FUNDAMENTALS OF INTERFEROMETRIC GRAVITATIONAL WAVE DETECTORS

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Preface

This book is an attempt to communicate the basic logic of interferometric gravitational wave detectors to people who are new to the field. With the recent start of construction of the Laser Interferometer Gravitational Wave Observatory in the United States, and similar ventures elsewhere, it seems likely that many people will want to know more about the subject, including both young people at the beginnings of their scientific careers and experienced scientists with a new interest in an expanding field. While I in no way expect that this book will displace the more specialized literature, I hope it will serve as an overview of a subject of tremendous intellectual excitement and technical sweetness. I assume the reader has a basic knowledge of physics, but no special familiarity with either general relativity or the special techniques of experimental physics.

... a grateful mind
By owing owes not, but still pays, at once
Indebted and discharged; what burden then?

Paradise Lost, IV:55-7

The inspiration for this book came from Abhay Ashtekar. As he promised, the Physics Department at Syracuse University has proven to be a wonderful home at which to teach, to learn, and to write. The outline of this book was worked out during a seminar I conducted there with my research group. Large parts of the manuscript were read carefully by Stan Whitcomb and by Gabriela Gonzalez; crucial parts were also read by John Price, David Shoemaker, and Robert Spero. Yinglei Huang, Alex Evako, Joe Adams, and especially Gabriela Gonzalez were of tremendous help in improving the presentation, tracking down important facts and ferreting out mistakes. Bess Collins taught me how to look at the figures from the point of view of a graphic designer. Everyone named above made valuable suggestions for which I am grateful. I am of course responsible for all of the mistakes that remain.

I was privileged to be able to present the contents of Chapters 2 and 5 of the present volume as lectures in the School of Cosmology and Gravitation at Erice in May, 1993. My thanks go to Joseph Weber for the invitation to lecture there, and to the School's organizer Venzo de Sabbata and the staff at Erice for creating such a pleasant ambiance for the study of physics.

It is with profound gratitude that I thank here those who have helped to shape me as a physicist. I was inspired to study physics by Edward Purcell, and guided along the way by Paul Bamberg, David Layzer, and Herbert Gursky. My graduate apprenticeship was in the hands of David Wilkinson, a model of what a scientist should be. In the Gravity Group at Princeton, the inspiring example of Robert Dicke was never far away. I learned the trade of gravitational wave detection from Rainer Weiss; Rai not only initiated me into the mysteries of the field, but also set standards of intellectual rigor and personal responsibility that are a continuing model.

I have also learned a great deal from the graduate students alongside of whom I have been privileged to work; at MIT, this included Dan Dewey, Jeff Livas, David Shoemaker, Michelle Stephens, Joe Kovalik, Nelson Christensen, Peter Fritschel, and Joe Giaime. Throughout my career, it has been my privilege to be surrounded by colleagues and students who were a constant source of insight and inspiration; the talent and openness of the members of the gravitational physics community are exemplary. Thanks to you all.

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For support in all possible forms over many years, I thank my parents, Dr. Stanley H. Saulson and Helen M. Saulson. I owe my deepest thanks to my wife, Sarah F. Saulson, for her love, encouragement, and fine editorial ear.