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Magnetohydrodynamics of Plasma Relaxation

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MAGNETOHYDRODYNAMICS OF PLASMA RELAXATION

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Preface

This book describes what the authors have learned about the causes and consequences of plasma relaxation over the past twenty years. Much of this knowledge has come from the field of controlled fusion research, in particular the Reversed-field Pinch (RFP) concept. Thus, a certain bias toward plasma phenomena inherent in this experiment is inevitable. However, it is the authors' belief that these phenomena are sufficiently generic that the concepts presented herein will be useful to researchers in other fields. Failing that, the book is at least an account of a successful world-wide research program that has led to a fairly complete, self-consistent description of a particular type of hot, magnetized plasma.

Early laboratory experiments discovered that axisymmetric toroidal pinch plasma discharges exhibited favorable confinement properties when operated in one of two regimes: the tokamak, in which the toroidal component of the magnetic field was approximately uniform and much larger than the poloidal component; and, the RFP, in which the toroidal field reversed direction in the outer regions of the plasma, and was comparable to the poloidal field. Furthermore, it was found that these states could be formed spontaneously by controlling only a few global parameters, such as the total toroidal current and magnetic flux. This behavior was explained in a general theoretical way by J. B. Taylor in 1974. Taylor used a variational principle with constraints to show that these states were the preferred states of the magnetoplasma system. These states were called relaxed states, and the process of attaining them was called relaxation. Relaxation concepts have since been applied in a variety of situations by a number of researchers to explain the natural states of plasmas and fluids.

The variational theory has proven useful for predicting the time-asymptotic average steady state of many systems, but it does not directly address the fundamental physical processes that give rise to plasma relaxation. Thus, in the ensuing years the fundamental questions shifted from explaining the existence of preferred states to understanding *how* and *why* relaxation occurs. Answering these questions has required a synergism between detailed experimental measurements, large scale numerical computations, and theoretical plasma physics. The results of these efforts are the subject of this book.

Taylor's theory requires only that the dynamical processes responsible for plasma relaxation be describable by the equations of resistive magnetohydrodynamics (MHD). While the specific details of the dynamics do not enter the variational theory, Taylor envisioned plasma relaxation as

the result of MHD turbulence. These fluctuations are characterized by relatively high frequency and short spatial scales. Subsequent experiments showed that the ubiquitous characteristic fluctuations in RFP discharges have low frequency and broad spatial extent. These fluctuations were reproduced in large scale computer simulations of RFP discharges. These simulations also exhibited plasma relaxation. Theoretical analyses of these results have shown how these long wavelength MHD fluctuations arise, and how they can produce not only the preferred relaxed states predicted by the variational theory, but can also account for many of the operating characteristics of these experiments. Further calculations have suggested that the dynamics that underlie plasma relaxation may also play an important role in determining the energy confinement properties of RFP plasmas.

The presentation of these results is (we hope) logical rather than historical, and relatively self-contained. Chapter 1 is devoted to a brief overview of plasma relaxation, toroidal pinches in general, and the RFP in particular. Chapter 2 presents the fundamentals of the resistive MHD model as required for describing plasma relaxation, including discussions of ideal and resistive instabilities and nonlinear phenomena. Taylor's variational theory is presented in detail in Chapter 3. In Chapter 4 we present phenomenological models of plasma relaxation as deduced from experimental observations and linear stability theory. Typical experimental results are also presented. The results of detailed, large scale numerical simulations of the magnetic field dynamics in RFP plasmas are presented in Chapter 5. It is these calculations that self-consistently explain both the presence of the experimentally observed long-wavelength fluctuations and their role in producing plasma relaxation. A brief discussion of the so-called RFP dynamo is also given here. The remainder of the book is devoted to the consequences of plasma relaxation, and to relaxation in situations other than fusion experiments. In Chapter 6 we show that the same fluctuations that are responsible for the attainment of relaxed states can also self-consistently account for anomalous plasma resistance, and enhanced loop voltage when discharges are operated with resistive shells and limiters. Chapter 7 contains a discussion of the possible consequences that the inherent fluctuations may have for energy transport and confinement. Chapter 8 describes calculations of similar dynamical events that may occur in the solar corona, the magnetized outer atmosphere of the sun. While the boundary conditions make it difficult to make a direct connection between these phenomena and the variational relaxation theory, similarities between the dynamics of the coronal and RFP plasmas are noted and discussed. A brief summary is found in Chapter 9.

Many researchers have contributed to the picture of plasma relaxation that is presented in this book. To attempt to acknowledge them all would be folly, for some deserving soul would surely be left off any list. Nonetheless,

the authors would like to especially thank their longtime colleagues H. A. B. Bodin, E. J. Caramana, J. Killeen, and R. A. Nebel for providing both insights and a stimulating scientific environment. We wish to acknowledge the specific contributions of our close collaborators V. Antoni, D. C. Barnes, S. Cappello, G. G. Craddock, D. S. Harned, Y. L. Ho, P. Martin, D. Merlin, Z. Mikić, and R. Paccagnella. We would also like to thank S. Prager and C. Gimblett for carefully reading a draft version of the manuscript and providing valuable comments that greatly improved the clarity and accuracy of the presentation. We thank CNR of Italy and the United States Department of Energy for supporting this research over the years. We are indebted to Ms. Anna Miklović for the seemingly endless task of formatting and preparing the camera-ready manuscript. This book exist because of her persistent and professional efforts. One of us (DS) thanks Prof. C. K. Rowdyshrub for spiritual enlightenment. Finally, we emphasize that the interpretation and synthesis of the results presented here is strictly the viewpoint of the authors.

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