**CONSTANTIN CARATHÉODORY:
AN INTERNATIONAL TRIBUTE
VOL. I**

Constantin Carathéodory (1873–1950)

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T R I B U T E

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Editor

Themistocles M. Rassias

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FOREWORD

This collection of articles by mathematicians from many countries is a tribute to the memory of Constantin Carathéodory. Among the leaders who in the first half of this century created a foundation for the future development of mathematics Carathéodory was one of the most original, and also one of the first to give new life to parts of classical mathematics that threatened to become stagnant.

His understanding of Riemann's mapping theorem was far ahead of his contemporaries', and his feat of promoting an obscure remark of H. A. Schwarz to become the famous Schwarz' Lemma was as important as it was generous. In the same area, I can personally never forget how impressed I was, and still am, by his invention of prime ends.

Carathéodory's contributions to the calculus of variations were probably the most important part of his work, but I wonder if he was not equally proud of what he did to real variables and measure theory. This was miles away from complex variables, but very much in the same spirit.

The initiative for this tribute was taken by Dr. Themistocles M. Rassias, who has also been the editor and the link with the contributors and the Publisher. As a fellow countryman of Carathéodory, he was the right person for the task and deserves everybody's gratitude.

Lars V. Ahlfors
Harvard University

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PREFACE

Constantin Carathéodory (1873–1950) was born in Berlin on 13 September 1873 of Greek parents, and he died in Munich on 2 February 1950. His work covered several subjects of mathematics, including the calculus of variations, function theory, measure and integration, as well as applied mathematics. Carathéodory has also done very fundamental work in mechanics, thermodynamics, geometrical optics and relativity theory.

The first important contribution of Carathéodory to the calculus of variations was his proposal of a theory of discontinuous curves. This was his doctoral thesis “Über die diskontinuierlichen Lösungen in der Variation-srechnung” written while he was a student at the University of Göttingen and published in 1904. This monumental work was the starting point for several fundamental contributions to the calculus of variations by himself and other mathematicians, as well as the inspiration for certain early results in optimal control. In function theory, one of Carathéodory’s significant achievements was a simplification of the proof of one of the most essential theorems of conformal representation. He succeeded in extending earlier results of Picard and Schwarz. In 1912, Carathéodory proved his celebrated kernel theorem on sequences of univalent functions. Later, in 1923 Charles Loewner introduced a new approach in function theory representing slit mappings in terms of a differential equation. Loewner, using the Carathéodory convergence theorem, proved the Bieberbach conjecture for the third coefficient. L. de Branges recently remarked that it was precisely this method which finally gave a full solution to the Bieberbach conjecture. In measure theory, as is well known, Carathéodory is the inventor of outer measures, and all that this implies in mathematics. In mechanics, thermodynamics, optics and relativity, besides his important scientific contributions, Carathéodory has significantly influenced the better understanding and the rigorous presentation of these fields.

An example of Carathéodory’s wide-ranging influence in the interna-

tional mathematical community was seen during the first Fields Medals awards at the International Congress of Mathematicians, Oslo, 1936. The selection committee consisted of G. D. Birkhoff, Elie Cartan, C. Carathéodory, F. Severi, and T. Takagi. Two medals were awarded, one to L. V. Ahlfors and one to Jesse Douglas. It was C. Carathéodory who presented both their works during the opening of the International Congress.

These two volumes contain a series of scientific articles dedicated to the memory of Constantin Carathéodory. These articles deepen our understanding of some of the current research problems and theories in modern topics of calculus of variations, complex analysis, real analysis, differential equations, geometry and their applications, which are related to the work of Carathéodory. This presentation of concepts and methods makes this tribute an invaluable reference for teachers and other professionals in mathematics who are interested in pure and applied research, philosophy of mathematics, and mathematics education.

It is my pleasure to express my warmest thanks to all of the scientists who contributed to these two volumes, and, I would particularly like to extend my special appreciation to Professor L. V. Ahlfors for writing the Foreword to these volumes. I would also like to acknowledge the superb assistance in editing and composition that the staff of World Scientific Publishing Co. has provided in the preparation of this publication.

Athens, Greece
November, 1990

Themistocles M. Rassias

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