Stability Analysis in Terms of Two Measures

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STABILITY ANALYSIS IN TERMS OF TWO MEASURES

V. Lakshmikantham

*Department of Applied Mathematics
Florida Institute of Technology*

X. Z. Liu

*Department of Applied Mathematics
University of Waterloo*

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Preface

The problems of modern society are both complex and multidisciplinary. In spite of the apparent diversity of problems, tools developed in one context are often adaptable to an entirely different situation. As an example, consider the well-known Lyapunov's second method, which has gained increasing significance and has offered decisive impetus for modern development of stability theory of differential equations. A manifest advantage of this technique is that it does not require the knowledge of solutions and therefore has great power in applications to the real world problems from many disciplines.

It is now recognized that the concept of Lyapunov function can be employed to investigate various qualitative and quantitative properties of dynamic systems. Lyapunov

function serves as a vehicle to transform a given complicated differential system into a relatively simpler system and as a result, it is enough to study the properties of solutions of the simpler system. It is also being realized that the same versatile tools are adaptable to discuss entirely different nonlinear problems, and these effective and fruitful techniques offer an exciting prospect for further advancement.

The concepts of Lyapunov stability have given rise to many new notions that are important in applications. For example, eventual stability, partial stability, relative stability, conditional stability, total stability, and corresponding boundedness and practical stability notions to name a few. Relative to each concept, there exists a sufficient literature parallel to Lyapunov's theory of stability. It is natural to ask whether we can find a notion and develop the corresponding theory which unifies and includes a variety of known concepts of stability in a single set up. The answer is yes and it is the development of stability theory in terms of two measures. It is in this spirit we see the importance of our present monograph. Its aim is to present a systematic account of recent developments in the stability theory in terms of two distinct measures, describe the current state of the art, show the essential unity achieved by wealth of applications, and provide a unified general structure applicable to several nonlinear problems.

Some important features of the monograph are as follows:

- (i) It is the first book that is dedicated to a systematic development of stability criteria in terms of two measures.
- (ii) It demonstrates the manifestations of general Lyapunov method in terms of two measures by showing how this effective technique can be adapted to study various apparently diverse nonlinear problems.
- (iii) It shows how in the present framework of two measures one can unify criteria of Lyapunov stability, practical stability and boundedness properties, and provides interesting applications of real world problems.

In view of several excellent books on Lyapunov second method, we have not included any material of the usual Lyapunov stability criteria. Since the interplay of the two different measures offers new insights into the stability problem and provides opportunity for new applications, and further advancement, we do hope that this monograph will stimulate further investigation of this fruitful technique of employing two different measures.

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V. Lakshmikantham

Xinzhi Liu

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