{a^{(r)}(\mathbf{p}), a^{(s)}(\mathbf{p}')\} = N_a \delta_{rs} \delta(\mathbf{p} - \mathbf{p}') , \quad (0.38')$$

and

$$\psi(x) = \frac{1}{(2\pi)^{3/2}} \int d^3p \frac{1}{N_\psi} \sum_{r=1,2} \left(a^{(r)}(\mathbf{p}) u^{(r)}(\mathbf{p}) e^{-ipx} + b^{(r)\dagger}(\mathbf{p}) v^{(r)}(\mathbf{p}) e^{ipx} \right) , \quad (0.39')$$

then the normalizations for $u^{(r)}(\mathbf{p})$ and $v^{(r)}(\mathbf{p})$ must be

$$u^{(r)\dagger}(\mathbf{p}) u^{(s)}(\mathbf{p}) = v^{(r)\dagger}(\mathbf{p}) v^{(s)}(\mathbf{p}) = \frac{|N_\psi|^2}{N_a} \delta_{rs} , \quad (0.40')$$

or

$$\bar{u}^{(r)}(\mathbf{p})u^{(s)}(\mathbf{p}) = -\bar{v}^{(r)}(\mathbf{p})v^{(s)}(\mathbf{p}) = \frac{|N_\psi|^2}{N_a} \frac{m}{E} \delta_{rs} . \quad (0.41')$$

The projection operators for positive and negative energy states are given by

$$\Lambda_+(\mathbf{p}) \equiv \sum_r u^{(r)}(\mathbf{p})\bar{u}^{(r)}(\mathbf{p}) = \frac{|N_\psi|^2}{N_a} \frac{\not{p} + m}{2E} , \quad (0.42')$$

and

$$\Lambda_-(\mathbf{p}) \equiv -\sum_r v^{(r)}(\mathbf{p})\bar{v}^{(r)}(\mathbf{p}) = -\frac{|N_\psi|^2}{N_a} \frac{\not{p} - m}{2E} , \quad (0.43')$$

respectively.

Although some of texts choose $N_\psi = \sqrt{E/m}$ or E/m , the choice is not useful for a treatment of massless Dirac particles. Therefore, in this book, we choose $N_\psi = \sqrt{2E}$ and $N_a = 1$ as well as those in boson fields.

Acronyms

Certain acronyms used frequently are as follows:

C.G. coefficient	Clebsch–Gordan coefficient
CKM mixing matrix	Cabibbo–Kobayashi–Maskawa mixing matrix
C.R.	Commutation relation
CVC	Conserved vector current
ETC	Equal-time commutator
ETCR	Equal-time commutation relation
G-M-O mass formula	Gell-Mann–Okubo mass formula
GMOR	Gell-Mann–Oakes–Renner
GWS model	Glashow–Weinberg–Salam model
GUT	Grand unification theory
l.h.s.	left-hand side
L.S.Z. reduction	Lehmann–Symanzik–Zimmermann reduction
KSFR relation	Kawarabayashi–Suzuki–Fayyazuddin–Riazuddin relation
PCAC	Partially conserved axial-vector current
PS-meson	Pseudoscalar meson
QCD	Quantum chromodynamics
QED	Quantum electrodynamics
r.h.s.	right-hand side
VEV	Vacuum expectation value

Experimental Data

All experimental values referred in this book except for those with specific reference numbers are quoted from Particle Data Group, *Review of Particle Properties*, Phys. Lett. **B239**, 1 (1990), which is referred in this book as Ref. [0].