NONLINEAR RANDOM WAVES

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To Gleb, José Luis, Luciana and Svetlana

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“Consider the work of God: for who can make that straight, which he hath made crooked?”

Ecclesiastes 7, v. 13

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ABBREVIATIONS USED IN THIS BOOK:

AKL	Ablowitz-Kruskal-Ladik
AL	Ablowitz-Ladik
FN	Furutsu-Novikov
FP	Fokker-Planck
GLM	Gelfand-Levitan-Marchenko
IST	Inverse Scattering Technique
KdV	Korteweg-de Vries
MFM	Mean Field Method
NKG	Nonlinear Klein-Gordon
NLS	Nonlinear Schrödinger
SG	Sine-Gordon
SV	Strauss-Vázquez
ZS	Zakharov-Shabat