PARALLEL SUBSTITUTION ALGORITHM

Theory and Application

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THEORY AND APPLICATION**

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Preface

This book sums up the results of some-year experience of authors' research¹ in the field of fine-grained parallelism and architectural design of cellular algorithm-oriented processing units. An original formal model called a Parallel Substitution Algorithm (PSA) has been proposed for representing computational processes in the cellular space. Founded on this model a theory of distributed computational processes and some methods of analysis and synthesis of the algorithm-oriented cellular architecture design have been developed. Having been aimed toward the VLSI implementation of array-like processing units, the scope of the methods has been called the Parallel Microprogramming [1 – 5].

Main ideas of the PSA model ascend to the concepts of Fon-Neumann's cellular Automaton [6] and Markov's Algorithm [7]. The first concept has served as a background for a number of modifications during more than thirty years. The earlier ones have been oriented to such theoretical problems as the possibility of reproduction [6, 8], completeness [9], complexity of typical algorithm realization [10, 11]. The later investigations are mainly directed to applications of the cellular automaton model to studying the dynamics of physical and biological systems [8, 12, 13, 14].

Computer technology development has brought about the consideration of a cellular automaton-like model as fine-grained parallel computation algorithm. A bright example of this is a concept of Bardzin's growing automaton [15], which combines the idea of cellular automaton with that of the Kolmogorov-Uspensky's algorithm [16]. The latter, although representing a sequential computation, brings up, nevertheless, very important new trends in the theory of algorithms by introducing the spatial relations between operators.

The above-mentioned formal models constitute the conceptual foundation for evolving the PSA concepts and constructions representing the cellular computations and architectures.

Later on, two tendencies stimulated the development of formalisms for fine-grained parallelism investigation: the great success of the VLSI technology and the advance in higher algebra computation methods, which are intended for data array processing. These tendencies converge displaying the urgent necessity of getting devices for processing data array in parallel. As a result several architectural conceptions have appeared to be oriented to the interpretation of certain classes of

¹ The research is supported by the Russian Fund for Fundamental Research (93-012-1000).

parallel algorithms, such as associative processors, cellular image processors, systolic matrices, etc. Yet no theoretical generalization or an advanced mathematical cellular computation model has been evolved till now except the PSA.

The following fundamental concepts form the background of the Parallel Substitution Algorithm.

Fine-grained parallelism. Each data-item is introduced being attached to a point in the computation space represented as a countable discrete set of names. The computation is an iterative procedure over a data array in this space. At each step certain data subarrays are replaced by other ones, these actions being done all over the whole data-array.

Decentralized control. No order of operation execution is defined in the model. Each substitution is performed when and where the ready conditions coincide with a data-pattern in the space. Thus, the associative mechanism of the time-spatial control of the computation process is performed, the spatial relations being explicitly represented by means of the functions defined in the computational space.

Synchronous mode of execution. An abstract outer clock is presumed to exist, making the computational process to obey the following rules:

- 1) all operations ready to be executed should be executed simultaneously with the first coming clock pulse,
- 2) no changes in the array are allowed out of the clock pulses. The asynchronous mode of execution is also of great theoretical interest. Its investigation is very important for understanding the behavioral properties of computations in the cellular spaces.

Interpretability by automata nets. A set of substitutions representing the cellular computation admits the direct mapping onto a net of automata. This allows one to construct methods and tools for architectural design of hardware implementation of cellular algorithms.

The direction of the development of the PSA theory is stipulated by the objective of its creation: to constitute the fundamentals for methods of synthesis of algorithm-oriented architectures of cellular processors. In accordance with this is the contents of the book. It consists of five chapters. The first two are fully theoretical. The third and the forth chapters are mostly methodological. The last chapter shows the experience in applications. The material is allocated among the chapters as follows:

In Chapter I, the main concepts and features of the model are presented. The most important classes of the PSAs are defined and studied. The PSA interpretation by automata nets is shown in a constructive form, which justifies the destination of the model.

In Chapter II, the behavioral properties of the computation process are studied at different modes of execution of a set of parallel substitutions. Particular emphasis has been placed on the asynchronous mode. Validity conditions for synchronous,

asynchronous and mixed modes are found out and methods for revealing determinacy and termination properties are elaborated.

In Chapter III, methods for PSAs composition are presented. The methods use parallel flowcharts and Petri Nets for representing the composition structure and synthesizing an algorithm for controlling the PSAs interactions. Much attention is being given to the generation of the termination signals in PSAs with local interactions between cells.

In Chapter IV, three types of equivalent transformations of a PSA are presented. The first one allows one to obtain a PSA processing a 3D data array which is result-equivalent to a given one primarily designed to process a 2D data array. Such a transformation is oriented towards the electro-optic implementation. The second type of transformation converts a PSS, intended for the synchronous execution, into a result-equivalent one, but acting asynchronously. This is done by constructing a computation process which simulates the original one being stretched out in time. The third transformation reduces the size of the data-array at the expense of the processing time. The idea is to fix a hyperplane in computation space turning out the remaining array into a shift register, which makes the data move towards the processing hyperplane where they are processed sequentially.

Chapter V is an illustrative one. It shows the authors' experience in PSA application. Firstly, a brief description of a computer simulating system is given, and then PSA-based methods are shown at work on examples of cellular arithmetic and logical processors. The emphasis is made on the 3D arrays and non-traditional algorithms such as Knuth's complex numbers multiplication, 3D pipelined neuro-processor, and obtaining an image skeleton.

The authors believe that the advance in VLSI technology and most intensive investigations in electro-optics would increase the interest in fine-grained parallelism. The most urgent need in design methods, and, hence, in a formal theory, will be pronounced when special-purpose massively parallel processors will find use in computer systems.

The authors hope that this book would attract attention of specialists in the field of fine-grained parallelism. Any critics and other remarks are welcomed and would be appreciated.

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