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A
CONCISE GUIDE
TO
SEMIGROUPS
AND
EVOLUTION
EQUATIONS

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τῇ Δημήτρει
ὕφ' ἧς καίπερ ἀποτρεπόμενος
οὐδὲν ἦττον
τοῦτο τὸ βιβλίον ἐπέρανα

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PREFACE

This book is a simple and concise introduction to the theory of semigroups and of evolution equations, both in the linear and in the semilinear case. The subject is presented by a discussion of two standard boundary value problems from particle transport theory and from population theory, and by showing how such problems can be rewritten as evolution problems in suitable Banach spaces.

Each section of the book is completed with some notes, where the relevant notions of functional analysis are explained. Some other definitions and theorems of functional analysis are discussed in the Appendices, (so that the only prerequisites to read the book are classical differential and integral calculus).

The book is based on lectures given by the author to graduate students of the Mathematical School of Bari, Firenze, and Napoli Universities, and on some seminars given at the Mathematical Institute of Oxford University. It is intended for undergraduates or graduate students of mathematics, physics, engineering, and for applied mathematicians, physicists, and engineers.

The author would like to thank the colleagues and friends Prof. G. Busoni of Florence University and Prof. A. Pagliarini of Bari University for many useful

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A. B.-M.

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